Parabolas and Optics

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NCTM's Principles and Standard for School Mathematics (2000) state that rich problem contexts involve interdisciplinary connections, for example, in science, art, and the social sciences. The study of the parabola provides an opportunity to find such connections between the mathematics studied in the classroom and its application to the real world and in the daily experience of students and professionals in many diverse fields.

Students typically begin their study of the parabola as the graph of a quadratic function. This investigation begins with the graph of the function $f(x) = x^2$ and continues with various transformations of that function. Later on, as students progress into the study of analytic geometry, they study the parabola as a conic section, the intersection of a cone and a plane, where the plane is parallel to a plane which is tangential to its surface.

For a construction of the parabola along with an explanation of its basic reflective properties, click **here**. The reflective properties of parabolas are also indicative of their refractive properties. Rays that are parallel to the paraboloid's axis are refracted either toward or away from its focus.

Here, we delve into the application of the parabola to the field of optics, particularly thin lenses. We will look at the mathematics used in this area as well as the various optical instruments used today that take advantage of the properties of the parabola.

A *converging (convex) lens* is thicker in the center than it is at the edge, and refracts a beam of parallel light to converge at a real focus point.

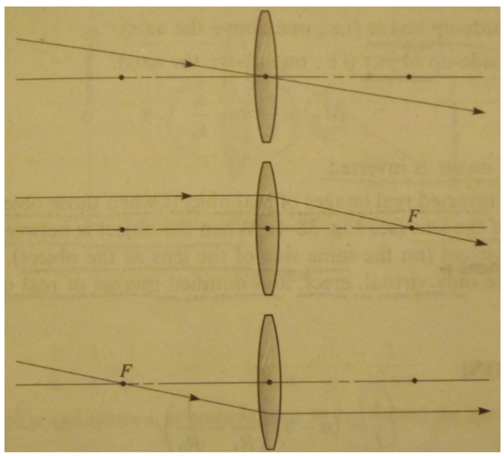
A *diverging (concave) lens* is thinner in the center than it is at the edge, and refracts a beam of parallel light away from a virtual focus point.

The focal point (F) is the point where rays parallel to the central axis are brought to (or away from) a focus. Again, this point is real for a converging lens and virtual for a diverging lens. The focal length is the distance of the focal point from the lens.

Since each lens can be reversed without changing the rays, each lens actually has two symmetric focal points. When a ray passes through a lens it refracts once when entering the lens and again when leaving the lens. But since both refractions result in symmetric focal points, all of the bending is assumed to occur along a vertical plane running down the middle of the lens.

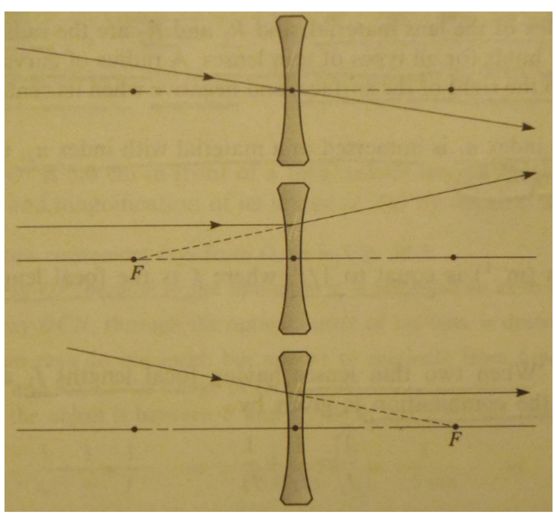
Any two rays coming from a point on the object and passing through the lens can be used to locate the image of that point. There are three particular rays that are convenient to use because we know how they will pass through the lens without having to make any calculations.

The following are the three rays passing through a converging lens:



(Bueche & Hecht, 2006)

And here are the same three rays passing through a diverging lens:

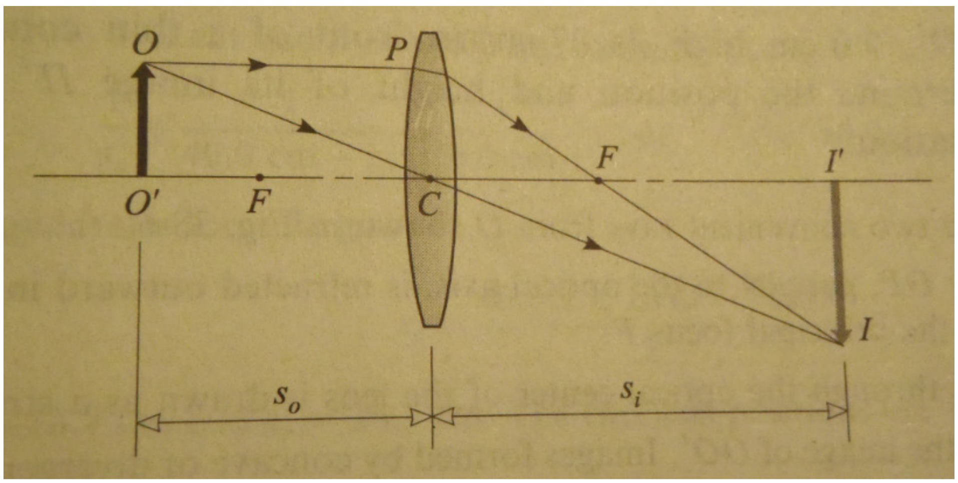


(Bueche & Hecht, 2006)

For both converging and diverging lenses, the Lens Power, measured in diopters (m^{-1}) is equal to $\frac{1}{f}$ where f is the lens' focal length. Also we have the following relationship:

$$\frac{1}{f} = \frac{1}{So} + \frac{1}{Si}$$

where s_o is the distance from the lens to the object and s_i is the distance from the lens to the image.



(Bueche & Hecht, 2006)

Using algebra to solve for f yields:

$$f = \frac{So \cdot Si}{So + Si}$$

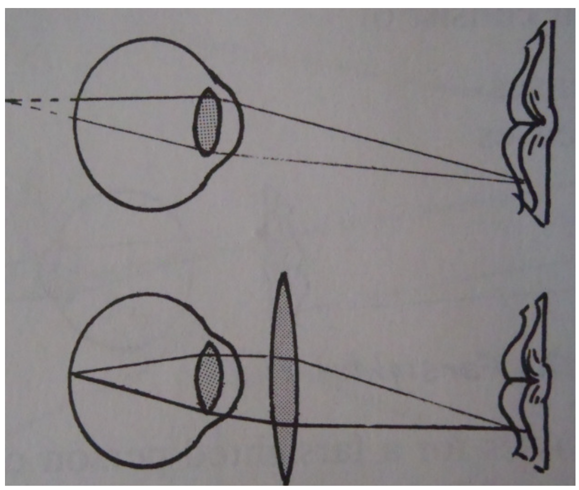
which is equal to one half the harmonic mean of the object distance and the image distance.

Lens Applications

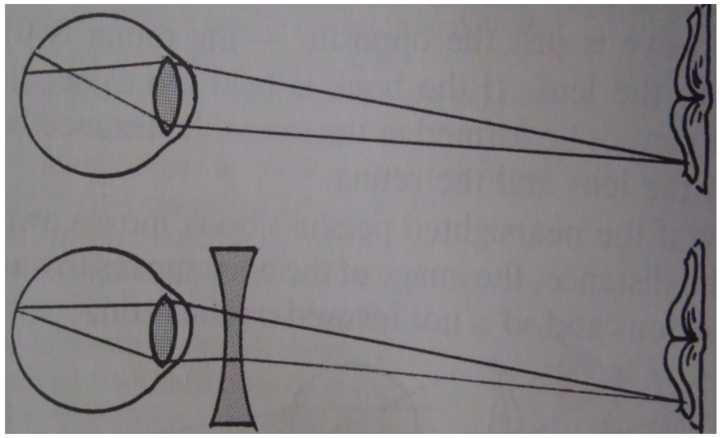
The following are examples of applications of optics that take advantage of the properties of a parabola.

Human Eye and Corrective Lenses

The human eye uses the cornea and a variable-focus lens to refract incoming light and project an image onto the retina. The closest distance to the eye from which an object can be viewed clearly is normally about 25cm. Farsighted persons can clearly see only objects that are far from the eye while nearsighted persons can only clearly see objects that are close to the eye. Corrective lenses, eyeglasses or contact lenses, can be used to correct vision problems when the focal point of the cornea and lens is either in front of or behind the retina.



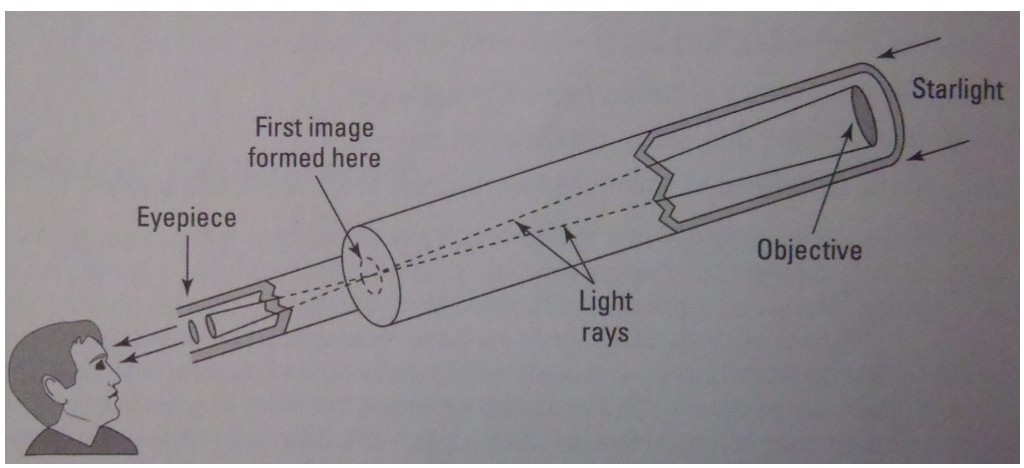
For a farsighted person, a convex (positive) lens is used to correct the eye's refraction of light that would normally converge behind the retina. (Epstein, 1983)



For a nearsighted person, a concave (negative) lens is
used to correct the eye's refraction of light that
would normally converge in front of the retina. (Epstein, 1983)

<u>Telescopes</u>

Telescopes take light from a distant source and focus it in a way that allows the eye to see an image that is bigger and brighter than what it could normally see. Since their creation in 1608, most telescopes are designed with the same basic elements, gathering light through an objective lens (or mirror) and viewing the image through the eyepiece which is a lens that refracts the image into a form that your eye can clearly see.



(Duree, 2011)

Within recent years, various telescopes have explored the depths of space using light and other forms of electromagnetic radiation:

Telescope	**Type of EM Radiation**
Hubble Space Telescope	Ultraviolet, Visible Light
Galaxy Evolution Explorer	Ultraviolet
Compton Gamma-Ray Observatory	Gamma Ray
Fermi Gamma-Ray Space Telescope	Gamma Ray
Chandra X-ray Observatory	X-ray
Spitzer Space Telescope:	Infrared
Herschel Space Observatory	Infrared
Planck Telescope	Microwave

The parabola has many varied applications in the real world. These examples show how the study of mathematical concepts do not just result in the acquisition of seemingly inconsequential abstract information, but in a better understanding of the world around us. The presentation of applications of mathematical concepts can aid in increasing student interest and engagement in mathematical study to create a more mathematically proficient student body.

References:

Bueche, F. J., & Hecht, E. (2006). *College physics*. (10th ed., pp. 365,367). New York, NY: McGraw-Hill.

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Epstein, L. (1983). *Thinking physics*. (2nd ed., p. 350). San Francisco, CA: Insight Press.