

Benjamin Roper

EMAT 6450 Final Project

Introduction

Each section of a roller can be written mathematically as some type of function, conic section, or curve in three-dimensional space. A graph that plots height against length can be created to understand how these parts connect to one another in the whole ride. Since very few high school students learn how to graph curves in the x-y-z plane, this project will only feature two-dimensional roller coasters; thus, there will not be any type of turns in them.

Additionally, roller coasters must be designed such that they obey certain laws of physics. While there are many different aspects of roller coaster physics that can be explored, such as centripetal acceleration and g-forces, this project focuses on the conservation of energy throughout the ride. When a roller coaster is lifted up a track by a mechanical lift, it builds potential energy. Once the car reaches the top of the first hill, it has created the total energy that will be used for the ride. During the ride, potential energy is converted into kinetic energy and vice versa; however, the sum of these two values never changes. Since this project will be assigned to students in a math class who may or may not have learned physics, I place the emphasis in the evaluation on the functions and their graphs. I think that enough information is provided in the instructions so that a student with no physics background can complete the task by analyzing the formulas and applying algebraic skills.

Implementation

This assignment would best be used at the end of the semester (or a unit) in an Analytic Geometry class or an equivalent one where students have learned the necessary concepts to complete it. Due to the number of parts involved in this project, it would be appropriate for students to work in pairs or groups of three to complete it. If time allows, students can be given class to work on the assignment, so they can plan with their partners and use available materials.

Presentation

Each group will graph their roller coaster on one piece of poster paper with clearly labeled axes and a title. Each section of the ride should be clearly labeled in some way to correspond to the appropriate functions and graph restrictions, which will also be on the poster paper. Additionally, the students in a group will turn in their own work on a

separate sheet of notebook paper that contains the algebra and calculations from the continuity and physics portions of the assignment.

Materials Needed

Markers and a piece of poster paper will be provided by the teacher for each group. Students will use their own notebook paper and writing instruments to show their work.

Standards Addressed

- MCC9-12.G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- MCC9-12.F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.
- MCC9-12.F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- MCC9-12.F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- MCC9-12.F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Objectives

- Students will create a graph that consists of various functions and conic sections and provide equations for the corresponding sections of the graph.
- Students will use algebra to show that the graph is continuous as it changes from one function to the next.
- Students will analyze their thrill rides using rudimentary physics to determine if they would be operable.

Prior Knowledge Required

- Graph various functions and conic sections and write their corresponding equations.
- Understand the definition of a continuous graph and how to show it algebraically.

- Solve for missing variables using algebra in simple equations.

Assessment/Evaluation

	1	2	3	4	Points Awarded
Graphs and Presentation	None or only one of the required sections is graphed.	The graph only contains two of the required sections, and the graph is not labeled correctly.	Three of the four required sections are graphed, but the graph may be missing some labels.	All four of the required sections are graphed, and the graph is clearly labeled.	/4 points
Equations	None or only one of the equations provided matches the graphs on the students' poster.	Two of the four required equations correctly correspond to their graphs and/or most of the limits are incorrect.	Three of the four required equations correctly correspond to their graphs and/or a few of the limits are incorrect.	All four of the required equations correctly correspond to their graphs.	/8 points
Continuity	The graph looks continuous but is not shown algebraically.	Continuity is proven for one of the transitions in the graph. There are mistakes or no work is shown in the others.	Continuity is proven for two of the transitions in the graph. There are mistakes or no work is shown in the other.	Continuity is proven for all of the transitions in the graph.	/4 points

Physics	The roller coaster shows no evidence of functional physics.	Energy is conserved at only one or two points, and there are several calculation errors.	Energy is conserved through most of the ride, but there are a few calculation errors at points.	The students' work shows that energy is conserved at each required point.	/4 points
Total					/20 points

References

Georgia Department of Education. (2012). Common Core Georgia Performance Standards (CCGPS) Mathematics 9-12. Retrieved from <https://www.georgiastandards.org/CommonCore/Documents/CCGPS-Gr9-12-MathStandards.pdf>.

Teachers' Domain. (2007, April 19). *Energy in a roller coaster ride*. Retrieved from <http://www.teachersdomain.org/resource/hew06.sci.phys.maf.rollercoaster/>.

The Physics Classroom. (2013). *Energy transformation on a roller coaster*. Retrieved from <http://www.physicsclassroom.com/mmedia/energy/ce.cfm>.

Roller Coaster Project

You will create a piecewise function that will graph a two-dimensional roller coaster. Your roller coaster must have AT LEAST one of each of the following:

- A linear launching ramp
- A circular loop
- A parabolic hill
- A higher-order polynomial or exponential function of your choice

You must draw your graphs on a piece of poster board that you will present to the class. You are free to be as creative as you like in the presentation, so long as your pair meets all the requirements for the project. However, your creativity will not affect your grade unless the project is illegible. When graphing, let the x-axis represent the length of your track and the y-axis represent the height of your roller coaster. Assume that each unit on the coordinate plane is ten feet. Each section of the graph should be clearly labeled with the corresponding equation.

In addition to your graph, you must also explore the realism of your ride. On a separate piece of paper, you must use algebra to show through algebra that your piecewise function is continuous as it transitions from one function to the next (except for the circular loop). Additionally, you must use basic physics to show that your roller coaster is operable. Once a roller coaster reaches the top of the launching ramp, it has accumulated its total energy for the ride in potential energy. After this point, the constant value of total energy is calculated by the sum of the potential and kinetic energy (assuming there is no friction). You must calculate the potential and kinetic energies, as well as the velocity when kinetic energy exists, at the top and bottom of each hill or loop. Then show that your total energy is constant throughout the ride. You will need to use the following formulas, where h is the height and v is the velocity at a given point:

$$KE = \frac{1}{2} * m * v^2$$

$$PE = m * g * h$$

Assume that your roller coaster car weighs 1200 lbs (m) and the acceleration due to gravity (g) is 32.2 ft/s^2 .