Günhan ÇAĞLAYAN, EMAT 6690

ESSAY THREE - PROBABILITY EXPERIMENTS USING GSP AND SPREADSHEET

Probability of an event is a number between zero and one. This is the starting point of this essay. Every event can be associated with a real number over the interval [0,1]. I planned not to mention much about the theoretical framework. Instead, it will be more meaningful to actually perform probability experiments and compare these observed values with the theoretical ones.

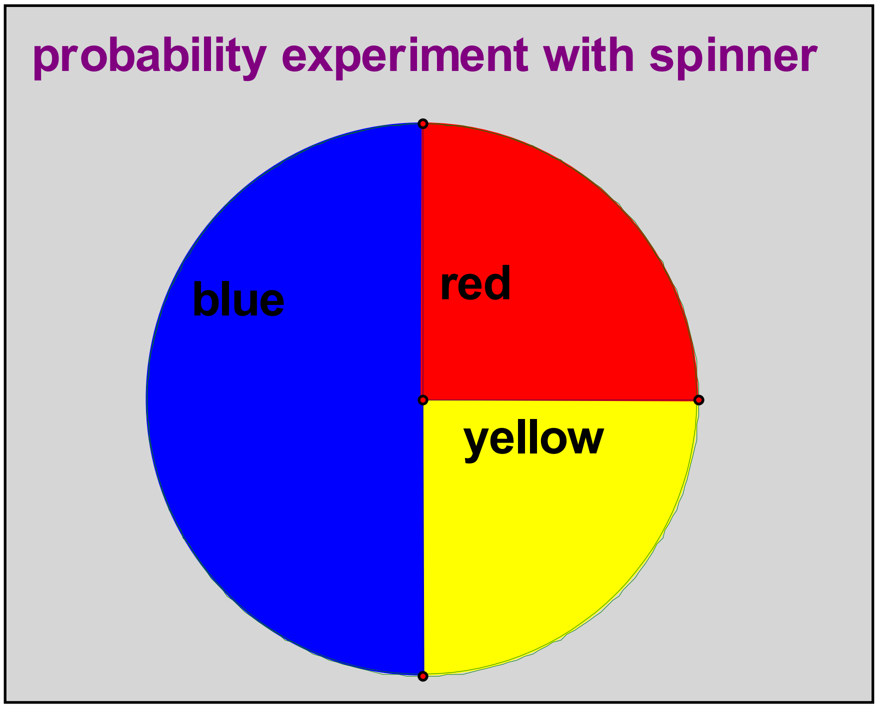
PART ONE: RANDOM NUMBER GENERATION IN SPREADSHEET

Using **rand()** function in spreadsheet with no argument, we can generate as many random numbers as we want. This function generates uniform random numbers, that is, uniformly distributed numbers from the interval [0,1]. For example, suppose that I want randomly zeros and ones. First, I generate, say, 500 random numbers using rand( ) function in column B. Next, we can double those random numbers, and then take their whole number part using the **int()** function: Everything below 0.5 would become 0, and everything above 0.5 would become 1. In this way, we get randomly zeros and ones. This could be used with coin tossing experiements since the outcome is either head (1) or tail (0).

As another example, suppose that I want randomly ones, twos, threes, fours, fives, and sixes. First, I generate, say, 500 random numbers using **rand()** function in column B. Next, we can multiply those random numbers by 6, and then take their whole number part using the int ( ) function: We also need to add 1 in this case. Here is what we write: **=int(6*rand())+1**. In this way, we get randomly ones, twos, threes, fours, fives, and sixes. This could be used with rolling one die experiements since the outcome is 1, 2, 3, 4, 5, or 6. It remains to count these: If we want to count the number of ones, then we use the **countif** function to count “1” s. Therefore, we write: **= countif (F2 : F501 , “1 ”)**. Similarly, you can count the other numbers.

Günhan ÇAĞLAYAN, EMAT 6690

One can also simulate spinner experiments in spreadsheet. Now we generate angles as for the numbers. Consider this spinner with three sectors:



We write: **= int (360 * rand ()) + 1**. When it comes to counting, we separately count the number of hits for each region. To count the number of times we hit the red, in cell J2, we write: **=COUNTIF(H2:H501;"<90")**. For the yellow, we write in cell J4: **=COUNTIF(H2:H501;">=270")**. And finally, for the blue sector, in cell J3, we write: **=COUNTIF(H2:H501;">90") - J4**. (Observe that we don't want to count yellow twice).

Günhan ÇAĞLAYAN, EMAT 6690

Here is what we did so far:

RAND()	10	int		dice	coin	angles	spinner	# spins =
0.13	7.17	4		4	0	145	0<red<90	112
0.25	0.4	3		1	0	109	90<=blue<270	250
1	7.69	3		1	0	181	270<=yellow<=360	138
0.14	3.77	6		1	1	219		
0.54	4.12	6		1	1	79		
0.06	0.14	7		6	1	290	One dice	# rolls =
0.51	2.08	8		3	1	305	Ones	74
0.67	8.1	1		5	1	224	Twos	87
0.76	0.87	2		3	0	4	Threes	77
0.29	7.4	4		5	0	180	Fours	95
0.28	5.46	3		1	1	178	Fives	94
0.28	1.78	9		2	0	57	Sixes	73
0.44	4.25	3		6	0	116		
0.61	4.28	2		3	0	301		
0.12	7.61	5		5	0	31	One coin	# tosses=
0.87	2.55	2		3	1	146	Head	237
0.26	6.7	4		5	0	332	Tail	263
0.1	6.13	1		1	1	43		
0.44	7.34	5		6	1	67		
0.73	3.06	9		3	1	62		

Go to the site to download the spreadsheet file to play with these experiments.

PART TWO: PROBABILITY EXPERIMENTS and the LAW OF LARGE NUMBERS

It looks like the more you do the experiment, the more you get close to the theoretical probabilities.

Let A be the event that the outcome is a head when one coin is tossed. Theoretical probability is P(A)=0.5. Here is what I got with 8 repeated experiments of tossing one coin 500 times:

0.490   0.446   0.486   0.482   0.504   0.512   0.536   0.504   —> AVERAGE = 0.495

In this way, we recorded an average for 4000 experiments. The experimental value 0.495 is very close to the theoretical value 0.500.

Similarly, the more you repeat the experiment and record your data, the more the number of counts converges to the expectation value.

For example, let B be the event that the outcome is a 5 when you roll one die. Theoretical probability is P(B)=1/6. Also let X denote the number of times we get a 5. Here is what I got with 8 repeated experiments of rolling one die 500 times:

96 70 83 93 89 78 98 69 —> TOTAL = 676

In this way, we recorded a value for X for 4000 experiments. The experimental value 676 is now very close to the expectation value = 8000/6~667.

There are other things one can do...

Let C be the event that the outcome is a prime number when one die is rolled. Theoretical probability is P(A)=0.5.

Here is what we got with 8 repeated experiments of rolling one die 500 times

0.487 0.502 0.507 0.473 0.504 0.502 0.529 0.474 —> AVERAGE = 0.497

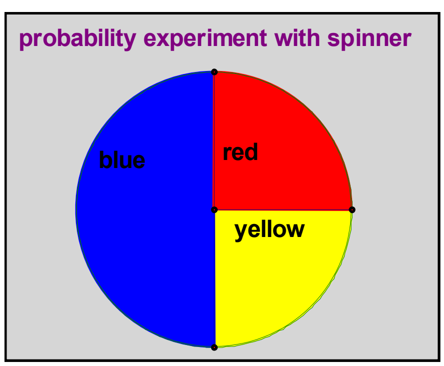
In this way, we recorded an average for 4000 experiments! Once again the observed value 0.497 is very close to the theoretical value 0.500.

I did this another time and recorded the number of counts:

256 245 227 240 231 260 270 247 —> TOTAL = 1976

In this way, we recorded a value for X for 4000 experiments. The experimental value 1976 is once again very close to the theoretical expectation value = 4000/2=2000.

Let's also define another event D related with the spinner experiment: D is the event that we hit the yellow sector.



Theoretical probability is P(D)=0.25. Let X denote the number of times we hit the

yellow sector. Now the expectation value of X must be the total number of spins divided by four. Let's compare this with the experimental values that I got:
117 112 131 123 121 123 126 128 —> TOTAL = 981 times out of 4000 spins. The expectation value is 1000.
Experimental probabilities recorded as: 0.2340 0.2240 0.2620 0.2460 0.2420 0.2460 0.2520 0.2560 —> AVERAGE = 0.2453
Once again the observed value 0.2453 is very close to the theoretical value 0.25.

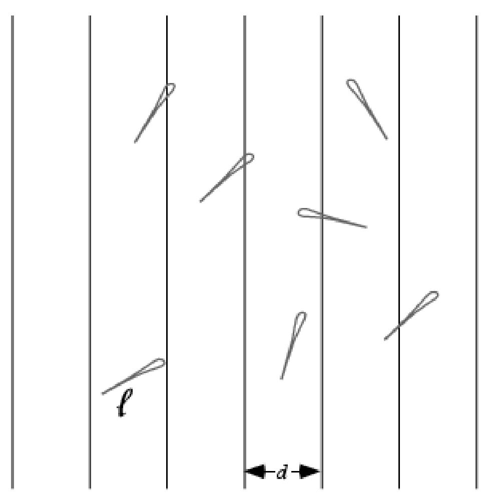
Go to the course website and click the link that will direct you to a site where you can play with spinner.Or you can copy the site address below:
http://www.shodor.org/interactivate/activities/spinner3/index.html

---

PART THREE: BUFFON'S NEEDLE PROBLEM IN GSP

Someone has already done this simulation with GSP. Go to course website and find the link that will direct you to the GSP file.

Problem (from MathWorld): Find the probability that a needle of length L will land on a line, given a floor with equally spaced parallel lines a distance D apart. The problem was first posed by the French naturalist Buffon in 1733.



When $L \le D$, the answer is $P = \frac{2L}{\pi D}$

When $L > D$, the probability is more complicated:

$$P = \frac{1}{\pi D}\{D[\pi - 2\arcsin\frac{D}{L}] + 2L(1 - \sqrt{1 - (\frac{D}{L})^2})\}$$

When L=D, the probability is $P = \frac{2}{\pi} = 0.6366$

The derivations can be found at MathWorld:
http://mathworld.wolfram.com/BuffonsNeedleProblem.html

Obviously, the needle size matters. Once again, similar to what we did before, let's perform the experiment, and compare with the theoretical values given above.

To perform this experiment, you must download the GSP file from Paul Kunkel's website http://whistleralley.com/buffon/buffon.htm

CASE ONE: L<D

With L=0.72cm and D=1.34cm, 1000 needles dropped and 343 of them intersected the lines. This means that the experimental probability is 343/1000=0.343. In fact, the theoretical probability is $P = \frac{2L}{\pi D} = \frac{2 \times 0.72}{\pi \times 1.34} = 0.34$. Very close...

LIMITING CASE: L=D

With L=1.00cm and D=1.00cm, 1000 needles dropped and 652 of them intersected the lines. This means that the experimental probability is 652/1000=0.652. In fact, the theoretical probability is $P = \frac{2}{\pi} = 0.6366$. Very close again...

CASE TWO: L>D

With L=2.25cm and D=1.00cm, 1000 needles dropped and 849 of them intersected the lines. This means that the experimental probability is 849/1000=0.849. In fact, the theoretical probability is $P = \frac{1}{\pi D}\{D[\pi - 2\arcsin\frac{D}{L}] + 2L(1 - \sqrt{1 - (\frac{D}{L})^2})\} = 0.86$. Once again, very close...