Trigonometric Unit Lesson 2

The Law of Cosines

Lesson: Deriving the Law of Cosines

Grade Level: Mathematics 4A

Overview

Students will explore triangles and right-triangle trigonometry to derive the Law of Cosines. They will use Geometer Sketchpad (GSP) in their investigations. Instruction and assessment includes the appropriate use of technology. Topics are represented in multiple ways. Concepts are introduced and used in the context of realistic phenomena.

Prerequisite: Successful completion of Mathematics 3 or Accelerated Mathematics 2

Georgia Performance Standards

Mathematics 4

GPSMM4A6: Students will solve trigonometric equations both graphically and algebraically.

a. Solve trigonometric equations over a variety of domains, using technology as appropriate.

c. Apply the law of sines and the law of cosines

NCTM Standard Grade 9-12: Use trigonometric relationships to determine lengths and angle measures.

NCTM Standard Grade 9-12: Establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others.

Learning outcomes

After completing this lesson students will be able to:

Explain the Law of Cosines.

Use the Law of Cosines in solving measurement problems.

Procedure

For this lesson, students will have access to computing technology using the GSP program and partially pre-constructed GSP sketches. Topics will be discussed in whole class and collaborative groups. The teacher will guide the outcomes of the lessons by interacting with groups (including the whole class) and individuals to assist and to assess progress toward desired learning outcomes.

This lesson is an investigation of triangle ABC to derive the Law of Cosines. First use GSP to construct the triangle in figure 1. Next, draw a line segment from B perpendicular to side AC as in figure 2. Line segment h divides side b into two parts, label the parts x and (b – x).

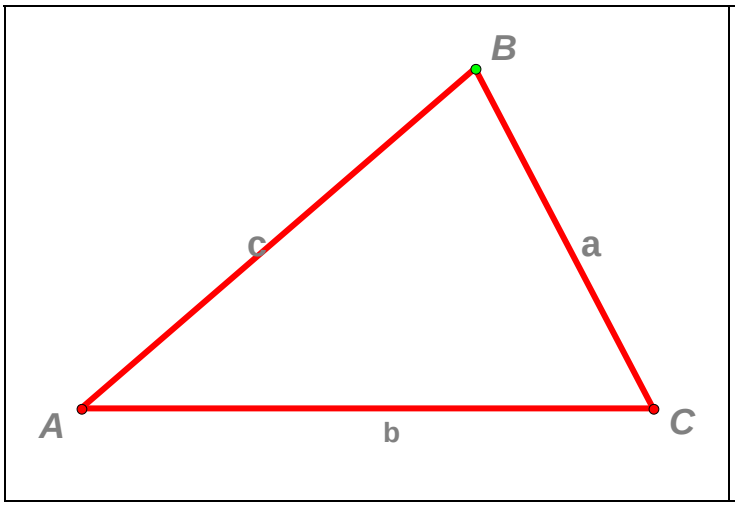


Figure 1

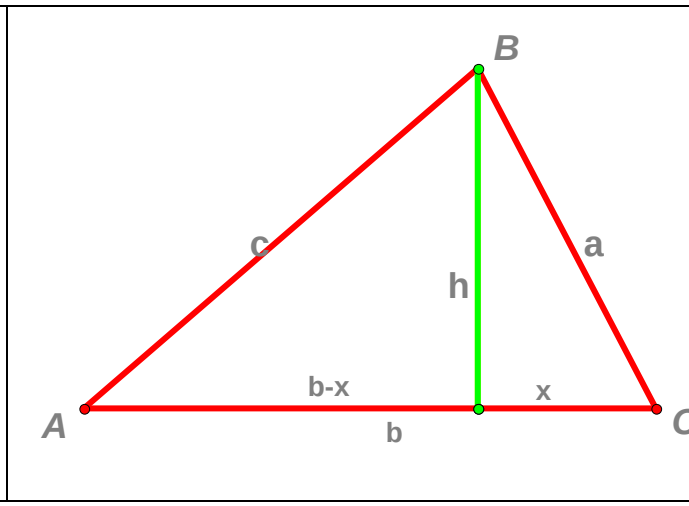


Figure 2

Using the Pythagorean Theorem we have

$c^2 = (b-x)^2 + h^2$, and $a^2 = x^2 + h^2$

so that $c^2 - (b-x)^2 = h^2$ and $a^2 – x^2 = h^2$

By the transitive property $c^2 - (b-x)^2 = a^2 – x^2$

Expanding $(b-x)^2$

$c^2 – b^2 + 2bx – x^2 = a^2 – x^2$

$c^2 = a^2 + b^2 – 2bx$

Now eliminate x from the equation by substituting a value of x which involves the cosine of C.

CosC = x/a-----> x = aCosC

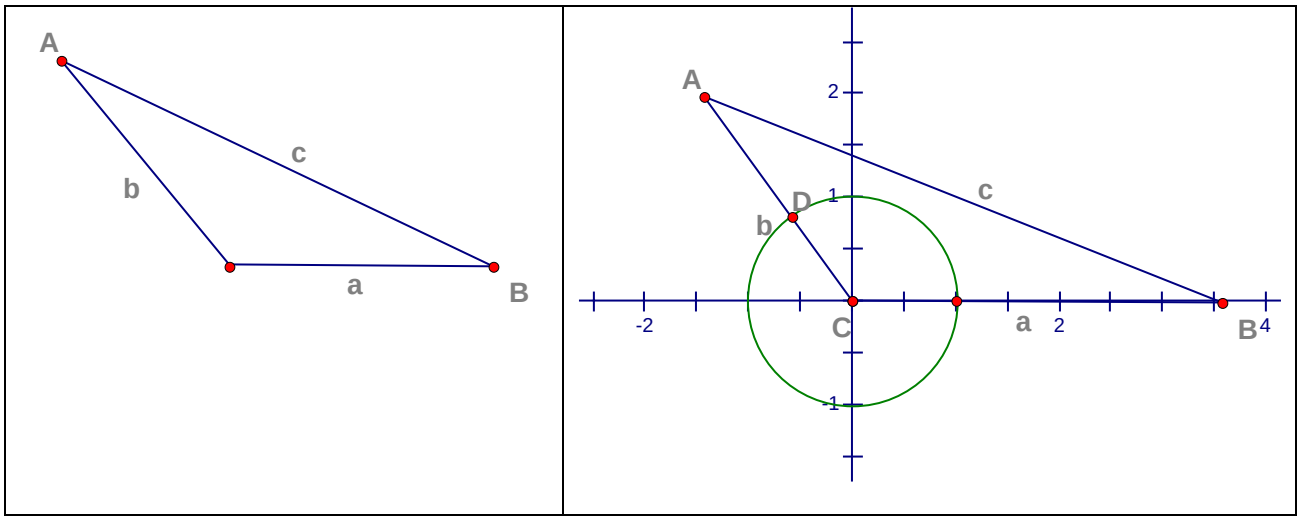
$c^2 = a^2 + b^2 – 2\, b(aCosC)$

$c^2 = a^2 + b^2 – 2\, abCosC$

Alternate proof of the Law of Cosines

Theorem: The Law of Cosines

Use GSP to place right triangle ABC on a coordinate system with C at (0,0) and B = (a, 0) on the positive ray of the x-axis. Let D be the intersection of side AB and the unit circle (Adjust your circle so that CD equals 1unit). From the definitions of sine and cosine, D = (cos A, sin A). Since AC = b, A can be considered the image of D under the scale change. Change (x, y)--> (bx, by), so that A = (bcosC, bsinC). Now use the distance formula,



Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$c = \sqrt{(bcosC - a)^2 + (bsinC - 0)^2}$

Rewrite the equation above so that it has the form in which it appears in the theorem (Law of Cosines).

Square both sides.

$c^2 = \{(bcosC - a)^2 + (bsinC - 0)^2\}$

Expand the binomials

$c^2 = b^2cos^2C - 2abcosC + a^2 + b^2sin^2C$

Apply the commutative property of addition.

$c = a^2 + b^2sin^2C + b^2cos^2C - 2abcosC$

Factor.

$c = a^2 + b^2(\sin^2 C + \cos^2 C) - 2ab\cos C$

Use the Pythagorean Identity: $\sin^2 C + \cos^2 C = 1$

$c = a^2 + b^2 - 2ab\cos C$

Assessment

Students will maintain a portfolio of their work in electronic files. The files will be reviewed and evaluated by the teacher. The teacher will provide feedback to students on their progress toward desired learning outcomes.