Trigonometric Unit Lesson 1
The Law of Sines

Lesson: Deriving the Law of Sines from the area formula of a triangle
Grade Level: Mathematics 4A

Overview
Students will explore triangles and right-triangle trigonometry to derive the Law of Sines. They will use Geometer Sketchpad (GSP) in their investigations. Instruction and assessment includes the appropriate use of technology. Topics are represented in multiple ways. Concepts are introduced and used in the context of realistic phenomena.

Prerequisite: Successful completion of Mathematics 3 or Accelerated Mathematics 2

Georgia Performance Standards
Mathematics 4

GPS MM4A7: Students will verify and apply \( A = \frac{1}{2}ab\sin(C) \) to find the area of a triangle.
GPSMM4A6: Students will solve trigonometric equations both graphically and algebraically.
   a. Solve trigonometric equations over a variety of domains, using technology as appropriate.
   c. Apply the Law of Sines

NCTM Standard Grade 9-12: Use trigonometric relationships to determine lengths and angle measures.

NCTM Standard Grade 9-12: Establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others.

Learning outcomes
After completing this lesson students will be able to:
Explain the Law of Sines.
Use the Law of Sines in solving measurement problems.

Procedure
For this lesson, students will have access to computing technology including calculators and the GSP program. Topics will be discussed in whole class and collaborative groups. The teacher will guide the outcomes of the lessons by modeling, guiding whole class discussion, and interacting with groups and individuals to assist and to assess progress toward desired learning outcomes.
This lesson is an investigation of triangles with the help of GSP. It is used to prove the Law of Sines. The lesson, along with its activities, verifies the Theorem SAS Area Formula

\[ \text{Area} = \frac{1}{2} bc(\sin A) \]

(In any triangle the area is one half the products of any two sides and the sine of their included angle) for a triangle ABC. Then the Law of Sines is derived from these results.

First use GSP to construct the triangle in figure 1. Next, draw a line segment from B perpendicular to side AC as in figure 2. Line segment h divides side b into two parts and forms two triangles.

![Figure 1](image1.png)  ![Figure 2](image2.png)

The area K of triangle ABC in figure 1 is given by the formula

\[ K = \frac{1}{2} bh. \]

To find the height h using triangle ABD,

\[ \sin A = \frac{h}{c}. \]

So \( h = csinA \)

Substituting this value into the area formula gives,

\[ K = \frac{1}{2} bcsinA \]

Similarly, \( \sin C = \frac{h}{a} \). So \( h = asinC \) and a second formula is \( K = \frac{1}{2} absinC \).
As a class activity, students can derive a third formula for the area of triangle ABC. Use Figure 3 or Figure 4 as a starting point. Construct a height from C or from A. Derive an area formula that involves the sine of B using the procedure above.

![Figure 3](image1)

![Figure 4](image2)

From the preceding results, the Theorem Law of Sines, which states that in any triangle ABC, \( \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \) can be proven.

To complete the proof, simplify the area formula ratios \( \frac{1}{2}bc(\sin A) = \frac{1}{2}ac(\sin B) = \frac{1}{2}ab(\sin C) \), by multiplying each term by 2 and dividing each term by abc. This yields \( \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \).

**Extension**

Use the Law of Sines to prove that the base angles of an isosceles triangle are congruent. This extension can be used as an assignment.

![isosceles triangle](image3)

**Proof:**

Given \( c = a \), because triangle ABC is isosceles.

\( \frac{\sin A}{a} = \frac{\sin C}{c} \), by the Law of Sines

Substitute \( a \) for \( c \).

\( \frac{\sin A}{a} = \frac{\sin C}{a} \)

Multiply both sides of the equation by \( a \).

\( \sin A = \sin C \)

Use a calculator to find the inverses of \( \sin A \) and \( \sin C \).
$$\sin^{-1} A = \sin^{-1} C$$

$$A = C$$ //

**Activity**

Use GSP to create oblique triangles, measure SAA and labor each. Solve the triangles and check the solutions using the measure feature of GSP.

**Assessment**

Students will maintain a portfolio of their work in electronic files. The files will be reviewed and evaluated by the teacher. The teacher will provide feedback to students on their progress toward desired learning outcomes.