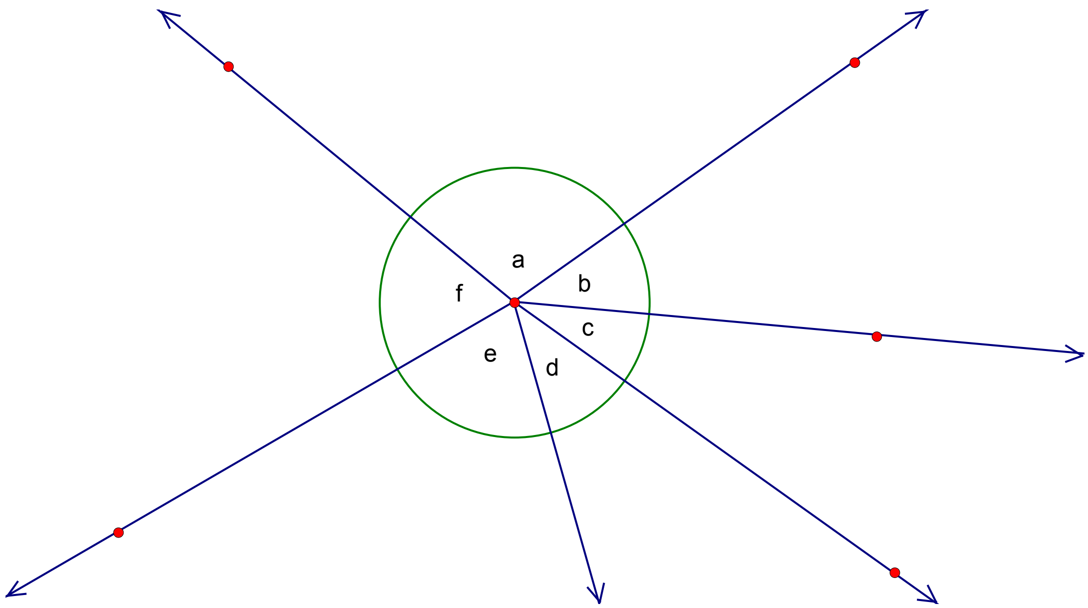
Geometric Proof

In this essay, we look at the Euclidean approach to the mathematical proof of geometrical facts. We will start with basic facts called axioms, or from other previously proven facts. Using these we will establish a chain of reasoning that demonstrate the truth of a particular statement or proposition.

The first basic assumption we will make is that a full turn is 360^o.



Theorem 1 Angles at a point add to 360^o

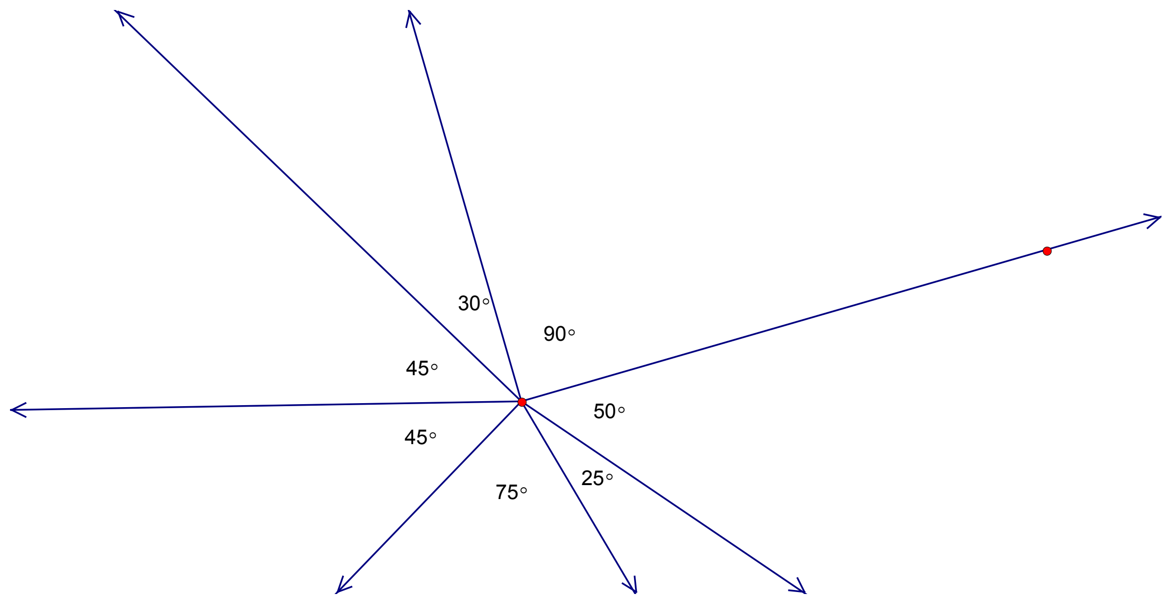
Proof

In the GSP sketch, the angles make up a full turn, and a full turn is 360^o, so

$a + b + c + d + e + f = 360^o$

This argument would hold for any number of angles at a given point. We have illustrated it for six angles.

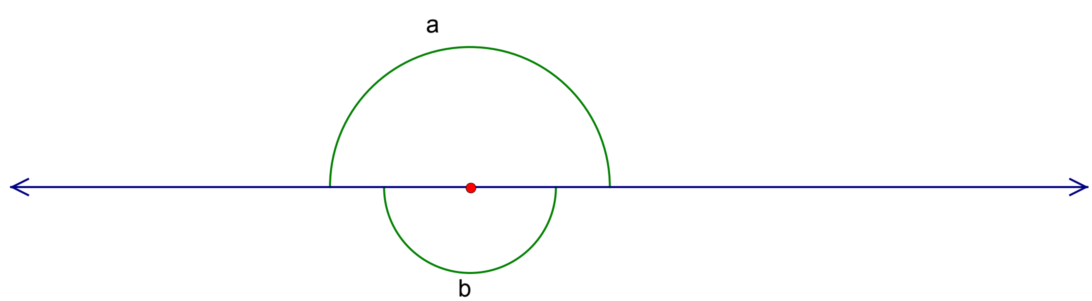
Example



Definition: A corollary is a fact that results from a significant theorem.

We can use Theorem 1, and the fact that angles on either side of a straight line are equal, to deduce a.

Corollary 1: The angle on a straight line is 180^o.



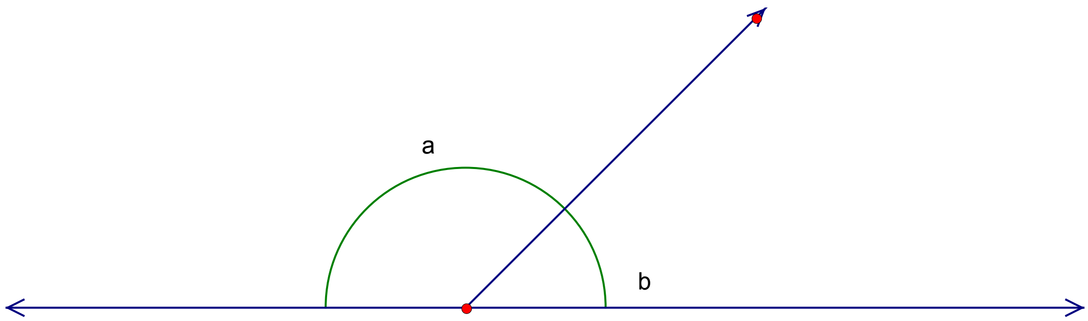
Proof:

In the GSP sketch, a and b are angles at a given point (common vertex) so that a + b = 360^o, by Theorem 1.

But a = b, so a + a = 360 degrees or 2a = 360 degrees, which gives a = 180^o. QED.

Example

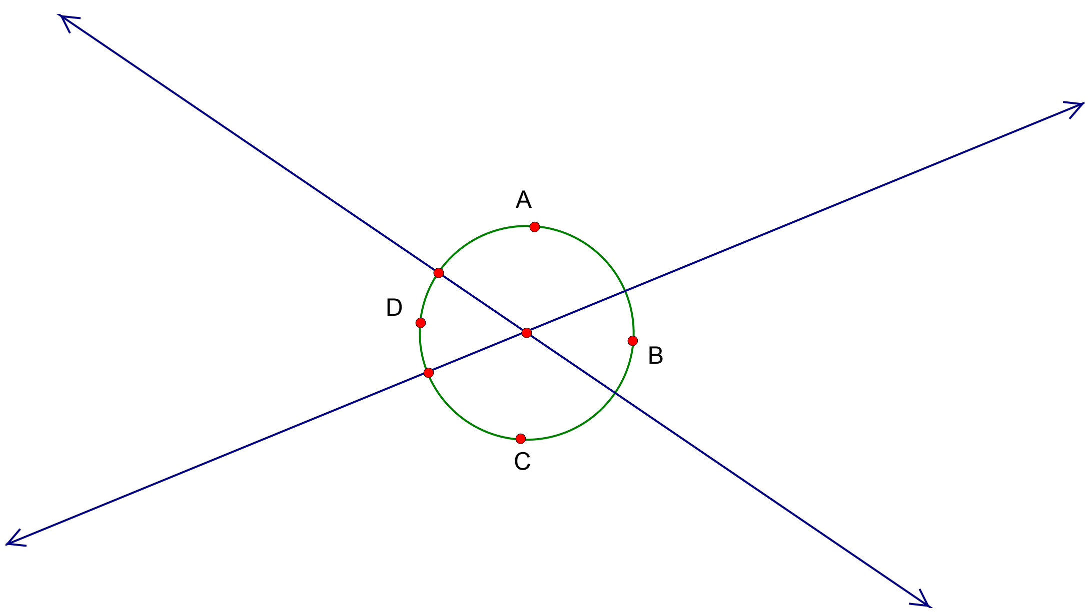
Corollary 2: The sum of angles on a straight line is 180^o



Proof:

In the diagram above, a and b are angles on a straight line, so a + b = 180^o by corollary 1.

Theorem 2: Vertically opposite angles are equal



Proof:

In the diagram above, angles a and b make up a straight line.

So, $a + b = 180^o$, by corollary 2. Angles a and c also make up a straight line.

$a + c = 180^o$, by corollary 2

Therefore $a + b = a + c$

Subtracting a from both sides of the equation yield, $b = c$.

The vertical opposite angles b and c are equal.

Similarly,

$a + b = 180^o$ (angles on a straight line)

$d + b = 180^o$ (angles on a straight line)

Therefore, $a + b = d + b$

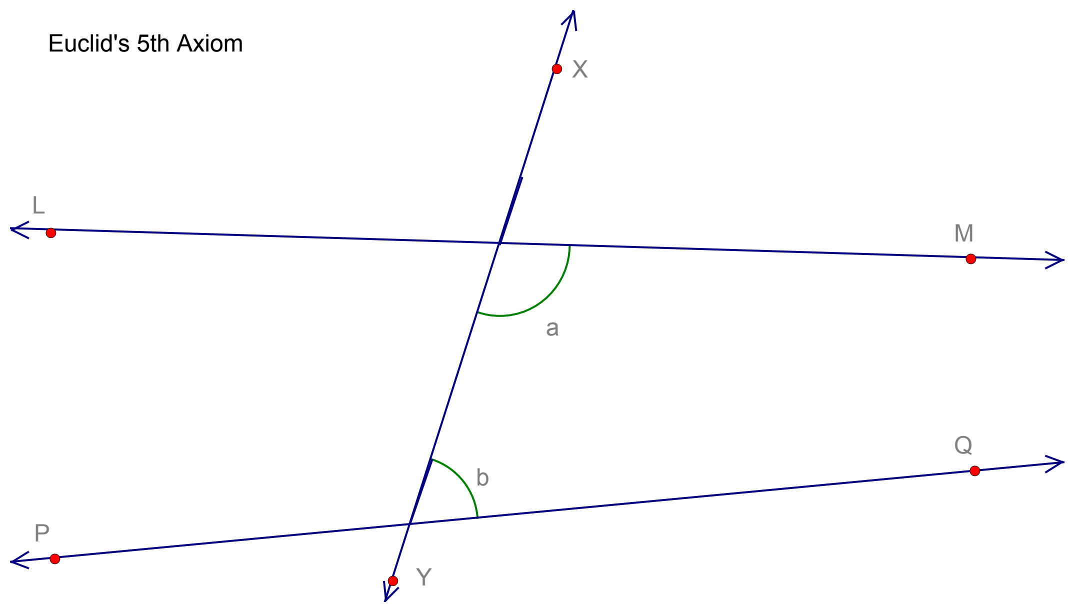
Subtracting b from both sides we get, $a = d$. QED.

Parallel Lines

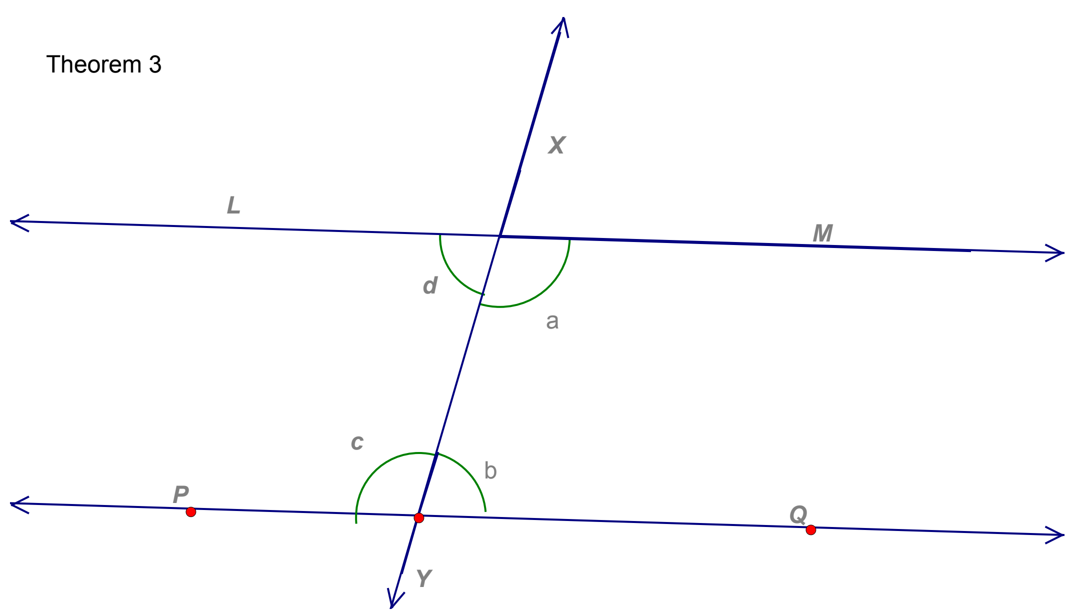
Definition: Parallel lines are lines that are always equal distance apart. Parallel lines never meet.

Euclid's 5th Axiom

If a straight line XY meets two other straight lines LM and PQ (see diagram below) so that a + b is not = to 180^o, then LM and PQ will meet, that is line LM and line PQ are not parallel.



Theorem 3: If LM and PQ are parallel lines intersected by a third line (transversal) XY, then alternate interior angles are equal. That is: angle a = c and angle b = d.



Proof:

In the diagram above, the fact that LM and PQ are parallel implies that,

a + b = 180 (by Euclid's 5^{th} axiom)

However, c and d are angles on a straight line, so

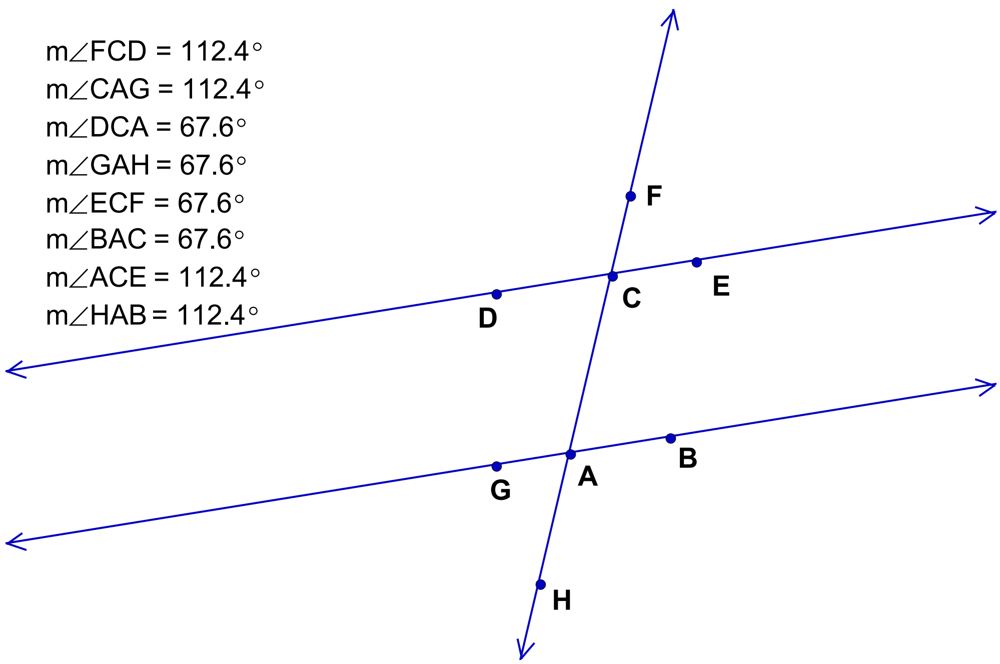
C + b =180^{o} (by corollary 2)

Therefore, a + b = c + b

Subtracting b from both sides of the equation yields

a = c. QED.

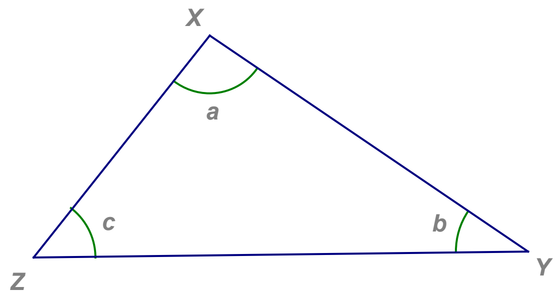
Example



Theorem 4: The sum of the angles of every triangle is 180^{o}

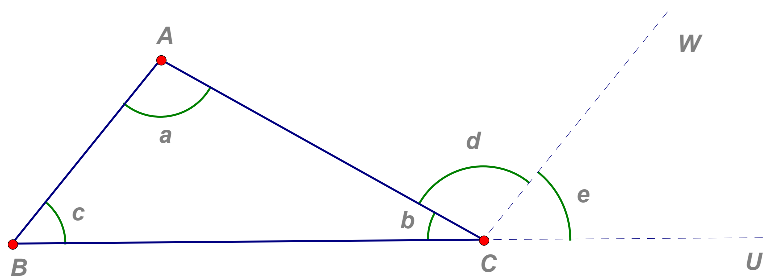
That is, a + b + c = 180^{o}

Theorem 4



Proof

Theorem 4 The angles of every triangle add to 180°; a + b + c = 180°



m∠ABC = 50.63°

m∠CAB = 99.71°

m∠ACB = 29.66°

m∠ABC+m∠CAB+m∠ACB = 180.00°

Proof
In the sketch above we extend the line segment BC to U and draw line segment CW parallel to AB.
We have
a = d
c = e
But b + d + e = 180°
Therefore, a + b + c = 180° QED.