Introduction and Overview

Probability and Statistics is a topic that is quickly growing, has become a major part of our educational program, and has a substantial role in the NCTM Standards. While covering this topic, you may find it useful to use some of the exercises, ideas, or topics found in this resource package. The main objective for collaborating our efforts to form these exercises is to give teachers the chance to allow their students to observe and get comfortable with "real life" situations that involve statistics. These exercises however focus especially on investigating "chance" situations, for example, game show games or games that involve different principles of gambling. It also allows students to focus on the basic concepts of probability, such as the odds of flipping a coin and having it land on heads or tails, or rolling die and having, for example, a five come up. While using these resources we've provided, students will be able to learn and get comfortable with the idea of statistics without feeling the pressure of learning.

After these activities the student should be more prepared to understand outcomes and odds of different mathematical situations. They should feel comfortable figuring out the predicted outcomes of small situations like the role or a die, flip of a coin, etc

Throughout these lessons there will be a few separate activities covered. First, there will be an activity that goes along with a story that takes place in a carnival. The activity will involve working with real life examples of probability and consequences. A second activity will focus on game show situations and will focus on the experimental probability. A third activity will involve exploring how experimental probability compares to theoretical probability. These activities are designed to show students the basic concepts of probability, help students see how probability relates to the students every day life, and they're designed to be a fun way to interact with the idea of probability.

Grades 8 and 9

Lesson Plan

Day	1
Goal	Students will be able to correctly create a fair game by the end of the lesson.
Objective(s)	**Relate probability to the students' lives.** **Understand the probability of a chance situation.** **Analyze a given situation to be either fair or unfair.** **Create an example of a fair game that exhibits understanding of basic probability.**

GPS	**M8D3 Students will use the basic laws of probability.** a. Find the probability of simple independent events. b. Find the probability of compound independent events.

Elements	**Procedures/Activities**	**Materials**	**Time**
Focus (e.g., hook, attention-getter, introduction of topic, review) **Before Activity**	Chalk Talk Draw Diagram 1 on the board. Explain the following activity is a silent one. No one can talk. You can add to the chalk talk at any time, but everyone must make at least 2 contributions. When you comment on an existing idea draw a line to connect it to the previous statement. There will only be 3 minutes to "chalk talk." Pass out as many markers as possible to the students and retain one for the teacher. The teacher will interact, circling interesting ideas, adding comments, and write questions to the comments. Follow with a brief discussion about how topic of probability and chance relates to everyday life and discuss student ideas of fairness.	Whiteboard, Markers	3 min 2 min
Activity 1 **During Activity**	Hand out copies of "The Case of the Carnival Probability Game." Select 3 readers to read 1 page each aloud to the class. Instruct the rest of the class to read along silently. Place students in groups of 3 or 4. *You could use a sorted deck of cards to assign groups evenly based on suit.	Class set of copies of pgs. 5-8 of Whodunit Math Puzzles by Bill Wise Deck of Cards	12-15 min
Transition	Allow students individually time to think through story and decide whether the game is fair or not. Students should think about what makes this game unfair.	Calculator	3 min

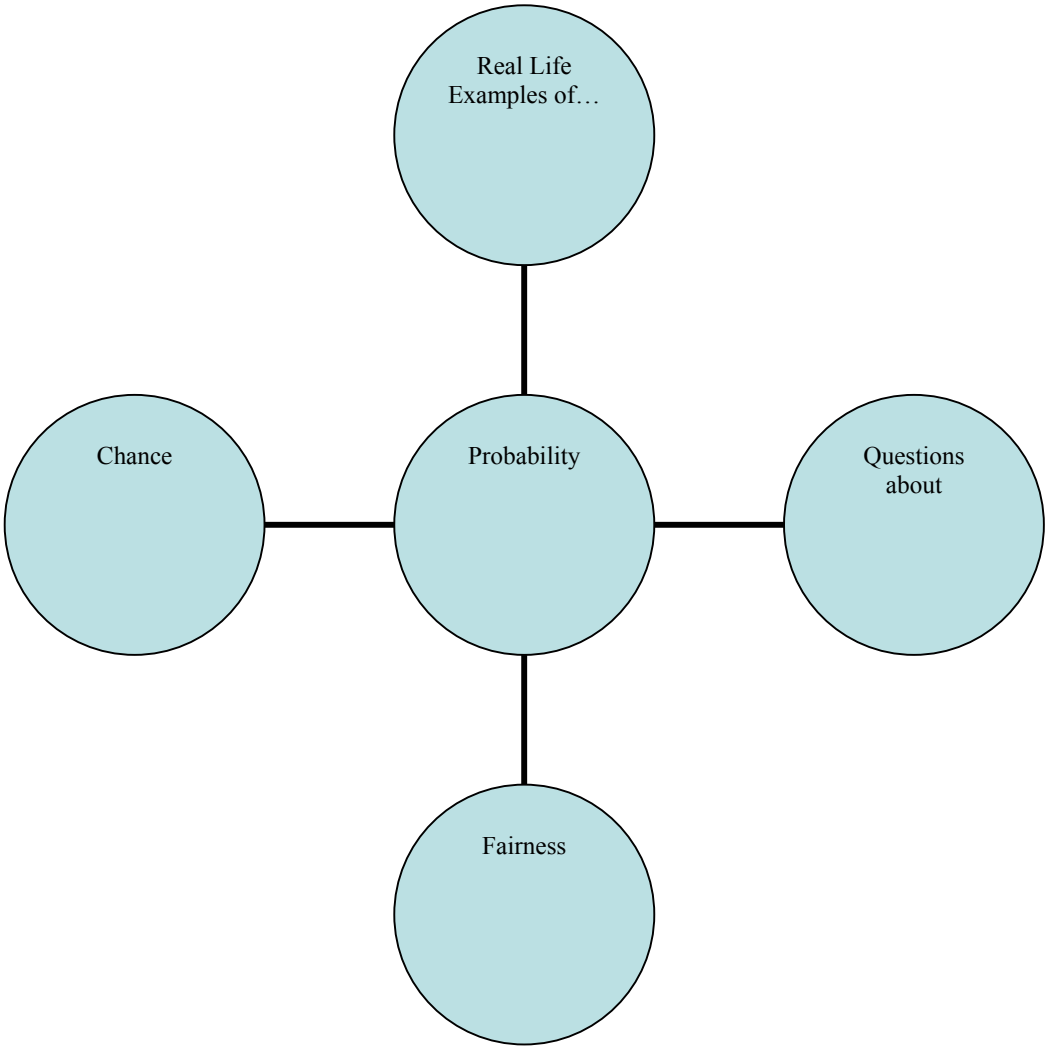
Activity 2 **Assessment** (if appropriate)	Ask students to come together to discuss their discoveries. Ask questions to stimulate student's thinking and help students to figure out an answer as you circulate among groups. Write out thoughts on the whiteboard, and discuss the different probabilities. participation	Whiteboard Calculator	10 min 5 min
Transition	If there's not enough time to finish, have students think about this example for homework. Also, have students think about other examples of situations like this.		
Focus (e.g., hook, attention-getter, introduction of topic, review) **After Activity**	Describe to students a game that involves two teams throwing a dart at dartboards that have evenly distributed sections but different circumferences. Next, describe to students a game that involves two teams rolling a die with faces labeled identically but being different sizes. Last ask students to vote on which game is fair or not fair.		2-3 min
Activity 3 **Assessment** (if Appropriate)	Students will focus back on their groups of 3 or 4 students. They will then be asked to come up with two cool games of their own similar to the ones just mentioned. One game would have to be an example of a fair game and the other of an unfair one. They will write their ideas on a chart passed out by the teacher. The chart provides space for the description of the game, why the student(s) feel it is fair or unfair, and space for a picture that will go with their game. *if students cannot complete both an unfair game and a fair game in the time allotted, they can choose one or the other. Walk around class to make sure students are on task. Assign full or partial credit for the charts based on evident student understanding	Make Your Own Game Chart, pencil & color pencils.	20-25 min

	of task.		
Transition	If theirs is not enough time to finish as a group, have students independently come up with a game idea in order to complete their chart as homework.		1 min
Closure (procedural and content; homework)	Discuss the importance of understanding probability and chance. Help students to see how practical this topic is to everyday life.		
Lesson Extender	Students can think through and come up with their own idea of a game that is similar to the one in the story. They can think through in groups about how to create unfair odds.		
Accommodations and/or Modifications	The book "Whodunit Math Puzzles" will be available as well as teacher for any questions or help needed.		
Strategies for managing the classroom environment during this lesson	To help with the discussions, it is important to ask questions that all students can think about. The activities should keep the students attention. The teacher should walk around as much as possible to assist students and ask questions that might stimulate more learning.		

Make Up Your Own Game

Description of Game
Fair Game: **Unfair Game:**
Explanations:
For Game Being Fair: **For Game Being Unfair:**
Illustration for Games:
For Fair Game: **For Unfair Game:**

Diagram 1:


Real Life Examples of…
Chance
Probability
Questions about
Fairness

Whodunit Math Puzzles

Bill Wise

Illustrated by Lucy Corvino

SCHOLASTIC INC.
New York Toronto London Auckland Sydney
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THE CASE OF THE CARNIVAL PROBABILITY GAME

The smells of buttered popcorn, cotton candy, and fried dough filled the air as Midville Police Chief Arthur Smart and his partner, 12-year-old junior detective Cal Q. Leiter, passed through the gates of the 85th annual Midville Fair. The Chief had been asked to participate in the pie-eating contest, and Cal had tagged along to support him.

"You know, it's gonna be difficult to stick to my diet today," said the Chief, eyeing the food stands that lined both sides of the walkway. He stopped in front of Perry's Pizza booth. "Maybe I'll have just a tiny slice of pizza before the contest."

"Do you really think that's a good idea, Chief?"

"Give me one good reason why I shouldn't."

Before Cal could reply, the Chief blurted out, "Hey, look. That's my cousin Norman arguing with that vendor over there."

Cal looked across the way and spotted a short, pudgy man who could pass for the Chief's twin gesturing wildly and stomping up and down. The Chief and Cal rushed over to see what the commotion was all about.

The vendor, a lanky man with long sideburns, stood

behind his counter, shaking his head smugly. "You lost, fair and square," he said, pointing at Cousin Norman.

The Chief patted Norman on the back. "Norman, what's going on here? I've never seen you this angry."

"I'll tell you what's going on here," Norman said through gritted teeth. "A big-time ripoff! This guy here is running some sort of crooked scam. I just can't figure out how he's doing it."

Cal looked up at the sign over the booth. It read: "CHANCES ARE — A GAME OF LUCK. Toss three coins. One head, you win $3. Two heads and the vendor wins. Three heads or no heads, no one wins, and we try again. Cost to play: $2."

"Let me get this straight," said the Chief. "Let's say you play once and win. It cost you $2 to play, and you won $3, so you really made $1. Isn't that right, Cal?"

Cal nodded and began drawing a diagram in his math notebook.

"And if you lose, you lose the $2 you paid to play, right?"

Again Cal nodded.

The vendor gave his side of the story. "Look," he said. "It's twice as hard to get two heads as it is to get one head. That's why it's a fair deal that if I win, I get twice as much as when I lose."

The Chief thought for a moment. "Makes sense to me."

"Wait," screamed Cousin Norman. "I've played this game 48 times. I've won 16 times, he's won 18 times; neither of us won 14 times. I've lost $20 bucks to this schemer. That shouldn't happen, should it?"

"Hey, haven't you heard of a lucky streak?" asked the vendor, smirking.

The Chief picked up the three coins from the counter and carefully examined each of them. "Nothing's wrong with the coins, Norman. Look, I'm sorry you lost, but it appears that this just wasn't your day. That's all."

"It doesn't make sense," said Norman. "Probability says I should have won twice as often as he did and that we should have broken even money-wise, doesn't it?"

Finally, Cal spoke up. "No, it doesn't," he said. "In fact, things pretty much went according to probability." Cal looked up at the Chief. "Chief, your cousin is not correct about probability, but he is correct about one thing: This game is a ripoff. Big time. This vendor has a guaranteed winner with his game."

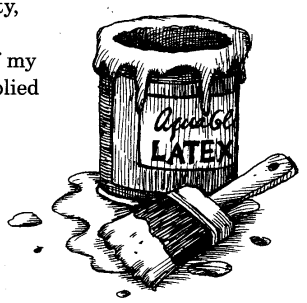
"Well, that's enough for me," bellowed the Chief. He turned to the vendor. "You, sir, are shutting down and coming downtown with us."

How is this game unfair? What did Cal figure out? ***(Solution on page 89.)***

THE CASE OF THE MAYOR'S RED OFFICE

Midville Police Chief Arthur Smart and his 12-year-old sidekick Cal Q. Leiter stood staring in shock at the mayor's office. Everything — the ceiling, the walls, the carpet, the furniture — had been slopped with bright red paint. It was an unbelievable scene.

Just then, the honorable Midville mayor, Linda Fuller, stormed into the room. "We must find out who did this," she said, turning to the Chief. "I want you to make finding this vandal your number-one priority, Chief."



"I've already got two of my best officers on this," replied the Chief.

Cal took out his trusty notebook. "When did this happen, Chief?"

"This morning between 10:30 and 11:00. The entire staff was with the mayor at a dedication."

Lesson Plan Format

Day	**2**
Goal	**Understand the probability of a chance situation**
Objective(s)	**The students will be able to determine the number of outcomes related to a given event.** **The students will be able to use the basic laws of probability**
GPS or QCC	**M8D2. a. Use tree diagrams to find the number of outcomes.** **b. Apply the addition and multiplication principles of counting.** **M8D3. a. Find the probability of simple independent events.** **b. Find the probability of compound independent events.**

Elements	**Procedures/Activities**	**Materials**	**Time**
Focus (e.g., hook, attention-getter, introduction of topic, review)	**Discussion: Review yesterday's class and the probability game and the ideas of fairness. Talk about real-life situations where probability is related. Gambling and game shows should be discussed. Introduce today's lesson with probability in game shows.** **We will be working in centers. Each group will be divided by having them draw a card from a deck of cards. The groups will form from the four different suits. Each group will work on a different activity in centers and then move to the next center when time is announced.**	**Deck of Cards** **Timer**	**5 min**
Activity 1 **Let's Make a Deal!** **Assessment** (if	Students report to assigned centers. They are to read the instructions and complete the game show games as directed in groups. They should fill out the worksheets and keep their sheets in their notebooks as they move from center to center. Informal check of completed worksheets.	**Calculators** **Activity Directions** **Corresponding worksheets** **2 Aces and 4 Jacks from a**	**14 min**

Appropriate)		**deck of cards**	
Transition	Remind students to take all of their work from previous centers and save it in their notebook. Switch quickly so they can play all of the games.		
Activity 2 **The Grand Game** **Assessment** (if appropriate)	Students report to assigned centers. They are to read the instructions and complete the game show games as directed in groups. They should fill out the worksheets and keep their sheets in their notebooks as they move from center to center. Informal check of completed worksheets.	**Monopoly Money for a cute display** **Activity Directions** **Calculator** **Corresponding worksheets**	**14 min**
Transition	Remind students to take all of their work from previous centers and save it in their notebook. Switch quickly so they can play all of the games.		
Activity 3 **Modified Craps** **Assessment** (if Appropriate)	Students report to assigned centers. They are to read the instructions and complete the game show games as directed in groups. They should fill out the worksheets and keep their sheets in their notebooks as they move from center to center. Informal check of completed worksheets	**Dice** **Activity Directions** **Corresponding worksheets** **Calculator**	**14 min**
Closure (procedural and content; homework)	**Talk about anything the students noticed during today's games. Bring all worksheets to class tomorrow!**		**3 min**
Lesson Extender			

Let's Make a Deal!

Game description: This game is based on the old television show, "Let's Make a Deal" hosted by Monty Hall. At the end of each show, the contestant who had won the most money was invited to choose from among 3 doors: Door #1, Door #2, and Door #3. Behind one of the three doors was a great prize, like a new car! Behind the other 2 doors there was a goat (not too exciting). The contestant selected a door. Monty then revealed what was behind one of the OTHER doors (always a goat). The contestant was then offered a choice: stick with his current door, or switch to the remaining un-revealed door. The contestant wins what was behind his final choice of door.

Game props: playing cards, specifically two Ace and four Jacks
Worksheet- 1 per group

This activity is heavily borrowed from http://statweb.calpoly.edu/mcarlton/gameshows/index.htmlBefore beginning the simulation, ask yourself: intuitively, does it make any difference to the chance of winning a car if the contestant switches or not?

Part 1

Instructions: In your group, pick two people to be Monty Hall, two people to be contestants, and at least one person to record all of the results. Each pair (one Monty and one contestant) will simultaneously (meaning at the same time) run 10 trials each of the following procedure. The Monty Hall

will be arranging the 3 cards (two Jacks which represent goats and the ace which represents the new car) face down. The contestant will pick a card (door). Without showing the original selection, the show host shows one of the other cards (it must always be a goat). The contestant must now decide to switch or stick with his or her current selection. The recorder will write the decision in the chart on the next page. The card selected is shown and the win or loss should be recorded in the chart as well.

Contestant 1

Trial	Stick/ Switch?	Win/ lose?
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Contestant 2

Trial	Stick/ Switch?	Win/ lose?
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Combine the data:

# of trials switched _____ # of cars won after switching ____

# of trials "stuck" _____ # of cars won after "sticking" ____

total # of trials _____ # of cars won (grand total) _____

Questions:

1. What proportion of all trials resulted in a win?
2. What proportion of all "switch" trials resulted in a win?
3. What proportion of all "stick" trials resulted in a win?
4. What proportion of all WINS was the result of the switching strategy?

Do your findings match what you previously thought?

After doing the activity, if you were really a contestant on the show, would you switch?

The Grand Game

Game description: This is one of the many games played on the popular game show "The Price is Right" hosted by Bob Barker.

The contestant starts with a bank of $1.

Six grocery items are shown along with a target price for the grocery items. Four items are below the target price, two are above.

Each time the contestant picks an item under the target price, s/he earns a "0" in the bank (i.e., the contestant's bank goes from $10 to $100 to $1000 to a possible total of $10,000).

If the contestant picks all four items under the target price, s/he wins $10,000 in cash.

If the contestant picks one of the items above the target price, the game is over.

Should the contestant lose the game with $1, $10, or $100 in the bank, s/he wins that amount.

However, when the contestant gets to $1,000 (when there are one good item and two bad items left), Bob ups the stakes.

The contestant can quit and keep the $1,000 or risk it and go on for the win.

If the contestant picks the final item, obviously s/he wins the $10,000, but if s/he picks a bad item, s/he loses the game and s/he loses the $1,000 as well!

You need: 1 Worksheet per group

This activity is heavily borrowed from

http://statweb.calpoly.edu/mcarlton/gameshows/index.html

The Grand Game

Look at the following scenarios and answer the questions. Make sure to talk the answers over with the whole group!

A contestant has won $1,000. Two of the remaining products cost more than the target price, and only one costs less than the target price.

- What is the probability the contestant will pick the one remaining product that costs less than the target price?

- Based on this probability alone, would you risk your $1,000?

- The contestant wins $10,000 if he picks the right product, and s/he drops to $0 if s/he picks either of the wrong products. Let *x* represent the contestant's winnings, assuming s/he risks $1,000. Fill in the table below.

Possible amounts to win, x	$0	$10,000
Probability of winning this amount, p(x)		

- Based on the table, calculate the contestant's expected winnings if s/he risks $1,000.

Hint:
Expected Winnings = amount 1 x amount 1's probability + amount 2 x amount 2's probability

- Would you risk $1,000 in *The Grand Game* for a chance at $10,000?

Roll the Die

Rules and Directions:

This game is similar to the Las Vegas gambling game, craps. A few things have been modified, but many of the same rules of probability apply. To start off, the player will need to take the two fair, six-sided die and roll them. If the first roll equals a seven then the player automatically wins. However if a seven is not rolled then the student will need to take the die and role again. The student will be provided as many roles as needed to either role the number

they first rolled, or to roll a seven. If a seven is not rolled for the first roll but a five for example, then the goal is to now role a five before a seven is rolled. If a seven appears before a five then the student loses. However if a five is rolled again before a seven, then the student wins! Let's get started and good luck!

Example 1

John rolled a seven for his first role. John wins!

Example 2

Mark rolled a **three** for his first roll. His next four rolls were a four, a ten, a nine, and a six. His next roll however was a seven. Mark loses since he rolled a seven before he rolled a **three** again.

Example 3

Jaime rolled a **six** for her first roll. Her next two rolls were an eleven and a five. Her next role though was a six. Jaime wins since she rolled a **six** (the number she first got) before she rolled a seven.

Die #1 → Die #2	**1**	**2**	**3**	**4**	**5**	**6**
1						
2						
3						

4						
5						
6						

1. What is the probability that the sum of the die will be 2? _____
2. What is the probability that the sum of the die will be 3? _____
3. What is the probability that the sum of the die will be 4? _____
4. What is the probability that the sum of the die will be 5? _____
5. What is the probability that the sum of the die will be 6? _____
6. What is the probability that the sum of the die will be 7? _____
7. What is the probability that the sum of the die will be 8? _____
8. What is the probability that the sum of the die will be 9? _____
9. What is the probability that the sum of the die will be 10? _____
10. What is the probability that the sum of the die will be 11? _____
11. What is the probability that the sum of the die will be 12? _____

Day 3 Lesson Plan

Day	**3**
Goal	**Understand and apply basic concepts of probability.** **Understand the probability of a chance situation.**
Objective(s)	**To use large numbers of trials to approximate the theoretical probability.**
GPS or QCC	**[39] Topic:** Prediction, Outcome, Event **Standard:** Identifies possible outcomes of simple and compound experiments, and predicts or describes the probability of a given event, expressed as a rational number.

Elements	**Procedures/Activities**	**Materials**	**Time**
Focus (e.g., hook, attention-getter, introduction of topic, review)	Tell students that today's lucky number is six and that we are going to be rolling a die so many times that it will make the students lose interest in ever rolling a die again in their life which could be good thing since it might also prevent some of them from having future gambling problems.	Whiteboard	2-3 min
Activity 1 **Assessment** (if Appropriate)	Place students in pairs and ask them to record what the theoretical probability of rolling a 6 with a die would be. Pass out die for each pair of students and the chart #1. Next, ask students to observe and record the outcomes of rolling a die 40 times while timing them to complete this task within 3 minutes. Students will be given a chart that has it's first column labeled the toss # and second the outcome. Next, ask students to record their observed probability of rolling a 6 after ten tosses, twenty tosses, thirty tosses, and forty tosses.	Dice, Timer & Chart #1 Calculator	10-15 min
Transition	Ask students what is the most number of tosses they think would be possible to do in one minute?		30 seconds
Focus (e.g., hook, attention-getter, introduction of topic, review)	Ask students what they think if you would say that you can make100 tosses in a second?		2-3 min
Activity 2	• Use the excel worksheet on the overhead to demonstrate the probability of tossing a die 100 times. • Next, ask students to continue filling out chart #2 that asks for the outcomes and probability of rolling a 6 after 100 tosses, 500 tosses, 1000 tosses and 10000 tosses while you do them on the overhead. • Also, validate that using this program is the same as them rolling the die by showing them that list of outcomes are randomly changing with each toss.	Chart #2, Computer with Excel Software, & Excel worksheet for tossing a die	10 min

Assessment (if Appropriate)	• Last, ask students to write briefly what they noticed from completing the two charts. Ask students who might be tempted to talk to sit closer the front of the class.		
Transition	Ask students to staple their charts together and hand them in to the teacher for review.		
Focus (e.g., hook, attention-getter, introduction of topic, review)	Tell students that they are going to get a chance to play all the games they did at the Centers in Day Two (last class day) 100 times in a second or even 10,000 times in a second.		2-3 min
Activity 3 **Assessment** (if Appropriate)	Provide each pair of students a computer so that they can observe the excel program for the tossing a die. Next, allow them to make changes in order to demonstrate each of the games they did at the Centers on Day 2. Walk around the classroom to make sure that all students are on task.	Computers with Excel Software	25-30 min
Transition	Let each group save their work and submit them to the teacher for review.	Excel worksheet for each game from Day 2 to use as an answer key.	1-2 min
Closure (procedural and content; homework)	Discuss how doing more trials resulted in their observed probability approaching the theoretical probability.		1-2 min
Lesson Extender	Each pair will discuss and share with the class what manipulations they made to the excel worksheet for the die probability in order to demonstrate the games they did at the Centers on Day 2.		

Accommodations and/or Modifications	In the case of a class with an odd number of students you can accommodate this problem by putting 3 students in one of the groups. If theirs is not enough time to finish in pairs, have students independently make their changes on a print out of the excel worksheet (with visible formulas) for the die probability. Also, the teacher should notify the students the times that the computer lab would be available before, after, or during the school day (with assistance).		
Strategies for managing the classroom environment during this lesson	Pair students who have shown in the past to work productively with each other. Pair the students who are more familiar with using a computer with students who are not as familiar. It is important to ask questions that all students can think about. The activities should keep the students attention. The teacher should walk around as much as possible to assist students and ask questions that might stimulate more learning.		

Chart #1
Rolling a 6 with a Die

Toss #	Outcome
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	

19	
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21	
22	
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26	
27	
28	
29	
30	
31	
32	
33	
34	
35	
36	
37	

38	
39	
40	

Probability After 10 Tosses_____
Probability After 20 Tosses_____
Probability After 30 Tosses_____
Probability After 40 Tosses_____

Chart # 2

Toss #	Outcome
100	
500	
1000	
10,000	

Below briefly describe what you have noticed after completing Chart #1 and Chart #2?

These lessons were produced by: Melissa Casteel, Saniaz Amirjalayer, Dan Davis and John Simmons