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## Georgia Performance Standards Framework

### Mathematics I – Unit 1: Function Families

**INTRODUCTION:** In seventh and eighth grade, students learned about functions generally and about linear functions specifically. This unit explores properties of basic quadratic, cubic, absolute value, square root, and rational functions as well as new language and notation for talking about functions. The discussion of function characteristics includes further development of the language of mathematical reasoning to include formal discussion of the logical relationships between a statement and its converse, inverse, and contrapositive.

**ENDURING UNDERSTANDINGS:**

- Functions have three parts: (i) a domain, which is the set of inputs for the function, (ii) a range, which is the set of outputs, and (iii) some rule or statement of correspondence indicating how each input determines a unique output.
- The domain and rule of correspondence determine the range of a function.
- Graphs are geometric representations of functions.
- Functions are equal if they have the same domain and rule of correspondence.
- Function notation provides an efficient way to define and communicate functions.
- The variables used to represent domain values, range values, and the function as a whole, are arbitrary. Changing variable names does not change the function.
- Logical equivalence is a concept that applies to the form of a conditional statement. A conditional statement and its contrapositive are logically equivalent. Neither the converse nor inverse of a conditional statement is logically equivalent to the statement.

**KEY STANDARDS ADDRESSED:**

**MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.**

- a. Represent functions using function notation.
- b. Graph the basic functions  $f(x) = x^n$  where  $n = 1$  to  $3$ ,  $f(x) = \sqrt[n]{x}$ ,  $f(x) = |x|$ , and  $f(x) = 1/x$ .
- c. Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the  $x$ - and  $y$ -axes. [*Previewed in this unit.*]
- d. Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.
- e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.
- f. Recognize sequences as functions with domains that are whole numbers.
- g. Explore rates of change, comparing constant rates of change (i.e., slope) versus variable rates of change. Compare rates of change of linear, quadratic, square root, and other function families.

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**MM1G2. Students will understand and use the language of mathematical argument and justification.**

- a. Use conjecture, inductive reasoning, deductive reasoning, counterexamples, and indirect proof as appropriate.
- b. Understand and use the relationships among a statement and its converse, inverse, and contrapositive.

**RELATED STANDARDS ADDRESSED:**

**MM1P1. Students will solve problems (using appropriate technology).**

- a. Build new mathematical knowledge through problem solving.
- b. Solve problems that arise in mathematics and in other contexts.
- c. Apply and adapt a variety of appropriate strategies to solve problems.
- d. Monitor and reflect on the process of mathematical problem solving.

**MM1P2. Students will reason and evaluate mathematical arguments.**

- a. Recognize reasoning and proof as fundamental aspects of mathematics.
- b. Make and investigate mathematical conjectures.
- c. Develop and evaluate mathematical arguments and proofs.
- d. Select and use various types of reasoning and methods of proof.

**MM1P3. Students will communicate mathematically.**

- a. Organize and consolidate their mathematical thinking through communication.
- b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- c. Analyze and evaluate the mathematical thinking and strategies of others.
- d. Use the language of mathematics to express mathematical ideas precisely.

**MM1P4. Students will make connections among mathematical ideas and to other disciplines.**

- a. Recognize and use connections among mathematical ideas.
- b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- c. Recognize and apply mathematics in contexts outside of mathematics.

**MM1P5. Students will represent mathematics in multiple ways.**

- a. Create and use representations to organize, record, and communicate mathematical ideas.
- b. Select, apply, and translate among mathematical representations to solve problems.
- c. Use representations to model and interpret physical, social, and mathematical phenomena.

**UNIT OVERVIEW:** Prior to this unit, students need to have worked extensively with operations on integers, rational numbers, and square roots of nonnegative integers as indicated in the Grade 6 – 8 standards for Number and Operations.

In the unit students will apply and extend the Grade 7 – 8 standards related to writing algebraic expressions, evaluating quantities using algebraic expressions, understanding inequalities in one variable, and understanding relations and linear functions as they develop much deeper and more sophisticated

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understanding of relationships between two variables. Students are assumed to have a deep understanding of linear relationships between variable quantities. Students should understand how to find the areas of triangles, rectangles, squares, and circles and the volumes of rectangular solids.

The unit begins with intensive work with function notation. Students learn to use function notation to ask and answer questions about functional relationships presented in tabular, graphical, and algebraic form. The distinction between discrete and continuous domains is explored through comparing and contrasting functions which have the same rule of correspondence, but different domains. Through extensive work with reading and drawing graphs, students learn to view graphs of functional relationships as whole objects rather than collections of individual points and to apply standard techniques used to draw representative graphs of functions with unbounded domains.

The unit includes an introduction to propositional logic of conditional statements, their converses, inverses, and contrapositives. Conditional statements about the absolute value function and vertical translations of the absolute value function give students concrete examples that can be classified as true or false by examining graphs. Combining analysis of conditional statements with further exploration of basic functions demonstrates that conditional statements are important throughout the study of mathematics, not just in geometry where this material has traditionally been introduced.

Average rate of change of a function is introduced through the interpretation of average speed, but is extended to the context of rate of change of revenue for varying quantities of a product sold. Students contrast constant rates of change to variable ones through the concept of average rate of change. The basic function families are introduced primarily through real-world contexts that are modeled by these functions so that students understand that these functions are studied because they are important for interpreting the world in which we live. The work with vertical shifts, stretches and shrinks, and reflections of graphs of basic functions is integrated throughout the unit rather than studied as a topic in isolation. Functions whose graphs are related by one of these transformations arise from the context and the context promotes understanding of the relationship between a change in a formula and the corresponding change in the graph.

Students need facility in working with functions given via tables, graphs, or algebraic formulas, using function notation correctly, and learning to view a function as an entity to be analyzed and compared to other functions.

Throughout this unit, it is important to:

- Begin exploration of a new function by generating a table of values using a variety of numbers from the domain. Decide, based on the context, what kinds of numbers can be in the domain, and make sure you choose negative numbers or numbers expressed as fractions or decimals if such numbers are included in the domain.
- Do extensive graphing by hand. Once students have a deep understanding of the relationships between formulas and graphs, regular use of graphing technology will be important. For this introductory unit, graphing by hand is necessary to develop understanding.
- Be extremely careful in the use of language. Always use the name of the function, for example  $f$ , to refer to the function as a whole and use  $f(x)$  to refer to the output when the input is  $x$ . For example, when language is used correctly, a graph of the function  $f$  in the  $x$ ,  $y$ -plane is the graph of the equation  $y = f(x)$  since we graph those points, and only those points, of the form  $(x, y)$  where the  $y$ -coordinates are equal to  $f(x)$ .

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**TASKS:** The remaining content of this framework consists of student learning tasks designed to allow students to learn by investigating situations with a real-world context. The first leaning task is intended to launch the unit. Its primary focus is introducing function notation and the more formal approach to functions characteristic of high school mathematics. The second through seventh learning tasks extend students knowledge of functions through in depth consideration of domain, range, average rate of change, and other characteristics of functions basic to the study of high school mathematics. The last task is designed to demonstrate the type of assessment activities students should be comfortable with by the end of the unit.

### Exploring Functions with Fiona Learning Task

- While visiting her grandmother, Fiona Evans found markings on the inside of a closet door showing the heights of her mother, Julia, and her mother's brothers and sisters on their birthdays growing up. From the markings in the closet, Fiona wrote down her mother's height each year from ages 2 to 16. Her grandmother found the measurements at birth and one year by looking in her mother's baby book. The data is provided in the table below, with heights rounded to the nearest inch.

Age (yrs.)	$x$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Height (in.)	$y$	21	30	35	39	43	46	48	51	53	55	59	62	64	65	65	66	66

- Which variable is the independent variable, and which is the dependent variable? Explain your choice.
- Make a graph of the data.
- Should you connect the dots on your graph? Explain.
- Describe how Julia's height changed as she grew up.
- How tall was Julia on her 11<sup>th</sup> birthday? Explain how you can see this in both the graph and the table.
- What do you think happened to Julia's height after age 16? Explain. How could you show this on your graph?

In Math 1 and all advanced mathematics, *function notation* is used as an efficient way to describe relationships between quantities that vary in a functional relationship. In the remaining parts of this investigation, we'll explore function notation as we look at other growth patterns and situations.

- In function notation,  $h(2)$  means the output value when the input value is 2. In the case of the table above,  $h(2)$  means the  $y$ -value when  $x$  is 2, which is Julia's height (in inches) at age 2, or 35. Thus,  $h(2) = 35$ . Function notation gives us another way to write about ideas that you began learning in middle school, as shown in the table below.

<i>Statement</i>	<i>Type</i>
At age 2, Julia was 35 inches tall.	Natural language
When $x$ is 2, $y$ is 35.	Statement about variables
When the input is 2, the output is 35.	Input-output statement
$J(2) = 35$ .	Function notation

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As you can see, function notation provides shorthand for talking about relationships between variables. With function notation, it is easy to indicate simultaneously the values of both the independent and dependent variables. The notation  $h(x)$  is typically read “ $h$  of  $x$ ,” though it is helpful to think “ $h$  at  $x$ ,” so that  $h(2)$  can be interpreted as “height at age 2,” for example.

Note: Function notation looks like a multiplication calculation, but the meaning is very different. To avoid misinterpretation, be sure you know which letters represent functions. For example, if  $g$  represents a function, then  $g(4)$  is *not* multiplication but rather the value of “ $g$  at 4,” that is, the output value of the function  $g$  when the input is value is 4.

- a. What is  $h(11)$ ? What does this mean?
  - b. When  $x$  is 3, what is  $y$ ? Express this fact using function notation.
  - c. Find an  $x$  so that  $h(x) = 53$ . Explain your method. What does your answer mean?
  - d. From your graph or your table, estimate  $h(6.5)$ . Explain your method. What does your answer mean?
  - e. Find an  $x$  so that  $h(x) = 60$ . Explain your method. What does your answer mean?
  - f. Describe what happens to  $h(x)$  as  $x$  increases from 0 to 16.
  - g. What can you say about  $h(x)$  for  $x$  greater than 16?
  - h. Describe the similarities and differences you see between these questions and the questions in #1.
3. Fiona has a younger brother, Tyler, who attends a pre-kindergarten class. One of the math activities during the first month of school was measuring the heights of the children. The class made a large poster to record the information in a bar graph showing the ages and heights of all the children. The children used heights rounded to the nearest whole number of inches; however, the teacher also recorded their heights to the nearest half inch in an Excel spreadsheet as shown below.

**Table A: Class Height Information**  
**August 2007**

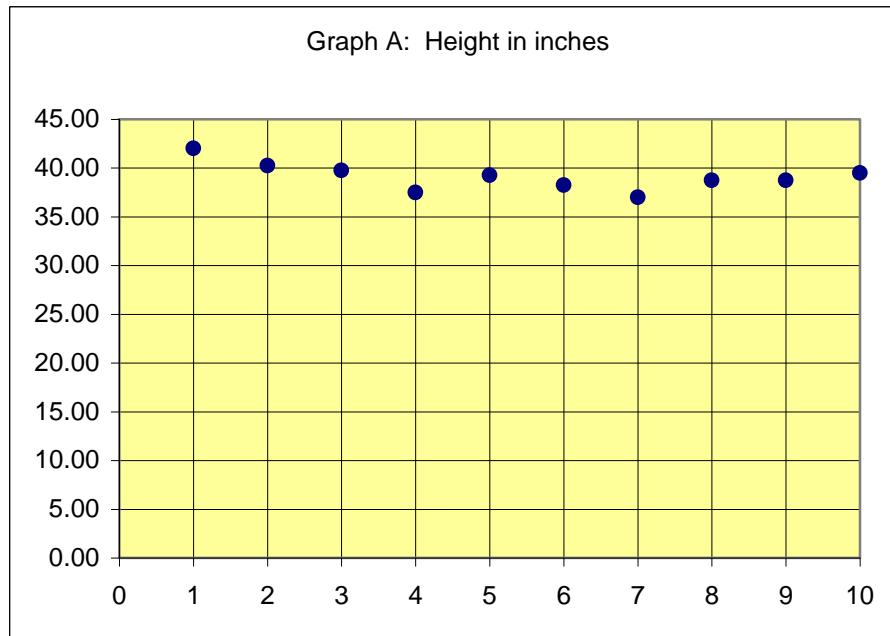
Student Number	Last name	First name	Height in inches
1	Barnes	Joshua	42.0
2	Coleman	David	40.5
3	Coleman	Diane	40.0
4	Drew	Keisha	37.5
5	Evans	Tyler	39.5
6	Hagan	Emily	38.0
7	Nguyen	Violet	37.0
8	Ruiz	Alina	38.5
9	Rader	Joshua	38.5
10	Vogel	Zach	39.5

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After making the Excel table, the teacher decided to also make an Excel version of the bar graph. While she was working on the bar graph, she had the idea of also graphing the information in the rectangular coordinate system, using the Student Number as the x-value and the height to the nearest half inch as the y-value. Here's her graph.



The relationship that uses Student Number as input and the height of the student with that student number as output describes a function because, for each student number, there is exactly one output, the height of the student with that number.

- The graph was drawn with an Excel option named *scatter plot*. This option allows graphs of relationships whether or not the graphs represent functions. Sketch a scatter plot using student ages as inputs and heights of students as output. Explain why this relationship is not a function.
- In the plot in Graph A, the pre-kindergarten teacher chose an Excel format that did not connect the dots. Explain why the dots should not be connected.
- Using the information in the Excel spreadsheet about the relationship between student number and height of the corresponding student, fill in each of the following blanks.

The height of student 2 is equal to \_\_\_\_\_ inches.

The height of student 6 is equal to \_\_\_\_\_ inches.

The height of student \_\_\_\_\_ is equal to 37 inches.

For what student numbers is the height equal to 38.5 inches? \_\_\_\_\_

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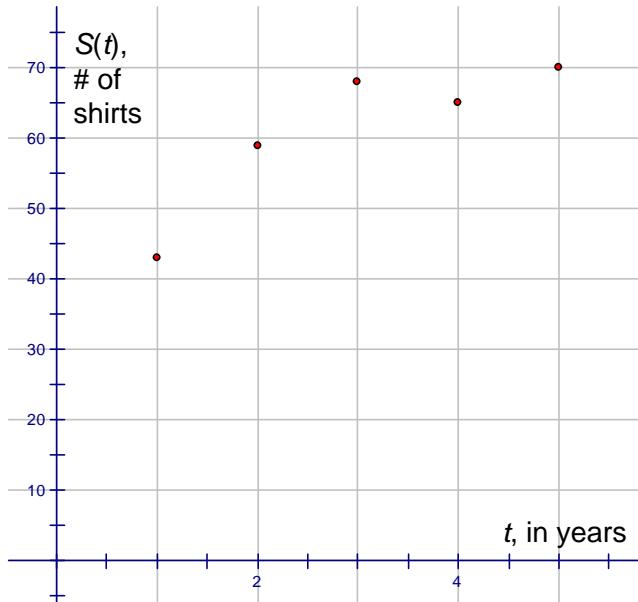
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- e. Using the function name  $h$ , we write  $h(1) = 42$  to indicate that the height of student 1 is equal to 42 inches. The following fill-in-the-blank questions repeat the questions from part d) in function notation. Fill in these blanks too.

$$h(2) = \underline{\hspace{2cm}} \quad h(6) = \underline{\hspace{2cm}} \quad h(\underline{\hspace{2cm}}) = 37$$

For what values of  $x$  does  $h(x) = 38.5$  ?  $\underline{\hspace{2cm}}$

4. Fiona attends Peachtree Plains High School. When the school opened five years ago, a few teachers and students put on FallFest, featuring contests, games, prizes, and performances by student bands. To raise money for the event, they sold FallFest T-shirts. The event was very well received, and so FallFest has become a tradition. This year Fiona is one of the students helping with FallFest and is in charge of T-shirt sales. She gathered information about the growth of T-shirt sales for the FallFests so far and created the graph below that shows the function  $S$ .



- What are the independent and dependent variables shown in the graph?
- For which years does the graph provide data?
- Does it make sense to connect the dots in the graph? Explain.
- What were the T-shirt sales in the first year? Use function notation to express your result.
- Find  $S(3)$ , if possible, and explain what it means or would mean.
- Find  $S(6)$ , if possible, and explain what it means or would mean.
- Find  $S(2.4)$ , if possible, and explain what it means or would mean.
- If possible, find a  $t$  such that  $S(t) = 65$ . Explain.
- If possible, find a  $t$  such that  $S(t) = 62$ . Explain.
- Describe what happens to  $S(t)$  as  $t$  increases, beginning at  $t = 1$ .
- What can you say about  $S(t)$  for values of  $t$  greater than 6?

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Note: As you have seen above, functions can be described by tables and by graphs. In high school mathematics, functions are often given by formulas, but it is important to remember that not all functions can be described by formulas.

5. Fiona's T-shirt committee decided on a long sleeve T-shirt in royal blue, one of the school colors, with a FallFest logo designed by the art teacher. The committee needs to decide how many T-shirts to order. Fiona was given the job of collecting price information so she checked with several suppliers, both local companies and some on the Web. She found the best price with Peachtree Plains Promotions, a local company owned by parents of a Peachtree Plains High School senior.

The salesperson for Peachtree Plains Promotions told Fiona that there would be a \$50 fee for setting up the imprint design and different charges per shirt depending on the total number of shirts ordered. For an order of 50 to 250 T-shirts, the cost is \$9 per shirt. Based on sales from the previous five years, Fiona was sure that they would order at least 50 T-shirts and would not order more than 250. If  $x$  is the number of T-shirts to be ordered and  $y$  is the total dollar cost of these shirts, then  $y$  is a function of  $x$ . Let's name this function  $C$ , for cost function. Fiona started the table below.

$x$	50	100	150	200	250
$9x$	450	900	1350		
$y = C(x)$	500	950			

- a. Fill in the missing values in the table above.
- b. Make a graph to show how the cost depends upon the number of T-shirts ordered. You can start by plotting the points corresponding to values in the table. What points are these? Should you connect these points? Explain. Should you extend the graph beyond the first or last point? Explain.
- c. Write a formula showing how the cost depends upon the number of T-shirts ordered. For what numbers of T-shirts does your formula apply? Explain.
- d. What does  $C(70)$  mean? How can you find the single number value equal to  $C(70)$ ? Did you use the table, the graph, or the formula?
- e. If the T-shirt committee decides to order only the 67 T-shirts that are pre-paid, how much will it cost? Show how you know. Express the result using function notation.
- f. If the T-shirt committee decides to order the 67 T-shirts that are pre-paid plus 15 more, how much will it cost? Show how you know. Express the result using function notation.

Note: As you have seen above, functions can be described by tables, by graphs, and by formulas. Although there are functions that cannot be described by formulas, many of the functions studied in high school mathematics are given by formulas. We explore such an example below.

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6. Fiona is taking physics. Her sister, Hannah, is taking physical science. Fiona decided to use functions to help Hannah understand one basic idea related to gravity and falling objects. Fiona explained that, if a ball is dropped from a high place, such as the Tower of Pisa in Italy, then there is a formula for calculating the distance the ball has fallen. If  $y$ , measured in meters, is the distance the ball has fallen and  $t$ , measured in seconds, is the time since the ball was dropped, then  $y$  is a function of  $t$ , and the relationship can be approximated by the formula  $y = d(t) = 4.9t^2$ . Here we name the function  $d$  because the outputs are distances.

$t$ (in seconds)	0	1	2	3	4	5	6	...
$t^2$	0	1	4	9				...
$y = d(t) = 4.9t^2$	0	5	20					...

- a. Fill in the missing values in the table above.
  - b. Suppose the ball is dropped from a building at least 100 meters high. Measuring from the top of the building, draw a picture indicating the position of the ball at times indicated in your table of values.
  - c. Draw a graph of  $t$  versus  $y$  for this situation. Should you connect the dots? Explain.
  - d. What is the relationship between the picture (part b) and the graph (part d)?
  - e. Explain what happens to the speed of the ball as the ball falls. Use your table and your picture to help you justify your reasoning.
  - f. What is  $f(4)$ ? What does this mean?
  - g. Estimate  $t$  such that  $f(t) = 50$ . Explain your method. What does it mean?
  - h. In this context,  $y$  is proportional to  $t^2$ . Explain what that means. How can you see this in the table?
7. Fiona is paid \$7 per hour in her part-time job at the local Dairy Stop. Let  $t$  be the amount time that she works, in hours, during the week, and let  $P(t)$  be her gross pay (before taxes), in dollars, for the week.
- a. Make a table showing how her gross pay depends upon the amount of time she works during the week.
  - b. Make a graph illustrating how her gross pay depends upon the amount of time that she works. Should you connect the dots? Explain.
  - c. Write a formula showing how her gross pay depends upon the amount of time that she works.
  - d. What is  $P(9)$ ? What does it mean? Explain how you can use the graph, the table, and the formula to compute  $P(9)$ .
  - e. If Towanda works 11 hours and 15 minutes, what will her gross pay be? Show how you know. Express the result using function notation.
  - f. If Towanda works 4 hours and 50 minutes, what will her gross pay be? Show how you know. Express the result using function notation.
  - g. One week Towanda's gross pay was \$42. How many hours did she work? Show how you know.
  - h. Another week Towanda's gross pay was \$57.19. How many hours did she work? Show how you know.

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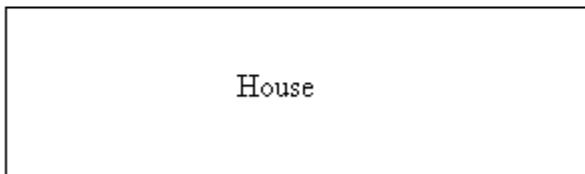
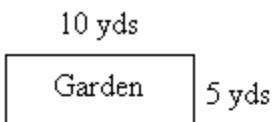
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### Fences and Functions Learning Task

- Claire wanted to plant a rectangular garden in her back yard using 30 pieces of fencing that were given to her by a friend. Each piece of fencing was a vinyl panel 1 yard wide and 6 feet high. Claire needed to determine the possible dimensions of her garden, assuming that she used all of the fencing and did not cut any of the panels. She began by placing ten panels (10 yards) parallel to the back side of her house and then calculated that the other dimension of her garden would be 5 yards, as shown in the diagram below.

Claire looked at the 10 fencing panels lying on the ground and decided that she wanted to consider other possibilities for the dimensions of the garden. In order to organize her thoughts, she let  $x$  be the garden dimension parallel to the back of her house, measured in yards, and let  $y$  be the other dimension, perpendicular to the back of the house, measured in yards. She recorded the first possibility for the dimensions of the garden as follows: When  $x = 10$ ,  $y = 5$ .



- Explain why  $y$  must be 5 when  $x$  is 10.
- Make a table showing the possibilities for  $x$  and  $y$ .
- Find the perimeter of each of the possible gardens you listed in part b. What do you notice? Explain why this happens.
- Did you consider  $x = 15$  in part b? If  $x = 15$ , what must  $y$  be? What would the garden look like if Claire chose  $x = 15$ ?
- Can  $x$  be 16? What is the maximum possible value for  $x$ ? Explain.
- Write a formula relating the  $y$ -dimension of the garden to the  $x$ -dimension.
- Make a graph of the possible dimensions of Claire's garden.
- What would it mean to connect the dots on your graph? Does connecting the dots make sense for this context? Explain.
- As the  $x$ -dimension of the garden increases by 1 yard, what happens to the  $y$ -dimension? Does it matter what  $x$ -value you start with? How do you see this in the graph? In the table? In your formula? What makes the dimensions change together in this way?

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2. After listing the possible rectangular dimensions of the garden, Claire realized that she needed to pay attention to the area of the garden, because area determines how many plants can be grown.
  - a. Does the area of the garden change as the  $x$ -dimension changes? Make a prediction, and explain your thinking.
  - b. Use grid paper to make accurate sketches for at least three possible gardens. How is the area of each garden represented on the grid paper?
  - c. Make a table listing all the possible  $x$ -dimensions for the garden and the corresponding areas. (To facilitate your calculations, you might want to include the  $y$ -dimensions in your table or add an area column to your previous table.)
  - d. Make a graph showing the relationship between the  $x$ -dimension and the area of the garden. Should you connect the dots? Explain.
  - e. Write a formula showing how to compute the area of the garden, given its  $x$ -dimension.
3. Because the area of Claire's garden depends upon the  $x$ -dimension, we can say that the area is a function of the  $x$ -dimension. Let's use  $G$  for the name of the function that uses each  $x$ -dimension an input value and gives the resulting area of the garden as the corresponding output value.
  - a. Use function notation to write the formula for the garden area function. What does  $G(11)$  mean? What is the value of  $G(11)$ ? What line of your table, from #2, part c, and what point on your graph, from #2, part d, illustrates this same information?
  - b. The set of all possible input values for a function is called the **domain** of the function. What is the domain of the garden area function  $G$ ? How is the question about domain related to the question about connecting the dots on the graph you drew for #2, part d?
  - c. The set of all possible output values is called the **range** of the function. What is the range of the garden area function  $G$ ? How can you see the range in your table? In your graph?
  - d. As the  $x$ -dimension of the garden increases by 1 yard, what happens to the garden area? Does it matter what  $x$ -dimension you start with? How do you see this in the graph? In the table? Explain what you notice.
  - e. What is the maximum value for the garden area, and what are the dimensions when the garden has this area? How do you see this in your table? In your graph?
  - f. What is the minimum value for the garden area, and what are the dimensions when the garden has this area? How do you see this in your table? In your graph?
  - g. In deciding how to lay out her garden, Claire made a table and graph similar to those you have made in this investigation. Her neighbor Javier noticed that her graph had symmetry. Your graph should also have symmetry. Describe this symmetry by indicating the line of symmetry. What about the context of the garden situation causes this symmetry?
  - h. After making her table and graph, Claire made a decision, put up the fence, and planted her garden. If it had been your garden, what dimensions would you have used and why?

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4. Later that summer, Claire's sister-in-law Kenya mentioned that she wanted to use 30 yards of chain-link fence to build a new pen for her two pet pot bellied pigs. Claire experienced déjà vu and shared how she had analyzed how to fence her garden. As Claire explained her analysis, Kenya realized that her fencing problem was very similar to Claire's.
- Does the formula that relates the  $y$ -dimension of Claire's garden to the  $x$ -dimension of her garden apply to the pen for Kenya's pet pigs? Why or why not?
  - Does the formula that relates the area of Claire's garden to its  $x$ -dimension apply to the pen for Kenya's pet pigs? Why or why not?
  - Write a formula showing how to compute the area of the pen for Kenya's pet pigs given the  $x$ -dimension for the pen.
  - Does the  $x$ -dimension of the pen for Kenya's pet pigs have to be a whole number? Explain.
  - Let  $P$  be the function which uses the  $x$ -dimension of the pen for Kenya's pet pigs as input and gives the area of the pen as output. Write the formula for  $P(x)$ .
  - Make a table of input and output values for the function  $P$ . Include some values of the  $x$ -dimension for the pen that could not be used as the  $x$ -dimension for Claire's garden.
  - Make a graph of the function  $P$ .
  - Does your graph show any points with  $x$ -values less than 1? Could Kenya have made a pen with  $\frac{1}{2}$  yard as the  $x$ -dimension? If so, what would the other dimension be? How about a pen with an  $x$ -dimension 0.1 of a yard? How big is a pot bellied pig? Would a pot bellied pig fit into either of these pens?
  - Is it mathematically possible to have a rectangle with  $x$ -dimension equal to  $\frac{1}{2}$  yard? How about a rectangle with  $x$ -dimension 0.1 yard?
  - Of course, Kenya would not build a pen for her pigs that did not give enough room for the pigs to turn around or pass by each other. However, in analyzing the function  $P$  to decide how to build the pen, Kenya found it useful to consider all the input values that could be the  $x$ -dimension of a rectangle. She knew that it didn't make sense to consider a negative number as the  $x$ -dimension for the pen for her pigs, but she asked herself if she could interpret an  $x$ -dimension of 0 in any meaningful way. She thought about the formula relating the  $y$ -dimension to the  $x$ -dimension and decided to include  $x = 0$ . What layout of fencing would correspond to the value  $x = 0$ ? What area would be included inside the fence? Why could the shape created by this fencing layout be called a "degenerate rectangle"?
  - The value  $x = 0$  is called a *limiting case*. Is there any other limiting case to consider in thinking about values for the  $x$ -dimension of the pen for Kenya's pigs? Explain.
1. Return to your graph of the function  $P$ . Adjust your graph to include all the values that could mathematically be the  $x$ -dimension of the rectangular pen even though some of these are not reasonable for fencing an area for pot bellied pigs.
- (Note: You can plot points corresponding to any limiting cases for the function using a small circle. To *include* the limiting case, *fill in the circle* to make a solid dot ●. To *not include* the limiting case, but just use it to show that the graph does not continue beyond that point, *leave the circle open* ○.)

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5. Use your table, graph, and formula for the function  $P$  to answer questions about the pen for Kenya's pet pigs.
  - a. Estimate the area of a pen of with an  $x$ -dimension of 10 feet (not yards). Explain your reasoning.
  - b. Estimate the  $x$ -dimension of a rectangle with an area of 30 square yards. Explain your reasoning.
  - c. What is the domain of the function  $P$ ? How do you see the domain in the graph?
  - d. What is the maximum area that the pen for Kenya's pigs could have? Explain. What do you notice about the shape of the pen?
  - e. What point on the graph corresponds to the ordered pair for the maximum area of the pen? What is unique about this point?
  - f. What is the range of the function  $P$ ? How can you see the range in the graph?
  - g. How should you move your finger along the  $x$ -axis (right-to-left or left-to-right) so that the  $x$ -value increases as you move your finger? If you move your finger along the graph of the function  $P$  in this same direction, do the  $x$ -values of the points still increase? As the  $x$ -dimension of the pen for Kenya's pigs increases, sometimes the area increases and sometimes the area decreases. Using your graph, determine the  $x$ -values such that the area increases as  $x$  increases. For what  $x$ -values does the area decrease as  $x$  increases?

When using tables and formulas we often look at a function a point or two at a time, but in high school mathematics, it is important to begin to think about "the whole function," which is to say all of the input-output pairs. We've started working on this idea already by using a single letter such as  $f$ ,  $G$ , or  $P$  to refer to the whole collection of input-output pairs.

We say that **two functions are equal** (as whole functions) if they have exactly the same input-output pairs. In other words, two functions are equal if they have the same domain and if the output values are the same *for each input value in the domain*. From a graphical perspective, two functions are equal if their graphs have exactly the same points. Note that the graph of a function consists of all the points which correspond to input-output pairs, but when we draw a graph we often can show only some of the points and indicate the rest. For example, if the graph of the function is a line, we show part of the line and use arrowheads to indicate that the line continues without end.

6. The possibilities for the pen for Kenya's pet pigs and for Claire's garden are very similar in some respects but different in others. These two situations involve different *functions*, even though the *formulas* are the same.
  - a. If Kenya makes the pen with maximum area, how much more area will the pen for her pet pigs have than Claire's garden of maximum area? How much area is that in square feet?
  - b. What could Claire have done to have built her garden with the same area as the maximum area for Kenya's pen? Do you think this would have been worthwhile?
  - c. Consider the situations that led to the functions  $G$  and  $P$  and review your tables, graphs, formulas related to the two functions. Describe the similarities and differences between Kenya's pig pen problem and Claire's garden problem. Your response should include a discussion of *domain* and *range* for the two functions.

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### From Wonderland to Functionland Learning Task

Consider the following passage from Lewis Carroll's *Alice's Adventures in Wonderland*, Chapter VII, "A Mad Tea Party."

"Then you should say what you mean." the March Hare went on.

"I do," Alice hastily replied; "at least -- at least I mean what I say -- that's the same thing, you know."

"Not the same thing a bit!" said the Hatter, "Why, you might just as well say that 'I see what I eat' is the same thing as 'I eat what I see'!"

"You might just as well say," added the March Hare, "that 'I like what I get' is the same thing as 'I get what I like'!"

"You might just as well say," added the Dormouse, who seemed to be talking in his sleep, "that 'I breathe when I sleep' is the same thing as 'I sleep when I breathe'!"

"It is the same thing with you," said the Hatter, and here the conversation dropped, and the party sat silent for a minute.

Lewis Carroll, the author of *Alice in Wonderland* and *Through the Looking Glass*, was a mathematics teacher who had fun playing around with logic. In this activity, you'll investigate some basic ideas from logic and perhaps have some fun too.

We need to start with some basic definitions.

A ***statement*** is a sentence that is either true or false, but not both.

A ***conditional statement*** is a statement that can be expressed in "if ... then" form.

A few examples should help clarify these definitions. The following sentences are statements.

- Atlanta is the capital of Georgia. (*This sentence is true.*)
- Jimmy Carter was the thirty-ninth president of the United States and was born in Plains, Georgia. (*This sentence is true.*)
- The Atlanta Falcons are a professional basketball team. (*This sentence is false.*)
- George Washington had eggs for breakfast on his fifteenth birthday. (*Although it is unlikely that we can find any source that allows us to determine whether this sentence is true or false, it still must either be true or false, and not both, so it is a statement.*)

Here are some sentences that are not statements.

- What's your favorite music video? (*This sentence is a question.*)
- Turn up the volume so I can hear this song. (*This sentence is a command.*)
- This sentence is false. (*This sentence is a very peculiar object called a self-referential sentence. It creates a logical puzzle that bothered logicians in the early twentieth century. If the sentence is true, then it is also false. If the sentence is false, then it is also true. Logicians finally resolved this puzzling issue by excluding such sentences from the definition of "statement" and requiring that statements must be either true or false, but not both.*)

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The last example discussed a sentence that puzzled logicians in the last century. The passage from *Alice in Wonderland* contains several sentences that may have puzzled you the first time you read them. The next part of this activity will allow you to analyze the passage while learning more about conditional statements.

Near the beginning of the passage, the Hatter responds to Alice that she might as well say that “I see what I eat” means the same thing as “I eat what I see.” Let’s express each of the Hatter’s example sentences in “if … then” form.

“I see what I eat” has the same meaning as the conditional statement “If I eat a thing, then I see it.” On the other hand, “I eat what I see” has the same meaning as the conditional statement “If I see a thing, then I eat it.”

1. Express each of the following statements from the Mad Tea Party in “if … then” form.
  - a. I like what I get. \_\_\_\_\_
  - b. I breathe when I sleep. \_\_\_\_\_
2. We use specific vocabulary to refer to the parts of a conditional statement written in “if … then” form. The ***hypothesis*** of a conditional statement is the statement that follows the word “if.” So, for the conditional statement “If I eat a thing, then I see it,” the hypothesis is the statement “I eat a thing.” Note that the hypothesis does **not** include the word “if” because the hypothesis is the statement that occurs after the “if.”

Give the hypothesis for each of the conditionals in 1a and 1b above.

3. The ***conclusion*** of a conditional statement is the statement that follows the word “then.” So, for the conditional statement “If I eat a thing, then I see it,” the conclusion is the statement “I see it.” Note that the conclusion does **not** include the word “then” because the conclusion is the statement that occurs after the word “then.”

Give the conclusion for each of the conditionals in 1a and 1b.

Now, let’s get back to the discussion at the Mad Tea Party. When we expressed the Hatter’s example conditional statements in “if … then” form, we used the pronoun “it” in the conclusion of each statement rather than repeat the word “thing.” Now, we want to compare the hypotheses (*note that the word “hypotheses” is the plural of the word “hypothesis”*) and conclusions of the Hatter’s conditionals. To help us see the key relationship between his two conditional statements, we replace the pronoun “it” with the noun “thing.” This replacement doesn’t change the meaning, and it does help us analyze the relationship between hypotheses and conclusions.

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4. a. List the hypothesis and conclusion for the revised version of each of the Hatter's conditional statements given below.

Hypothesis

Conclusion

If I eat a thing, then I see the thing. \_\_\_\_\_

If I see a thing, then I eat the thing. \_\_\_\_\_

- b. Explain how the Hatter's two conditional statements are related.

5. There is a term for the new statement you get by swapping the hypothesis and conclusion in a conditional statement. This new statement is called the **converse** of the first

- a. Write the **converse** of each of the conditional statements in 1a and 1b.

- b. What is the converse of each of the converses in 5a?

6. The March Hare, Hatter, and Dormouse did not use "if ... then" form when they stated their conditionals. Write the converse for each conditional statement below without using "if ... then" form.

Conditional: I breathe when I sleep.

Converse: \_\_\_\_\_

Conditional: I like what I get.

Converse: \_\_\_\_\_

Conditional: I see what I eat.

Converse: \_\_\_\_\_

Conditional: I say what I mean.

Converse: \_\_\_\_\_

7. a. What relationship between breathing and sleeping is expressed by the conditional statement "I breathe when I sleep"? If you make this statement, is it true or false?
- b. What is the relationship between breathing and sleeping expressed by the conditional statement "I sleep when I breathe"? If you make this statement, is it true or false?
8. The conversation in passage from *Alice in Wonderland* ends with the Hatter's response to the Dormouse "It is the same thing with you." The Hatter was making a joke. Do you get the joke? If you aren't sure, you may want to learn more about Lewis Carroll's characterization of the Dormouse in Chapter VII of *Alice in Wonderland*.

We want to come to some general conclusions about the logical relationship between a conditional statement and its converse. The next steps are to learn more of the vocabulary for discussing statements and to see more examples.

In English class, you learn about compound sentences. All of the sentences that we put in "if ... then" form in 1 – 4, as well as their "if ... then" forms of these sentences, are compound sentences. As far as English is concerned, what makes them "compound"?

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In logic, there are several specific ways to combine statements to create a ***compound statement***, or ***compound proposition***. Compound statements formed using “and” and “or” are important in the study of probability. In this task, as we focus on compound propositions that are conditional statements, we need a way to talk about the general idea of forming an “if ... then” statement no matter the particular statements we used for the hypothesis and conclusion. In order to do this, we need to use variables to represent statements as a whole. This use is demonstrated in the formal definition that follows.

**Definition:** If  $p$  and  $q$  are statements, then the statement “if  $p$ , then  $q$ ” is the ***conditional statement***, or ***implication***, with hypothesis  $p$  and conclusion  $q$ .

We call the variables used above, ***statement***, or ***propositional variables***. We seek a general conclusion about the logical relationship between a conditional statement and its converse that is true no matter what particular statements we substitute for the statement variables  $p$  and  $q$ . That’s why we need to see more examples.

Our earlier discussions of function notation, domain of a function, and range of a function have included conditional statements about inputs and outputs of a function. For the next part of this activity, we consider conditional statements about a particular function, the ***absolute value function***  $f$ , defined as follows:

$f$  is the function with domain all real numbers such that  $f(x) = |x|$ .

(Note. To give a complete definition of the absolute value function, we must specify the domain and a formula for obtaining the unique output for each input. It is not necessary to specify the range because the domain and the formula determine the set outputs.)

9. We’ll explore the graph of the absolute value function  $f$  and then consider some related conditional statements.

- a. Complete table of values given below.

$x$	0	$\frac{1}{2}$	$-\frac{1}{2}$	1	-1	3.8	-3.8	-5			
$f(x) =  x $	0	$\frac{1}{2}$	$\frac{1}{2}$					5	$\frac{15}{2}$	$\frac{15}{2}$	10

- b. Most graphing calculators have a standard graphing window which shows the portion of the graph of a function corresponding to  $x$ -values from -10 to 10 and  $y$ -values from -10 to 10. On grid paper, set up such a standard viewing window and then use the table of values above to draw the part of the graph of the function  $f$  for this viewing window. Does your graph show all of the input/output pairs listed in your table of values? Does your graph show the input/output pairs for those  $x$ -values from -10 to 10 that were not listed in the table? Explain.

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- c. Think of your graph as a picture. Do you see a familiar shape? Describe the graph to someone who cannot see it. If you extended your graph to include points corresponding to additional values of  $x$ , say to include  $x$ -values from  $-1000$  to  $1000$ , would the shape change? What can you do to your sketch to indicate information about the points outside of your “viewing window”?
- d. For what  $x$ -values shown on your graph of  $f$  does the  $y$ -value increase as  $x$  increases, that is, for what  $x$ -values is it true that, as you move your finger along your graph so that the  $x$ -values increase, the  $y$ -values also increase? (Be sure to give the  $x$ -values where this happens.) For what  $x$ -values does the  $y$ -value decrease as  $x$  increases, that is, for what  $x$ -values is it true that, as you move your finger along your graph so that the  $x$ -values increase, the  $y$ -values decrease? (Be sure to give the  $x$ -values where this happens.) Considering your answers to the questions in part c, for the whole function  $f$ , determine (i) those  $x$ -values such that the  $y$ -value increases as  $x$  increases and (ii) those  $x$ -values such that the  $y$ -value decreases as  $x$  increases.
10. Evaluate each of the following expressions written in function notation. Be sure to simplify so that there are no absolute value signs in your answers. Use your graph to verify that each of your statements is true.
- a.  $f(0) = \underline{\hspace{2cm}}$    b.  $f(-5) = \underline{\hspace{2cm}}$    c.  $f(-\pi) = \underline{\hspace{2cm}}$    d.  $f(\sqrt{3}) = \underline{\hspace{2cm}}$
11. The statements in 6, a – d, are written using the equals sign. The same ideas can be expressed with conditional statements. Fill in the blanks to form such equivalent statements.
- a. If  $x = 0$ , then  $f(x) = \underline{\hspace{2cm}}$ .   b. If  $x = -5$ , then  $f(x) = \underline{\hspace{2cm}}$ .
- c. If the input for the function  $f$  is  $-\pi$ , then the output for the function  $f$  is  $\underline{\hspace{2cm}}$ .
- d. If the input for the function  $f$  is  $\sqrt{3}$ , then the output for the function  $f$  is  $\underline{\hspace{2cm}}$ .

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12. Write the converse of each of the conditional statements in 7. For each converse, use your graph to determine whether the statement is true or false. In doing so, keep in mind that to say that a conditional is true means that, whenever the hypothesis is true, then the conclusion must also be true; and to say that a conditional is false means that it is possible for hypothesis to be true and yet have a false conclusion.

It may be helpful to organize your work in a table such as the one shown below.

Conditional statement	Truth value	Converse statement	Truth value
a. If $x = 0$ , then $f(x) = \underline{\hspace{2cm}}$ .	True		
b.	True		
c.	True		
d.	True		

13. As indicated in the table above, whether a statement is true or false is called the ***truth value*** of the statement. In this item, you are asked to decide whether there is a general relationship between the truth value of a conditional statement and the truth value of its converse. Any particular conditional statement can be true or false, so you need to consider examples for both cases. Remember that the converse of the converse is the original statement.

Complete the following sentences to make true statements. Explain your reasoning. Are your answer choices consistent with **all** of the examples of converse that you have seen in this task so far?

- a. The converse of a true conditional statement is \_\_\_\_\_.
- A) always also true                              B) always false
- C) sometimes true and sometimes false because whether the converse is true or false does not depend on whether the original statement is true or false.
- b. The converse of a false conditional statement is \_\_\_\_\_.
- A) always also false                              B) always true
- C) sometimes true and sometimes false because whether the converse is true or false does not depend on whether the original statement is true or false.

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14. Whenever we talk about conditional statements in general, without having a particular example in mind, it is useful to talk about the *propositional form* “if  $p$ , then  $q$ ” and the converse form “if  $q$ , then  $p$ .”

We want to determine whether there is any relationship between the truth values for the two forms that holds for all the possible cases of substituting particular statements for the propositional variables. If two propositional forms result in statements with the same truth value for *all possible cases* of substituting statements for the propositional variables, we say that the forms are *logically equivalent*. Two propositional forms are *not logically equivalent* if there exists some group of statements that can be substituted into the propositional forms so that the two statements corresponding to the two forms have different truth values.

- a. Consider your answers to 13, a and b, and decide how to complete the following statement to make it true. Justify your choice.

The propositional form “if  $p$ , then  $q$ ” is/ is not (*choose one*) logically equivalent to its converse “if  $q$ , then  $p$ .”

- b. If you learn a new mathematical fact in the form “if  $p$ , then  $q$ ”, what can you immediately conclude, without any additional information, about the truth value of the converse?

A) no conclusion because the converse is not logically equivalent

B) conclude that the converse is true

C) conclude that the converse is false

15. Look back at the opening of the passage from *Alice in Wonderland*, when Alice hastily replied “I do, at least -- at least I mean what I say -- that’s the same thing, you know.” What statements did Alice think were logically equivalent? What was the Hatter saying about the equivalence of these statements when he replied to Alice by saying “Not the same thing a bit!”?

There are two other statements related to any given conditional statement. We introduce these by exploring another inhabitant of the land of functions.

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## Student Edition

## Georgia Performance Standards Framework for Mathematics I

16. Let  $g$  be the function with domain all real numbers such that  $g(x) = |x| + 3$ .

- a. Complete table of values given below.

$x$	0	$\frac{1}{2}$	$-\frac{1}{2}$	1	-1	3.8	-3.8		-5			
$g(x) =  x  + 3$	3	$\frac{7}{2}$	$\frac{7}{2}$					8		$\frac{21}{2}$	$\frac{21}{2}$	13

- b. On grid paper, draw the portion of the graph of  $g$  for  $x$ -values such that  $-10 \leq x \leq 10$ . To show all of the points corresponding to input/output pairs shown in the table, how much of the  $y$ -axis should your viewing window include? Is the part of the graph that you have drawn representative of the whole graph? Explain.
- c. For what  $x$ -values shown on your graph of  $g$  does the  $y$ -value increase as  $x$  increases? For what  $x$ -values does the  $y$ -value decrease as  $x$  increases? For the whole function  $g$ , determine (i) those  $x$ -values such that the  $y$ -value increases as  $x$  increases and (ii) those  $x$ -values such that the  $y$ -value decreases as  $x$  increases.
- d. What is the relationship between the graphs of  $f$  and  $g$ ? What in the formulas for  $f(x)$  and  $g(x)$  tells you that the graphs should be related in this way?

17. It is clear from the graphs of  $f$  and  $g$  that, for each input value, the two functions have different output values. For just one example, we see that  $f(4) = 4$  but  $g(4) = 7$ . If we wanted to emphasize that  $g$  is not the absolute value function and that  $g(4)$  is different from 4, we could write  $g(4) \neq 4$ , which is read “ $g$  of 4 is not equal to 4.” We now consider making a conditional from such a statement of inequality.

- a. Complete each of following conditional statements to indicate that  $g(4) \neq 4$ .

If the input of the function  $g$  is 4, then the output of the function  $g$  is not \_\_\_\_.

If the output of the function  $g$  is 4, then the input of the function  $g$  \_\_\_\_.

- b. Let’s consider another true statement about the function  $g$  using “is not equal to”; for example,  $g(-3) \neq 0$ . Use your graph to verify that this is true and to evaluate  $g(-3)$ .

Let  $p$  represent the statement “The input of the function  $g$  is -3.”

Let  $q$  represent the statement “The output of the function  $g$  is not 0.”

What statement is represented by “if  $p$ , then  $q$ ”? Does this statement tell you that  $g(-3) \neq 0$ ?

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- c. In logic, we form the ***negation*** of a statement  $p$  by forming the statement “It is not true that  $p$ .” When we replace  $p$  by a particular statement in English, we can usually state the negation in a more direct way. For example:

(i) when  $p$  represents the statement “The input of the function  $g$  is  $-3$ ,”  
then “not  $p$ ” represents “The input of the function of the function  $g$  is not  $-3$ ” and

(ii) when, as above,  $q$  represents “The output of the function  $g$  is not  $0$ ,”  
then not  $q$  represents “The output of the function  $g$  is not not  $0$ ,” or more simply  
“The output of the function  $g$  is  $0$ .”

d. What statement is represented by “If not  $p$ , then not  $q$ .”? This statement is called the ***inverse*** of “if  $p$ , then  $q$ .”. Does it tell you that  $g(-3) \neq 0$ ? Is this inverse statement true?

e. What statement is represented by “If not  $q$ , then not  $p$ .”? This statement is called the ***contrapositive*** of “if  $p$ , then  $q$ .”. Does it tell you that  $g(-3) \neq 0$ ? Is this contrapositive statement true?

Summarizing the information about forming related conditional statements, we see that the conditional “if  $p$ , then  $q$ ,” has three related conditional statements:

**converse:** “if  $q$ , then  $p$ ” (swaps the hypothesis/conclusion)

**inverse:** “if not  $p$ , then not  $q$ ” (negates the hypothesis/conclusion)

**contrapositive:** “if not  $q$ , then not  $p$ ” (negates hypothesis/conclusion, then swaps)

The next part of this activity investigates inverse and contrapositive further while exploring another function named  $h$ .

# **Student Edition**

## Georgia Performance Standards Framework for Mathematics I

18. Let  $h$  be the function with domain all real numbers such that  $h(x) = 2|x|$ .
- On grid paper, draw the portion of the graph of  $h$  for  $x$ -values such that  $-10 \leq x \leq 10$ .
  - For  $x = -7, -2, 0, 5, 8$ , compare  $f(x)$  and  $h(x)$ . Does this relationship hold for every real number? How do the graphs of  $f$  and  $h$  compare? Explain why the graphs have this relationship.
  - The table below includes statements about the functions  $f$  and  $h$ . Fill in the blanks in the table. Be sure that your answers for the truth value columns agree with the graphs for  $f$  and  $h$ .
- | Conditional statement          | Truth value | Inverse statement                    | Truth value |
|--------------------------------|-------------|--------------------------------------|-------------|
| If $x = 3$ , then $f(x) = 3$ . | True        | If $x \neq 3$ , then $f(x) \neq 3$ . |             |
|                                |             | If $x = -3$ , then $f(x) = 3$ .      | True        |
| If $h(x) = 6$ , then $x = 3$ . | False       |                                      |             |
|                                |             | If $h(x) \neq 6$ , then $x = 6$ .    | False       |
- For the statements in the table that you classified as false, give a value of  $x$  that makes the hypothesis true and the conclusion false.
  - Consider the results in the table above, and then decide how to complete the following statement to make it true. Justify your choice.
- The propositional form “if  $p$ , then  $q$ ” is/ is not (*choose one*) logically equivalent to its inverse “if not  $p$ , then not  $q$ .”
- If you learn a new mathematical result in the form “if  $p$ , then  $q$ ”, what can you immediately conclude, without any additional information, about the truth value of the inverse?
    - no conclusion because the inverse is not logically equivalent
    - conclude that the inverse is true
    - conclude that the inverse is false

### Student Edition

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19. a. The table below includes statements about inequalities involving less than or greater than. Verify the negation that is shown, and then complete the table below.

Statement	Negation
$x > 0$	$x \leq 0$
	$x \geq 0$
$x < -7$	
	$x > 11$

- b. The next table involves conditional statements about the function  $h$ . Fill in the blanks in the table. In doing so, keep in mind that to say that a conditional is true means that, whenever the hypothesis is true, then the conclusion must also be true. To say that a conditional is false means that it is possible for hypothesis to be true and yet have a false conclusion. Be sure that your answers are consistent with your graph of  $h$ .

Conditional statement	Truth value	Contrapositive statement	Truth value
If $x > 3$ , then $h(x) > 3$ .	False	If $h(x) \leq 3$ , then _____.	
		If $x \neq 4$ , then $h(x) = 2$ .	
If $h(x) \neq 6$ , then $x \neq 3$ .	True		
		If $x < 3$ , then $h(x) < 6$ .	

- c. For the statements in the table that you classified as false, give a value of  $x$  that makes the hypothesis true and the conclusion false.
- d. On each line of the table, how do the truth value of the conditional statement and its contrapositive compare?
- e. Suppose that you are given that “if  $p$ , then  $q$ ” is a true statement for some particular choice of  $p$  and  $q$ . For example, suppose that there is a function  $k$  whose domain and range are all real numbers and it is true that, if the input to the function is 17, then the output is –86. What is the hypothesis of the contrapositive statement? What is the conclusion of the contrapositive statement? Given that  $k(17) = -86$ , whenever the hypothesis of the contrapositive is true, must the conclusion of the contrapositive also be true? Explain.
- f. Suppose that you are given that the contrapositive statement “if not  $q$ , then not  $p$ ” is a true statement for some particular choice of  $p$  and  $q$ . For example, suppose that there is a function  $k$  whose domain and range are all real numbers and it is true that, if the output of the function is not 101, then the input is not 34. What is the hypothesis of the original conditional statement? What is the conclusion of the original conditional statement? Given that if  $k(x) \neq 101$ , then  $x \neq 34$ , whenever the hypothesis of the original is true, must the conclusion of the original conditional also be true? Explain.

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## Student Edition

## Georgia Performance Standards Framework for Mathematics I

- g. Consider your answers to d, e, and f, and decide how to complete the following statement to make it true. Justify your choice.

The propositional form “if  $p$ , then  $q$ ” is/ is not (*choose one*) logically equivalent to its contrapositive “if not  $q$ , then not  $p$ . ”

- h. If you learn a new mathematical result in the form “if  $p$ , then  $q$ ,” what can you immediately conclude, without any additional information, about the truth value of the contrapositive?

- A) no conclusion because the contrapositive is not logically equivalent  
 B) conclude that the contrapositive is true      C) conclude that the contrapositive is false

The absolute value function  $f$  and the functions  $g$  and  $h$  that you have worked with in this investigation are not linear. However, in your study of functions prior to Mathematics I, you have worked with many linear functions. We conclude this investigation with discussion about converse, inverse, and contrapositive using a linear function.

20. Consider the linear function  $F$  which converts a temperature of  $c$  degrees Celsius to the equivalent temperature of  $F(c)$  degrees Fahrenheit. The formula is given by

$$F(c) = \frac{9}{5}c + 32, \text{ where } c \text{ is a temperature in degrees Celsius.}$$

- a. What is freezing cold in degrees Celsius? in degrees Fahrenheit? Verify that the formula for  $F$  converts correctly for freezing temperatures.
- b. What is boiling hot in degrees Celsius? in degrees Fahrenheit? Verify that the formula for  $F$  converts correctly for boiling hot temperatures.
- c. Draw the graph of  $F$  for values of  $c$  such that  $-100 \leq c \leq 400$ . What is the shape of the graph you drew? Is this the shape of the whole graph?
- d. Verify that “if  $c = 25$ , then  $F(c) = 77$ ” is true. What is the contrapositive of this statement? How do you know that the contrapositive is true without additional verification?
- e. What is the converse of “if  $c = 25$ , then  $F(c) = 77$ ”? How can you use the formula for  $F$  to verify that the converse is true? What is the contrapositive of the converse? How do you know that this last statement is true without additional verification?
- f. There is a statement that combines a statement and its converse; it’s called a ***biconditional***.

**Definition:** If  $p$  and  $q$  are statements, then the statement “ $p$  if and only if  $q$ ” is called a ***biconditional statement*** and is logically equivalent to “if  $q$ , then  $p$ ” **and** “if  $p$ , then  $q$ .”

Write three true biconditional statements about values of the function  $F$ .

## Student Edition

## Georgia Performance Standards Framework for Mathematics I

### Sequences as Functions Learning Task

In your previous study of functions, you have seen examples that begin with a problem situation. In some of these situations, you needed to extend or generalize a pattern. In the following activities, you will explore patterns as sequences and view sequences as functions.

A *sequence* is an ordered list of numbers, pictures, letters, geometric figures, or just about any object you like. Each number, figure, or object is called a *term* in the sequence. For convenience, the terms of sequences are often separated by commas. In Mathematics I, we will focus mostly on sequences of numbers and will often use geometric figures and diagrams as illustrations and contexts for investigating various number sequences.

1. Some sequences follow predictable patterns, though the pattern might not be immediately apparent. Other sequences have no pattern at all. Explain, when possible, patterns in the following sequences:
  - a. 5, 4, 3, 2, 1
  - b. 3, 5, 1, 2, 4
  - c. 2, 4, 3, 5, 1, 5, 1, 5, 1, 5, 1, 7
  - d. S, M, T, W, T, F, S
  - e. 31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31
  - f. 1, 2, 3, 4, 5, ..., 999, 1000
  - g. 1, -1, 1, -1, 1, -1, ...
  - h. 4, 7, 10, 13, 16, ...
  - i. 10, 100, 1000, 10000, 100000, ...

The first six sequences above are *finite sequences*, because they contain a finite number of terms. The last three are *infinite sequences* because they contain an infinite number of terms. The three dots, called *ellipses*, indicate that some of the terms are missing. Ellipses are necessary for infinite sequences, but ellipses are used for large finite sequences, too. The sixth example consists of the counting numbers from 1 to 1000; using ellipses allows us to indicate the sequence without having to write all of the one thousand numbers in it.

2. In looking for patterns in sequences it is useful to look for a pattern in how each term relates to the previous term. If there is a consistent pattern in how each term relates to the previous one, it is convenient to express this pattern using a *recursive definition* for the sequence that gives the first term and a formula for how the  $n^{\text{th}}$  term relates to the  $(n-1)^{\text{th}}$  term. For each sequence below, match the sequence with a recursive definition that correctly relates each term to the previous term, and then fill in the blank in the recursive definition.

- |   |   |
|---|---|
| a. 1, 2, 3, 4, 5, ...   | I. $t_1 = \underline{\hspace{2cm}}, t_n = t_{n-1} + 5$    |
| b. 5, 10, 15, 20, 25, ...   | II. $t_1 = \underline{\hspace{2cm}}, t_n = t_{n-1} + 2$   |
| c. 1.2, 11.3, 21.4, 31.5, 41.6, 51.7, ...   | III. $t_1 = \underline{\hspace{2cm}}, t_n = t_{n-1} + 1$  |
| d. $3 + \sqrt{2}, 5 + \sqrt{2}, 7 + \sqrt{2}, 9 + \sqrt{2}, 11 + \sqrt{2}, \dots$ | IV. $t_1 = \underline{\hspace{2cm}}, t_n = -2 t_{n-1}$    |
| e. -1, 2, -4, 8, -16, ...   | V. $t_1 = \underline{\hspace{2cm}}, t_n = t_{n-1} + 10.1$ |

### Student Edition

## Georgia Performance Standards Framework for Mathematics I

3. Some recursive definitions are more complex than those given in #2. A recursive definition can give two terms at the beginning of the sequence and then provide a formula the  $n$ th term that depends on the two preceding terms,  $n - 1$  and  $n - 2$ . It can give three terms at the beginning of the sequence and then provide a formula the  $n$ th term that depends on the three preceding terms,  $n - 1$ ,  $n - 2$ , and  $n - 3$ ; and so forth. The sequence of Fibonacci numbers, 1, 1, 2, 3, 5, 8, 13, 21, ..., is a well known sequence with such a recursive definition.

- What is the recursive definition for the Fibonacci sequence?
- The French mathematician, Edouard Lucas discovered a related sequence that has many interesting relationships to the Fibonacci sequence. The sequence is 2, 1, 3, 4, 7, 11, 18, 29, ..., and it is called the Lucas sequence. For important reasons studied in advanced mathematics, the definition of the Lucas numbers starts with the 0<sup>th</sup> term. Examine the sequence and complete the recursive definition below.

$$t_0 = 2, \quad t_1 = 1,$$

$$t_n = \underline{\hspace{2cm}}, \quad \text{for all integers } n = 2, 3, 4, \dots$$

- What is the 8<sup>th</sup> term of the Lucas sequence, that is, the term corresponding to  $n = 8$ ?
- What is the 15<sup>th</sup> term?

4. Recursive formulas for sequences have many advantages, but they have one disadvantage. If you need to know the 100<sup>th</sup> term, for example, you first need to find all the terms before it. An alternate way to define a sequence uses a ***closed form*** definition that indicates how to determine the  $n$ th term directly, without the need to calculate other terms. Match each sequence below with a definition in closed form. Verify that the closed form definitions you choose agree with the five terms given for each sequence.

- 5, 10, 15, 20, 25, ...
- 1, 0, 2, 0, 3, 0, 4, ...
- 5, 9, 13, 17, 21, 25, ...
- 1, 8, 27, 64, 125, ...

I.  $t_n = n^3$ , for  $n = 1, 2, 3, \dots$

II.  $t_n = 5n$ , for  $n = 1, 2, 3, \dots$

III.  $t_n = 4n + 1$ , for  $n = 1, 2, 3, \dots$

IV. 
$$t_n = \begin{cases} \frac{n+1}{2}, & \text{if } n = 1, 3, 5, \dots \\ 0, & \text{if } n = 2, 4, 6, \dots \end{cases}$$

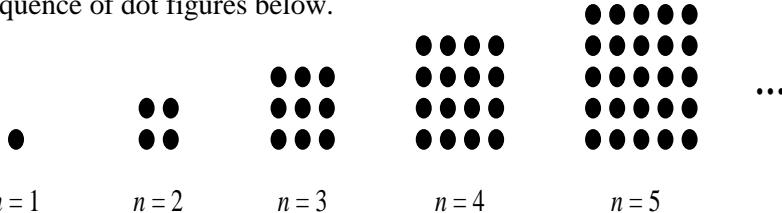
Sequences do not need to be specified by formulas; in fact, some sequences are impossible to specify with formulas. The two types of formulas that are commonly used are the ones used above: recursive and closed form. In #4, you probably noticed that a closed form definition for a sequence looks very much like a definition for a function. Whether or not you have a formula for the  $n$ -th term of a sequence, if you think of the index values as input values and the terms of the sequence as output values, then any sequence can be considered to be a function.

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## Student Edition

## Georgia Performance Standards Framework for Mathematics I

5. Consider the sequence of dot figures below.



- a. The sequence continues so that the number of dots in each figure is the next *square number*. Why are these numbers called square numbers?
  - b. What is the next figure in the sequence? What is the next square number?
  - c. What is the 25<sup>th</sup> square number? Explain.
  - d. Is 156 a square number? Explain.
  - e. Let  $g$  be the function that gives the number of dots in the  $n^{\text{th}}$  figure above. Write a formula for this function.
  - f. Make a table and a graph for the function  $g$ .
  - g. What is the domain of the function  $g$ ?
  - h. What is the range of the function  $g$ ?
6. Consider a function  $f$  that gives the area of a square of side-length  $s$ , shown below.
- a. What is the area of a square of side length 6?
  - b. Find a formula for the function  $f$ .
  - c. Make a table and a graph for the function  $f$ .
  - d. Use your formula to find the area of squares with the following side lengths: 7,  $1/2$ , 1.2,  $\sqrt{3}$ , and 200.
  - e. What is the domain of  $f$  (in the given context)?
  - f. What are the limiting case(s) in this context? Explain.
  - g. Extend the domain of  $f$  to include the limiting case(s).
7. Compare the functions  $g$  and  $f$  from #5 and #6.
- a. Explain how similarities and differences in the formulas, tables, and graphs arise from similarities and differences in the contexts.
  - b. Are these functions equal? Explain.



$s$

### Student Edition

**Georgia Performance Standards Framework for Mathematics I**

8. Look back at the finite sequences in 1, a and b. Consider the sequence in part a to be a function named  $h$  and the sequence in part b to be a function named  $k$ .
- Make a table and graph for function  $h$  and for function  $k$ .
  - How many points are included in each graph?
  - Determine the following:  
domain of  $f$  \_\_\_\_\_ range of  $f$  \_\_\_\_\_  
domain of  $k$  \_\_\_\_\_ range of  $k$  \_\_\_\_\_
  - Are these functions equal?
  - Write a formula for that gives  $h(n)$  for each number  $n$  in the domain of  $h$ .
  - Consider the linear function  $d$  which finds the difference of 10 and the real number  $x$ , that is the function given by the formula  
$$d(x) = 10 - x, \text{ for each real number } x.$$
Make a table and graph for the function  $d$ .
  - Compare the tables, graphs, and formulas for the functions  $h$  and  $d$ .

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**Student Edition**

## Georgia Performance Standards Framework for Mathematics I

### Learning Task 5: Walking, Falling, and Making Money

In previous mathematics courses, you studied the formula  $distance = rate \times time$ , which is usually abbreviated  $d = rt$ . If you and your family take a trip and spend 4 hours driving 200 miles, then you can substitute 200 for  $d$ , 4 for  $t$ , and solve the equation  $200 = r \cdot 4$  to find that  $r = 50$ . In this case we say that the average speed for the trip was 50 miles per hour. In this task, we develop the idea of *average rate of change* of a function, and see that it corresponds to average speed in certain situations.

- To begin a class discussion of speed, Dwain and Bryan want to stage a walking race down the school hallway. After some experimentation with a stop watch, and using the fact that the flooring tiles measure 1 foot by 1 foot, they decide that the distance of the race should be 40 feet and that they will need about 10 seconds to go 40 feet at a walking pace. They decide that the race should end in a tie, so that it will be exciting to watch, and finally they make a table showing how their positions will vary over time. Your job is to help Dwain and Bryan make sure that they know how they should walk in order to match their plans as closely as possible.

Time (sec.)	0	1	2	3	4	5	6	7	8	9	10
Dwain's position (ft.)	0	4	8	12	16	20	24	28	32	36	40
Bryan's position (ft.)	0	1	3	6	10	15	20	25	30	35	40

- Draw a graph for this data. Should you connect the dots? Explain.
- How can you tell that the race is supposed to end in a tie? Provide two explanations.
- Who is ahead 5 seconds into the race? Provide two explanations.
- Describe how Dwain should walk in order to match his data. In particular, should Dwain's speed be constant or changing? Explain how you know, using observations from both the graph and the table.
- Describe how Bryan should walk in order to match his data. In particular, should Bryan's speed be constant or changing? Explain how you know, using observations from both the graph and the table.
- In your answers above, sometimes you paid attention to the actual data in table. At other times, you looked at how the data was changing, which involved computing differences between values in the table. Give examples of each. How can you use the graph to distinguish between actual values of the data and differences between data values?
- Someone asks, "What is Bryan's speed during the race?" Kellee says that this question does not have a specific numeric answer. Explain what she means.
- Christie says that Bryan went 40 feet in 10 seconds, so Bryan's speed is 4 feet per second. But Kellee thinks that it would be better to say that Bryan's *average speed* is 4 feet per second. Is Christie's calculation sensible? What does Kellee mean?
- Sarah explains that to compute average speed over some time interval, you divide the distance during the time interval by the amount of time. Compute Dwain's average speed over several time intervals (e.g., from 2 to 5 seconds; from 3 to 8 seconds). What do you notice? Explain the result.
- What can you say about Bryan's speed during the first five seconds of the race? What about the last five seconds? Explain.

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### Student Edition

## Georgia Performance Standards Framework for Mathematics I

- k. Trey wants to race alongside Dwain and Bryan. He wants to travel at a constant speed during the first five seconds of the race so that he will be tied with Bryan after five seconds. At what speed should he walk? Explain how Trey's walking can provide an interpretation of Bryan's average speed during the first five seconds.

In describing relationships between two variables, it is often useful to talk about rates of change. When the rate of change is not constant, we often talk about average rates of change. If the variables are called  $x$  and  $y$ , and  $y$  is the dependent variable, then

$$\text{Average rate of change} = \frac{\text{the change in } y}{\text{the change in } x}.$$

When  $y$  is distance and  $x$  is time, the average rate of change can be interpreted as an average speed, as we have seen above.

2. Earlier you investigated the distance fallen by a ball dropped from a high place, such as the Tower of Pisa. In that problem,  $y$ , measured in meters, is the distance the ball has fallen and  $x$ , measured in seconds, is the time since the ball was dropped. You saw that  $y$  is a function of  $x$ , and the relationship can be approximated by the formula  $y = f(x) = 5x^2$ . You completed a table like the one below.

$x$	0	1	2	3	4	5	6	...
$x^2$	0	1	4	9				...
$y = f(x) = 5x^2$	0	5	20					...

- a. What is the average rate of change of the height of the ball during the first second after the ball is dropped? In what units is the average rate of change measured?
- b. What is the average rate of change of the height of the ball during the second second after the ball is dropped? after the third, fourth, and fifth? What do these answers say about the speed of the ball?
- c. What is the average rate of change over the first five seconds? Is this the same average speed as the average speed for the fifth second during which the ball is falling? Explain.
- d. What is the average rate of change over the first two seconds?
- e. What is the average rate of change from 2 to 5 seconds after the ball is dropped?
- f. What is the average rate of change from  $t = 1.5$  to  $t = 2$ ?
- g. Why is it important to use the phrase "average rate of change" or "average speed" for your calculations in this problem?

Average speed is a common application of the concept of average rate of change but certainly not the only one. There are many applications of analyzing the money a company can make from producing and selling products. The rest of this task explores average rate of change for functions related to the Vee Company and production and sale of a game called Zingo. The Vee Company is a small privately owned manufacturing company which sells exclusively to a national chain of toy stores. Zingo games are packaged and sold in crates holding 24 games each. Due to the size of the Vee Company work force, the maximum number of games per week that can be produced is 6000, which is enough to fill 250 crates.

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## Georgia Performance Standards Framework for Mathematics I

3. The table below shows data that the Vee Company has collected about the relationship between the wholesale price per game and the number of crates of Zingo that the toy store chain will order each week. In the business world, it generally happens that lowering the price of a product increases the number that will be bought; this holds true for the Zingo sales data. Also, it may seem a bit backwards, but in business analysis, price is usually expressed as a function of the number sold, as indicated in the table.

no. crates ordered per week, $x$	10	20	30	40	50	60	70	80	90	100
\$ price per Zingo game, $y = p(x)$	14.50	14.00	13.50	13.00	12.50	12.00	11.50	11.00	10.50	10.00

- When the number of crates ordered increases from 10 crates per week to 20 crates per week, is the change in price positive or negative? How much does the price change?
- Calculate the average rate of change for the price function as the number of crates ordered increases from 10 to 20 per week.
- What is the unit of measure for the average rate of change?
- Calculate the average rate of change for the increase from 20 to 40 crates ordered per week, for the increase from 40 to 70 crates ordered per week, and for the increase from 50 to 100 crates ordered per week.
- Is the average rate of change constant for all these increases in the number of crates ordered, or does it change depending on the initial number of crates involved?
- Based on your calculations of average rate of change, determine whether the relationship between  $x$  and  $p(x)$  is a linear relationship or a non-linear relationship. Explain.
- How much does the Vee Company need to change the price of a Zingo game to sell one more crate per week? Does your answer depend on how many crates are currently being sold? Explain.
- Write a formula to calculate  $p(x)$ .
- Could the function  $p$  be considered a finite sequence? Explain.
- Draw a graph of the function  $p$ .

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### Student Edition

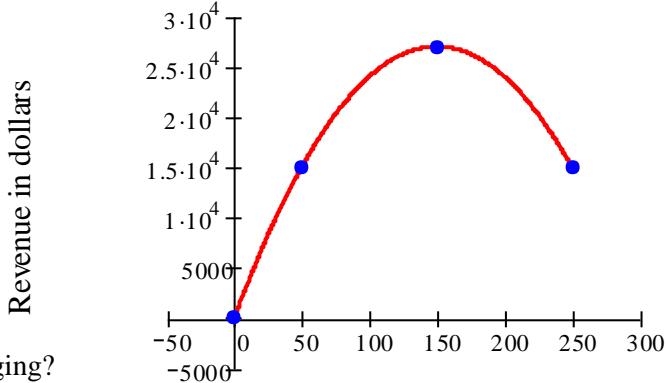
## Georgia Performance Standards Framework for Mathematics I

4. In business, the term *revenue* is used to indicate the money a company receives for sales of its products. In this context, revenue is a function of the number of the product made and sold. The Vee Company's financial analyst has determined that its revenue in dollars for production and sales of the Zingo game is given by the function  $R$  with the formula

$$R(x) = 360x - 1.20x^2,$$

where  $x$  is the number of crates sold, and  $R(x)$  is revenue measured in dollars.

- a. The graph of the function  $R$  is shown at the right.
- b. Is the relationship between  $x$  and  $R(x)$  a linear relationship or a non-linear relationship? Explain.
- c. Is the average rate change constant or changing?
- d. Using the formula for the price function in 3h, find the price per Zingo game when 150, 200, and 250 crates are sold. Find the price per crate when 150, 200, and 250 crates are sold. Do these values agree with the revenue function? Explain.
- e. Is the average rate of change for the increase from 50 to 150 crates per week sold positive or negative? Explain how to use the graph to find the answer without calculating the value.
- f. Is the average rate of change for the increase from 150 to 250 crates per week sold positive or negative? Explain how to use the graph to find the answer without calculating the value.
- g. What is the unit of measure for the average rate of change of function  $R$ ?
- h. Calculate the average rate of change for the increase from 50 to 150 crates per week sold to the toy store chain and for the increase from 150 to 250 crates per week. What is the relationship between the values? How is this shown in the graph?
- i. The average rate of change from 100 crates sold to 200 crates sold is 0. What feature of the graph causes this to be so? Explain and give other examples of the same phenomenon.



### Student Edition

## Georgia Performance Standards Framework for Mathematics I

### Southern Yard and Garden Learning Task

In this task, you'll investigate functions whose formulas involve the algebraic expressions  $\frac{1}{x}$  and  $\sqrt{x}$

by considering activities of Southern Yard and Garden, a Georgia-owned business that produces grass sod, garden plants, and trees for sale to nurseries and landscaping companies.

Last year, the research and development (R&D) team of the Grass Division at Southern Yard and Garden (SYnG – pronounced “sin gee”) was ready to plant test plots of several new drought resistant grasses that have been under development for the last six months. The R&D Team wanted each new grass variety planted at five different test plots at the experiment farm. Each plot would be the same area, 1200 square feet. In addition, to assure consideration of a variety of soil and sun/shade conditions, the plots could not be adjacent, and to prevent any grazing by wildlife in the area, each plot need to be surrounded by a special fence. When the plans were sent to the Chief Financial Officer (CFO) of SYnG for approval, she concluded that a separate fence for each grass test plot would be a major expense and asked the R&D Team how much fencing would be needed. The R&D Team sent a quick email that the length of fencing needed would depend on the perimeter of the plot, and the CFO replied with a request that they provide a detailed analysis of the possible perimeters for the test plots.

1. Before the CFO's request for an analysis of possible perimeters, the R&D Team had planned to use rectangular plots 24 feet wide and 50 feet long. They chose these dimensions because they would facilitate cutting of the sod for sale. All SYnG machinery is set to cut sod in 3' by 1' rectangular sections for sale to nurseries and landscaping companies.
  - a. Verify that a 24' by 50' plot has the required area.
  - b. How can such a plot be cut into 3' by 1' rectangular sections so that there are no leftover pieces of sod? How many of these sections will be produced in a 24' by 50' plot?
  - c. How much fencing is needed for a 24' by 50' grass test plot?
  
2. The Maintenance Division of SYnG is responsible for maintaining all experimental plots of plants, grass, and tree varieties. The R&D Team decided to consult the Maintenance Division as a first step in developing the analysis for the CFO. The mowing crew needs to mow grass test plots several times before the sod is established, and the gardeners monitor the watering and use of fertilizers and herbicides as directed by the R&D Team. So R&D asked the mowers and gardeners for input. After working with diagrams of mowing patterns for their 60-inch blade mowers, the mowers recommended that the plots be 25' by 48' and that the fences have gates in opposite corners so that they could enter in one corner, cut five 60-inch strips and exit at the opposite corner.
  - a. Use grid paper to represent a 25' by 48' rectangle and the mowers plans for cutting the grass.
  - b. Verify that a 25' by 48' rectangle has the required area and find its perimeter.

The gardeners suggested that the plots be 12' by 100' to simplify use of existing 12-foot wide equipment currently in use for watering and applying liquid fertilizers.

- c. Using the same scale from part a, represent a 12' by 100' rectangle on your grid paper.
- d. Find the perimeter of this rectangle.

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### Student Edition

## Georgia Performance Standards Framework for Mathematics I

3. The R&D Team at SYnG let  $w$  represent one dimension of a rectangle that has area 1200 square feet, let  $z$  represent the other dimension, and explored the relationship between the two variables.
  - a. Why is the relationship a functional relationship? Does it matter which dimension you consider to be the input and which to be the output? Explain.
  - b. Find a formula to write  $z$  as a function of  $x$ .
  - c. Use your formula to find the value of  $z$  when  $x = 5, 10, 15, 20, 30, 40$ . Organize your results in a table, and add the possible values for  $x$  and  $z$  from 1 and 2 above.
  - d. Because the 60-inch mowers must be able to cut the test plots, the smaller dimension of the rectangle must be at least 5'. In this context, what is the domain for the function from part a?
  - e. Draw a graph of the function from part using the same scale on each axis. The graph has a line of symmetry that is neither horizontal nor vertical. Where is the line?
  - f. There is a value of  $x$  such that  $x$  and  $z$  are the same. What special shape does the rectangle have when  $x = z$ ? What is the corresponding point on your graph?
  - g. For what  $x$ -values does the  $z$ -value increase as  $x$ -increases? For what  $x$ -values does the  $z$ -value decrease as  $x$ -increases? Explain your answer based on the graph and the formula.
4. The R&D Team at SYnG used their formula for  $z$  as a function of  $x$  to write a formula for the perimeter in terms of  $x$ . Let  $P$  be the perimeter function for this context.
  - a. Add a column for perimeter to your table from 3c above.
  - b. Using function notation, write a formula to calculate the perimeter for any choice of  $x$ -dimension of the rectangle.
  - c. What is the domain of  $P$ ? What is the range of  $P$ ?
  - d. Draw a graph of the function  $P$ . What is the line of symmetry for this graph?
  - e. What is the maximum value for the perimeter? What are the dimensions of any rectangle(s) that have the maximum perimeter?
  - f. What is the minimum value for the perimeter? What are the dimensions of any rectangle(s) that have the minimum perimeter?
5. The R&D Team provided the CFO with tables and graphs similar to those you have created in responding to 3 and 4 above. She estimates that the special fencing to surround the test plots will cost approximately \$15 per foot installed and that the gate for each fenced area will add \$200 to the cost.
  - a. Let  $C$  be the cost function where the input is the  $x$ -dimension of the rectangular plot and  $C(x)$  is the cost in dollars for fencing a 1200-square-foot plot with the given  $x$ -dimension. Using the CFO's cost estimates, write a formula for  $C(x)$ .
  - b. Draw a graph of the function  $C$ . What is the line of symmetry for this graph?
  - c. What is the domain of  $C$ ? What is the range of  $C$ ?
  - d. What is the maximum value for the cost? What are the dimensions of any rectangle(s) that have the maximum cost?
  - e. What is the minimum value for the cost? What are the dimensions of any rectangle(s) that have the minimum cost?
  - f. Redraw the graph of the perimeter function,  $P$ , from 4 above, on the same axes as the graph for  $C$ . How do the graphs compare?
  - g. Compare the formulas for the functions  $P$  and  $C$ . How do the relationships in the formulas show in the graphs?

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## Student Edition

## Georgia Performance Standards Framework for Mathematics I

The R&D Team at SYnG is also trying to develop new hardier and more drought tolerant varieties of flowering plants for use in landscaping. The experimental plants are studied for one growing season and are set in square raised beds with one plant for every square foot of area within the beds. Previously, the number of plats had been restricted to those sizes which allowed for a square arrangement of the plants, similar to the pattern of dots considered in the task “Sequences as Functions”, part 5. Last summer, the Maintenance Division, which is responsible for building the raised beds as well as maintaining them, asked for permission to experiment with a variety of sizes for the beds. They pointed out that as long as the number of plants was the same as the number of square feet enclosed by the raised box, they could find an arrangement that gave each plant a square foot of area, although that area for each plant would not necessarily have a square shape.

6. The R&D Team agreed to allow the Maintenance Division to experiment with the size of the raised beds. Pleased with the chance to experiment, the Maintenance Division decided to build beds for every number of plants from 4 to 400.
  - a. The length, in feet, of a side of a raised bed is a function of the number of plants that Maintenance will plant in the bed. Name this function  $s$ . If  $n$  represents the number of plants in a raised bed, then  $s(n) = \sqrt{n}$ .
  - b. What domain of values does Maintenance plan to use for the function  $s$ ?
  - c. Let  $f$  be the function defined for all real numbers  $x \geq 0$  by the formula

$$f(x) = \sqrt{x}.$$

Make a table of values for  $f$  and draw a graph that includes domain values for  $0 \leq x \leq 500$  and indicates the shape of the whole graph.

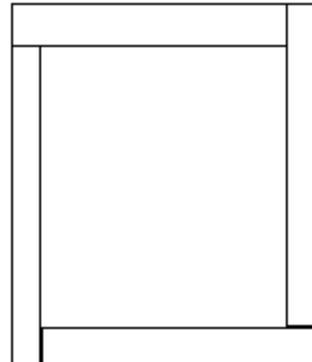
- d. The functions  $f$  and  $s$  both take the square root of the input to find the output. Are their domains the same? Are they equal functions? Explain.
- e. How can you use the graph of  $f$  to obtain the graph of  $s$ ?
- f. The function  $s$  could be considered as a sequence. Do you think it is helpful to think of the function this way? Why or why not?

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## Student Edition

## Georgia Performance Standards Framework for Mathematics I

7. Maintenance builds the raised beds using recycled railroad cross ties. They arrange the cross ties as shown in the figure at the right.
- The end of each cross tie is square approximately nine inches on a side. What is the length of each cross tie, with length measured in feet, needed to make a box for 4 plants?
  - Answer the question from part a for 9 plants, 16 plants, and 23 plants.
  - The Maintenance Division needs a cross-tie length function that inputs the number of plants,  $n$ , and outputs the length  $L$  for each cross tie needed to build the raised bed. Write the formula they need to calculate  $L$  given  $n$ .
  - How can you use the graphs of  $f$  and  $s$  from 6 above to draw the graph of the cross-tie length function?
  - Draw a plan for a raised bed for 24 plants. Specify the lengths of the cross-ties and how you would arrange the plants. Explain how you know that your arrangement gives each plant one square foot of growing area.



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## Student Edition

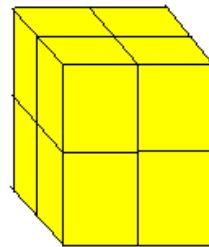
## Georgia Performance Standards Framework for Mathematics I

### **Painted Cubes\* Learning Task**

The Vee Company, which produces the Zingo game, is working on a new product: a puzzle invented by one of its employees. The employee, Martin, made a large cube from 1,000 smaller cubes, each having edge length one centimeter, by using temporary adhesive to hold the small cubes together. He painted the faces of the large cube, but when the paint had dried, he separated the large cube into the original 1000 centimeter cubes. The object of his puzzle is to put the cube back together so that no unpainted faces are showing.

The manager responsible for developing Martin's puzzle into a Vee Company product thought that 1000 cube puzzle might have too many pieces and decided that he should investigate the puzzle starting with smaller versions.

1. The cube at the right is made of smaller unpainted cubes, each having edge length 1 centimeter. All the faces of the large cube are painted yellow.
  - a. How many small cubes were used to make the large cube?
  - b. If you could separate the large cube into the original centimeter cubes, how many cubes would be painted on
    - i. three faces?
    - ii. two faces?
    - iii. one face?
    - iv. no faces?
  
2. Consider large cubes with edge lengths of 3, 4, 5, 6, and 7 centimeters by building and/or sketching models, and answer the same questions that you answered for the large cube of edge length 2. Organize all your answers in a table as shown below.



Edge length of large cube	Number of small centimeter cubes	Number of small centimeter cubes painted on			
		3 faces	2 faces	1 face	0 faces
2					
3					
4					
5					
6					
7					

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### Student Edition

## Georgia Performance Standards Framework for Mathematics I

3. Study the pattern in the table when the edge length of the large cube is used as input and the output is the number of small centimeter cubes used in constructing the large cube.
  - a. Denote this functional relationship  $N$ , for the number of small cubes, and let  $x$  represent the edge length of the large cube. Write an equation expressing  $N(x)$  in terms of  $x$ .
  - b. What is the domain of the function  $N$ , given that the input  $x$  is the edge length of a large cube made from small centimeter cubes?
  - c. Let  $f$  be the function such that the formula for  $f(x)$  is the same as the formula for  $N(x)$  but the domain is all real numbers. Make a table for values of  $f$ . Include the values from the function  $N$  and include values of  $x$  not in the domain of  $N$ , especially some negative numbers and fractions.
  - d. Sketch the graphs of  $f$  and  $N$  on the same axes. How does the graph of  $f$  help you understand the graph of  $N$ ? Explain.
4. Study the pattern in the table when the edge length of the large cube is used as input and the output is the number of small centimeter cubes that have 0 faces painted.
  - a. Denote this functional relationship  $U$ , for unpainted small cubes, and again let  $x$  represent the edge length of the large cube. Write an equation expressing  $U(x)$  in terms of  $x$ .
  - b. What is the domain of the function  $U$ , given that the input  $x$  is the edge length of a large cube made from small centimeter cubes?
  - c. Let  $g$  be the function such that the formula for  $g(x)$  is the same as the formula for  $U(x)$  but the domain is all real numbers. Make a table for values of  $g$ . Include the values from the function  $U$  and include values of  $x$  not in the domain of  $U$ , especially some negative numbers and fractions.
  - d. Sketch the graphs of  $g$  and  $U$  on the same axes. How does the graph of  $g$  help you understand the graph of  $U$ ? Explain.
5. Consider the context of the functions,  $N$  and  $U$ .
  - a. For any edge length  $x$  for the large cube, geometrically, what do the numbers  $N(x)$  and  $U(x)$  represent?
  - b. Do your formulas for the functions  $N$  and  $U$  show this relationship?
  - c. How does this relationship show up in the graphs? Explain the relationship in the graphs in terms of inputs and outputs to the two functions.
  - d. Consider the graphs of  $f$  and  $g$ . Are they related in the same way that the graphs of  $N$  and  $U$  are related?
6. Use the data in the table above to make a table where edge length of the large cube is used as input and the output is the number of small centimeter cubes that have 1 painted face.
  - a. Give a geometric explanation for why all of the output numbers are multiples of 6.
  - b. Denote this functional relationship  $S$ , where  $S$  represents the number *single* face painted, and again let  $x$  represent the edge length of the large cube. Write an equation expressing  $S(x)$  in terms of  $x$ .
  - c. What is the domain of the function  $S$ , given that the input  $x$  is the edge length of a large cube made from small centimeter cubes?
  - d. Let  $h$  be the function such that the formula for  $h(x)$  is the same as the formula for  $S(x)$  but the domain is all real numbers. Make a table for values of  $h$ . Include the values from the function  $S$  and include values of  $x$  not in the domain of  $S$ , especially some negative numbers and fractions.
  - e. Sketch the graphs of  $h$  and  $S$  on the same axes. How does the graph of  $h$  help you understand the graph of  $S$ ? Explain.

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## Student Edition

## Georgia Performance Standards Framework for Mathematics I

7. You have seen functions similar to the function  $h$  before. In an earlier learning task, you considered the function for the distance  $y$  in meters that a ball dropped from a high place falls in  $x$  seconds where  $y = 5x^2$ . In another learning task, you considered the function  $A$  for the area of a square as a function of the length of a side,  $s$ , where  $A(s) = s^2$ .

Consider the related functions whose relationships are specified by the equations below and which each have domain the set of all real numbers.

(i)  $y = x^2$               (ii)  $y = 5x^2$               (iii)  $y = 6x^2$

Each of these equations has the form  $y = ax^2$ , where  $a$  is a constant real number. In each of these situations, we say that  $y$  **varies directly as the square of  $x$** .

- a. On the same axes, sketch the graphs, with domain all real numbers, for equations (i) – (iii) above. What is the value of  $a$  for each of these equations. How does the value of  $a$  affect the graphs?
  - b. On the same axes, sketch the graphs of  $y = x^2$  and  $y = -x^2$ . What is the effect of changing the sign of  $a$ ? Graph  $y = 5x^2$  and  $y = -5x^2$  on the same axes. Is the effect the same?
  - c. Choose two values for  $a$  between 0 and 1, and graph  $y = ax^2$  for these two values and  $y = x^2$  on the same axes. Does the value of  $a$  affect these graphs in the same way that you described in part a above?
8. Typical braking distances,  $d$ , of an automobile for a given speed  $s$  are given in the following table. [Note that braking distance is different from stopping distance since there is a reaction time between the time when a driver decides to brake and the time when the brakes are actually applied.]

Speed (miles per hour)	$s$	10	20	30	40	50	60	70
Braking distance (feet)	$d$	5	20	45	80			

- a. The braking distance for an automobile varies directly as the square of the speed. What is the general form of the equation relating  $d$  and  $s$ ? Use the data in the table to find the specific formula for  $d$  as a function of  $s$ .
- b. Fill in the missing value in the table.
- c. Describe the graph of this function. What is the domain? How does a 10 MPH increase in speed change the braking distance?

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### Student Edition

## Georgia Performance Standards Framework for Mathematics I

9. Use the data in the table from #2 to make a table where edge length of the large cube is used as input and the output is the number of small centimeter cubes that have 2 faces painted.
- Give a geometric explanation for why all of the output numbers are multiples of 12.
  - Denote this functional relationship  $T$ , for *two* faces painted, and again let  $x$  represent the edge length of the large cube. Write an equation expressing  $T(x)$  in terms of  $x$ .
  - What is the domain of the function  $S$ , given that the input  $x$  is the edge length of a large cube made from small centimeter cubes?
  - Let  $k$  be the function such that the formula for  $k(x)$  is the same as the formula for  $T(x)$  but the domain is all real numbers.
  - Sketch the graphs of  $k$  and  $T$  on the same axes. What kind of functional relationship does each of these functions possess?
10. Use the data in the table from #2 to make a table where edge length of the large cube is used as input and the output is the number of small centimeter cubes that have 3 faces painted.
- Give a geometric explanation for why all of the output numbers are the same.
  - Denote this functional relationship  $H$ , for *half* of the faces painted, and again let  $x$  represent the edge length of the large cube. Write the equation for  $H(x)$ .
  - What is the domain of the function  $H$ , given that the input  $x$  is the edge length of a large cube made from small centimeter cubes?
  - Let  $j$  be the function such that the formula for  $j(x)$  is the same as the formula for  $H(x)$  but the domain is all real numbers.
  - Sketch the graphs of  $j$  and  $H$  on the same axes. Why are these called *constant* functions?

\* Adapted from the “Painted Cubes” section of *Frogs, Fleas, and Painted Cubes: Quadratic Relationships* in the *Connected Mathematics 2* series, Pearson Prentice Hall.

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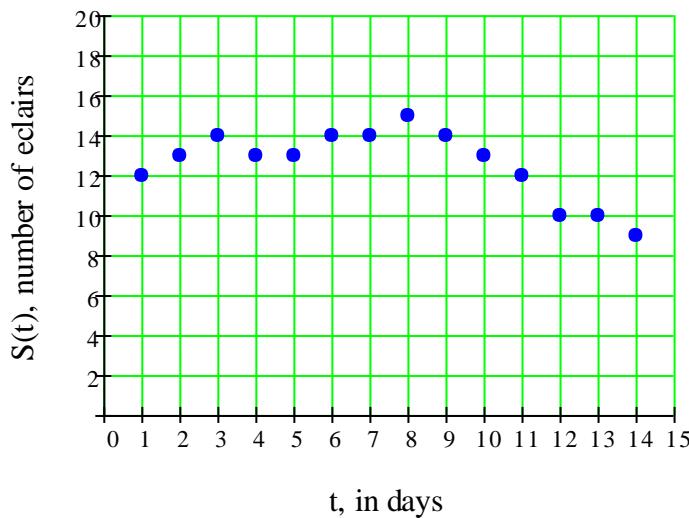
## Student Edition

## Georgia Performance Standards Framework for Mathematics I

### Ye Old Village Shoppes Culminating Task

Major cities around the world have areas of historical interest that attract local visitors and tourists. Nearby there is often an area of shops specializing in gift items and local products. In one major U.S. city, there is a mall of specialty stores housed in a large warehouse built in the 1800's. The mall also includes two restaurants, a deli, and a coffee shop.

- The coffee shop sells pastries and baked goods in addition to a wide selection of coffees and coffee-based drinks. The owner, Mr. Nguyen, regularly evaluates sales of the baked goods in order to discontinue items that do not sell well. The graph below shows daily sales of chocolate éclairs for the first two weeks of this past June.



- What are the independent and dependent variables shown in the graph?
- For which dates does the graph provide data?
- Does it make sense to connect the dots in the graph? Explain.
- Find  $S(10)$ , if possible, and explain what it means or would mean.
- Find  $S(15)$ , if possible, and explain what it means or would mean.
- If possible, find a  $t$  such that  $S(t) = 15$ . Explain the meaning.
- Find all values of  $t$  such that  $S(t) = 12$ . Explain the meaning.
- Describe what happens to  $S(t)$  as  $t$  increases, beginning at  $t = 1$ .
- Find the domain and range of the function  $S$ .

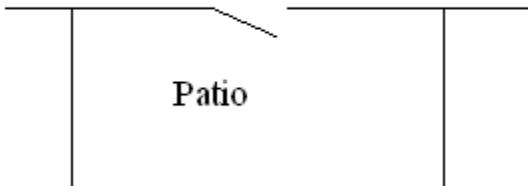
### Student Edition

## Georgia Performance Standards Framework for Mathematics I

2. Mr. Nguyen is planning to add an outdoor seating patio that will be accessible from the coffee shop when he reopens an existing door to the outside. His plans call for a 42-inch brick wall surrounding the patio on three sides with the wall of the building enclosing the fourth side, as shown in the diagram below.

**Inside the Coffee Shop**

Mr. Nguyen has found a supplier who can provide used brick that match the outside wall of the shop reasonably well and, with the help of a local antique dealer, has found some molding from the 1800's to trim the top of the wall. However, there is only enough molding to cover 80ft of brick wall.



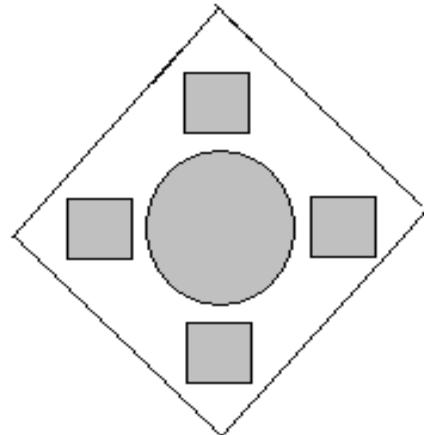
- a. Let  $x$  represent the dimension of the patio parallel to the outside wall of the coffee shop and let  $y$  represent the other dimension of the patio. Make a table and draw a graph to express  $y$  as a function of  $x$ .
- b. What domain for  $x$ -values is shown in your graph? Explain.
- c. Write a formula to express  $y$  as a function of  $x$ .
- d. Explain features of your table, your graph, and your formula that show that  $y$  is a linear function of  $x$ .
- e. Mr. Nguyen wants to have at least ten outdoor tables with umbrellas on the patio. The website for this brand of outdoor furniture recommends 64 square feet for each table and chair set. How many square feet of patio area are required for ten tables? Make a table showing some possible values for  $x$  and for the corresponding area of the patio,  $A(x)$ .
- f. Use the values in your table to draw a graph of the function  $A$ . What domain for  $x$ -values is appropriate for this context? What are the limiting values of the domain? How are these shown on the graph?
- g. Describe the line symmetry of your graph. What about the context causes this symmetry?
- h. For what  $x$ -values does the value of  $A(x)$  increase as  $x$  increases? For what  $x$ -values does the value of  $A(x)$  decrease as  $x$  increases?
- i. By examining your graph, determine whether it is possible to make a patio that has room for the ten outdoor table and chair sets that Mr. Nguyen would like to place there. Explain your reasoning.
- j. How would you advise Mr. Nguyen to build the patio and how many of the outdoor table and chair sets should he order? Explain your reasoning.

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## Student Edition

## Georgia Performance Standards Framework for Mathematics I

3. One day, when business in the coffee shop was slow, Mr. Nguyen was looking at the outdoor furniture website and began to think in more detail about the table and chair sets. The website included a diagram, similar to the one at the right, to illustrate the area requirements for each table and chair set. He noticed that the company offered a variety of outdoor dining sets with a range of square foot needs and began to muse about the relationship between the area required by a table and chair set and the length of the line needed to outline this square area.
- For the coffee shop patio, Mr. Nguyen wants to order table and chair sets that require 64 square feet of floor space. What is the length of the side of a square which has area 64 square feet? What is the perimeter of this square?
  - What is the perimeter of a square which has area 100 square feet?
  - Consider the function  $P$  where the independent variable is  $A$ , the area of a square measured in square feet, and  $P(A)$  is the perimeter of that square, measured in feet. Make a table of values for the function  $P$ , and include the areas from the outdoor furniture website: 49, 64, 81, 100, 144. Also include these values for  $A$ : 63, 75, 96.
  - Find a formula for  $P(A)$ .
  - Assuming a domain of all real number values of  $A$  such that  $49 \leq A \leq 144$ , graph the function  $P$ . How does this graph compare to the graph of the square root function?



Many of the stores in Ye Old Village Shoppes sell unique handmade items, and their customers often want their purchases gift wrapped or packaged securely for the journey home. Fortunately, there is also a shop just for wrapping, packing, and shipping; it is called Pack, Wrap, Etc.

One spring day a customer came into Pack, Wrap, Etc., to have a silk scarf gift wrapped. The scarf had a colorful geometric design, and the customer wanted wrapping paper with a similar theme. None of the wrapping paper on hand was suitable, so Juanita, the shop manager, used her computer, color printer, and some graphing software to create paper to satisfy the customer. Juanita used yellow paper and a combination of red, blue, and black inks. Use the instructions below to create your own smaller, handmade version of the paper Juanita designed.

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## Student Edition

## Georgia Performance Standards Framework for Mathematics I

4. On grid paper with quarter-inch grids, set up coordinate axes with a graphing window for  $-6 \leq x \leq 6$  and  $-6 \leq y \leq 6$  using one-inch for each unit. Graph all of the following functions in this one graphing window, using the colors indicated.

- |  |  |
|--|--|
| a. Function $f_1$ : $y =  x $ , red                        | i. Function $f_9$ : $y = - x $ , blue                      |
| b. Function $f_2$ : $y =  x  + 1.75$ , black               | j. Function $f_{10}$ : $y = - x  - 1.25$ , red             |
| c. Function $f_3$ : $y =  x  + 3$ , blue                   | k. Function $f_{11}$ : $y = - x  - 2.25$ , black           |
| d. Function $f_4$ : $y =  x  + 3.75$ , red                 | l. Function $f_{12}$ : $y = - x  - 3$ , blue               |
| e. Function $f_5$ : $y =  x  + 4.5$ , black                | m. Function $f_{13}$ : $y = - x  - 3.5$ , red              |
| f. Function $f_6$ : $y = \frac{2.25}{x}$ , $x > 0$ , blue  | n. Function $f_{14}$ : $y = - x  - 4.25$ , black           |
| g. Function $f_7$ : $y = \frac{2.25}{x}$ , $x < 0$ , red   | o. Function $f_{15}$ : $y = \frac{16}{x}$ , $x > 0$ , blue |
| h. Function $f_8$ : $y = \frac{4}{x}$ , $x \neq 0$ , black | p. Function $f_{16}$ : $y = \frac{16}{x}$ , $x < 0$ , red  |

Note: To get the best replica of Juanita's design, create your graph on grid paper and then trace it on plain paper without copying the coordinate axes. Graph functions  $f_1$  and  $f_9$  with a heavy line, approximately  $\frac{1}{16}$  inch wide, and use successively thinner lines as you graph functions  $f_2$  through  $f_5$  and  $f_{10}$  through  $f_{14}$ . Use medium weight lines for  $f_6$ ,  $f_7$ ,  $f_8$ ,  $f_{15}$ , and  $f_{16}$ .

5. Design your own wrapping paper and write instructions similar to those in #3. Exchange instructions with a classmate and reproduce each other's designs.

One of the Old Village Shoppes sells handmade jewelry. They stock only one size of jewelry gift box and provide customers with a box for each jewelry item purchased. On a particular summer day, the jewelry shop had several customers who each purchased multiple items, and all of these shoppers arrived at Pack, Wrap, Etc. simultaneously to have their jewelry purchases gift wrapped. For the next half hour, Jen and Brad, employees of Pack, Wrap, Etc., were busy wrapping the items.

6. Juanita, the manager, observed Brad as he worked. Several times during the half hour, she noted how many packages he had completed; the data is shown in the table below.

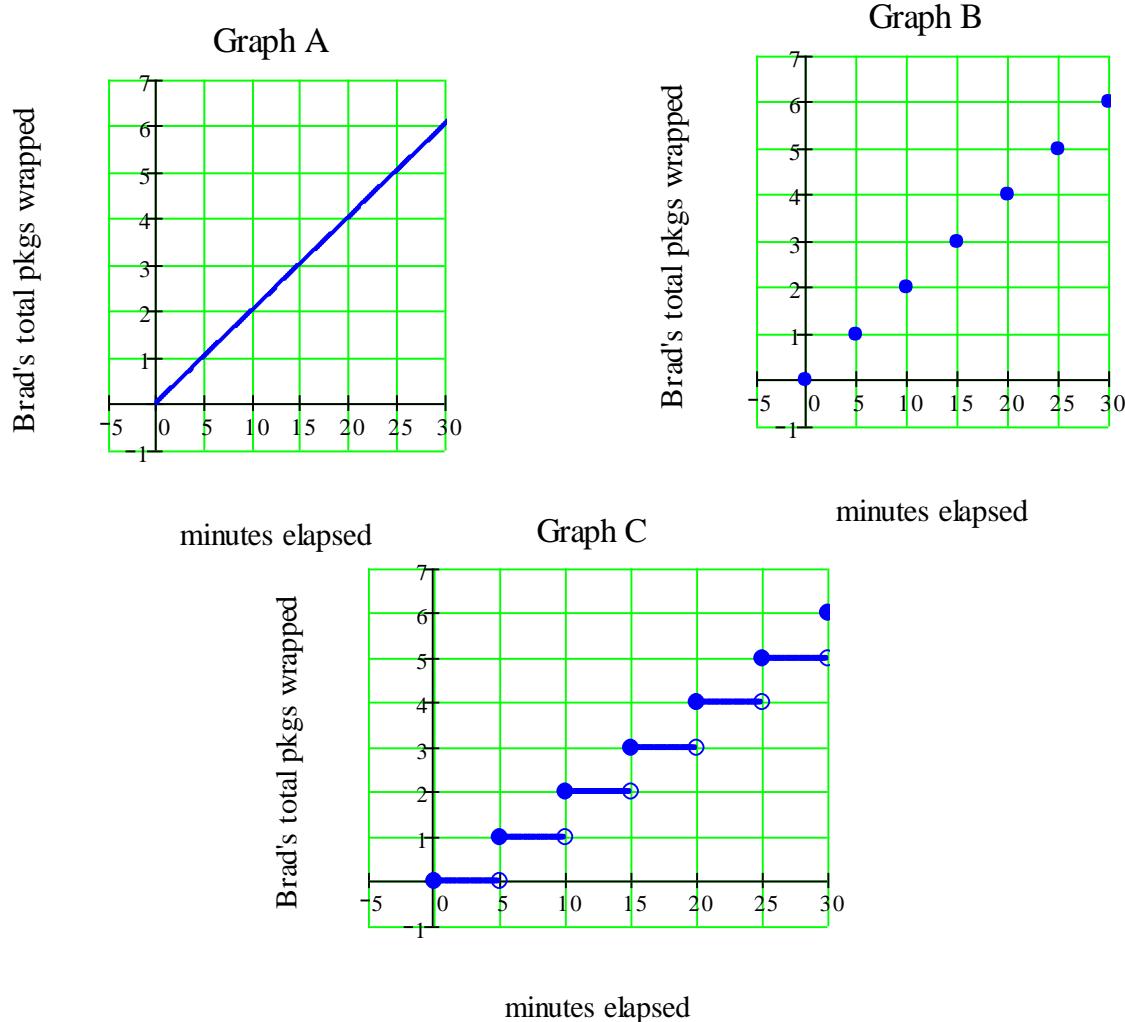
minutes elapsed, $m$	0	10	15	20	30
Brad's total number of boxes wrapped, $T(m)$	0	2	3	4	6

- a. For the first 10 minutes that Brad worked, what is the average rate of change of his total number of packages wrapped? Explain the meaning of this number.
- b. What is average rate of change of Brad's total number of packages wrapped for the last 15 minutes he worked? Is the rate of change of Brad's total number of packages wrapped constant or changing? Based on the data, how would you describe the way Brad worked?

## Student Edition

## Georgia Performance Standards Framework for Mathematics I

- c. While Brad is wrapping a package, it is difficult to determine exactly when he has one-fourth of the job completed or one-half of the job done, or, for that matter, any other fractional part of the work done. So, if we use a function to describe the relationship between minutes elapsed since Brad started wrapping and the number of packages Brad has wrapped in the time elapsed, it is reasonable just to count the whole number of packages wrapped at any point in time. Consider Graphs A, B, and C below, and decide which one could be the graph of the function  $T$ , where  $T(m)$  is the whole number of packages completed  $m$  minutes after Brad began wrapping. Explain.



## Student Edition

## Georgia Performance Standards Framework for Mathematics I

7. Juanita also observed Jen as she worked. The data for Juanita's observations of the total number of boxes that Jen wrapped during the half hour are given in the next table.

minutes elapsed, $m$	0	10	15	20	30
Jen's total number of boxes wrapped, $J(m)$	0	3	5	5	8

- a. For the first 10 minutes that Jen worked, what is the average rate of change of her total number of packages wrapped? Explain the meaning of this number.
- b. What is the average rate of change of Jen's total number of packages wrapped for the last 10 minutes she worked? for the last 15 minutes?
- c. Is the rate of change of Jen's total number of packages wrapped constant or changing? Based on the data, how would you describe the way Jen worked?
- d. For this context, what is the domain of the function  $J$ ?
- e. For this context, what is the range of the function  $J$ ?

One of the stores in Ye Old Village Shoppes sells hand-blown glass creations. Their holiday ornaments are especially popular. Each is of these is unique glass sphere, but the glass blowers consistently create orbs with diameters of approximately 3 inches. When an ornament is purchased, the glass shop wraps it in several layers of tissue paper and then places it in a cube-shaped box 4 inches tall. Customers concerned about their delicate treasures often head straight to Pack, Wrap, Etc. for more protection from accidental breakage.

- 8. Because of the popularity of the hand-blown glass ornaments, Pack, Wrap, Etc. keeps cubical packing boxes on hand. The heights, in inches, of the available box sizes are given by the terms of the following finite sequence: 5.0, 5.5, 6.0, 6.5, 7.0, ..., 20.0.
  - a. What type of sequence is 5.0, 5.5, 6.0, 6.5, 7.0, ..., 20.0?
  - b. Write a recursive formula for the sequence of packing box heights.
  - c. Write a closed formula for this sequence.
- 9. Customers may choose any one of the available cubical packing boxes and have the glass ornament, in its box from the glass store, packed in the larger box surrounded by polyester fiberfill to fill the packing box.
  - a. List the terms of the finite sequence of volumes of the packing boxes.
  - b. Consider the function  $V$ , where the input is the height, in inches, of a cubical packing box, and the output is the volume of the box, in cubic inches. If  $x$  is the height of a packing box, what is  $V(x)$ ?
  - c. Consider the function  $F$  which outputs the volume of polyester fiberfill surrounding the boxed glass ornament inside a cubical packing box of height  $x$  inches. Find a formula for  $F(x)$ .
  - d. In this context, what is the domain for the functions  $V$  and  $F$ ?
  - e. Graph  $V$  and  $F$  on the same coordinate axes and describe how the graphs are related geometrically.

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### Student Edition

**Georgia Performance Standards Framework for Mathematics I**

10. There are two signs of logical interest hanging in Pack, Wrap, Etc. The first hangs near the gift wrap cash register and reads “If we fail to give you a gift wrapping receipt, the service is free.”
  - a. Write the negation of the statement “We fail to give you a gift wrapping receipt” using good English syntax.
  - b. Write a conditional statement using the negation you wrote in part a which is logically equivalent to “If we fail to give you a gift wrapping receipt, the service is free.”
  - c. Does the conditional statement “If we fail to give you a gift wrapping receipt, the service is free” exclude Pack, Wrap, Etc. from providing free gift wrapping service for some reason other than failing to give a receipt?
  - d. Write the converse of “If we fail to give you a gift wrapping receipt, the service is free.” Is the converse logically equivalent to the original? Explain.
11. The second sign of logical interest is prominently displayed near the packing counter. It reads “Items left over 10 days will be donated to charity.”
  - a. Write a logically equivalent conditional statement.
  - b. Write the inverse of the statement in part a.
  - c. Does the original statement give any information about what Pack, Wrap, Etc. will do with items that are left for 9 days? 8 days? any number of days less than or equal to 10?
  - d. If Pack, Wrap, Etc. donated an item to charity when it had only been 7 days since the customer left it to be packaged, do you think the customer would be upset? Would they have violated their own policy: “Items left over 10 days will be donated to charity.”
  - e. What do you think Pack, Wrap, Etc. intends its policy to be, and how should they state it in order to fully disclose their policy about parcels left for later pick up?

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**Student Edition**