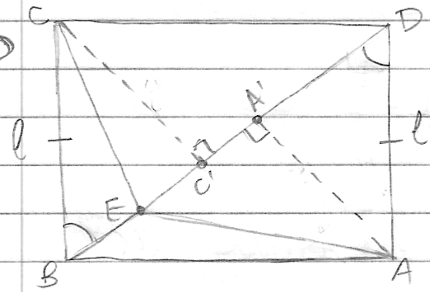
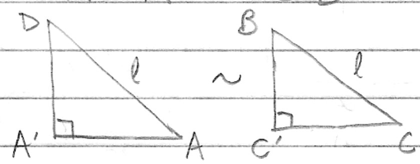
→ Areas of a Rectangle

① 

Conjecture: Areas are equal

$A_{tri} = \frac{1}{2}bh$. If we use $\overline{ED}$ as the base for both, then we just have to prove that $AA' = CC'$

$\measuredangle A'DA \cong \measuredangle C'BC$ (alternate interior angles)
And $\measuredangle DA'A \cong \measuredangle BC'C$ (both are right)
So $\triangle A'DA \sim \triangle C'BC$



$\frac{AA'}{l} = \frac{CC'}{l} \Rightarrow AA' = CC'$

So $\text{Area}_{\triangle EDA} = \frac{1}{2}(ED)(AA')$

$\text{Area}_{\triangle DEC} = \frac{1}{2}(ED)(CC') = \frac{1}{2}(ED)(AA') = \text{Area}_{\triangle EDA}$

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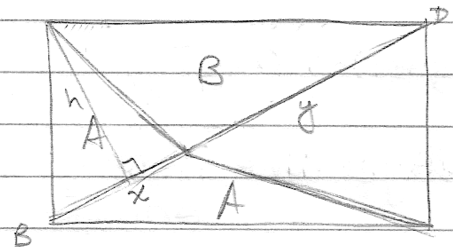
② $\triangle ABD \cong \triangle CDB$ by SSS
So $\text{Area}_{\triangle ABD} = \text{Area}_{\triangle CDB}$
So $\text{Area}_{\triangle ABE} + \text{Area}_{\triangle AED} = \text{Area}_{\triangle CEB} + \text{Area}_{\triangle CDE}$

equal by ①

So $\text{Area}_{\triangle ABE} = \text{Area}_{\triangle CEB}$

$\boxed{\text{or:}}$ equivalent to ① — just rotate rectangle 180°

3



$B = 2A$

$\frac{1}{2} y h = 2\left(\frac{1}{2} x h\right)$

$y = 2x$

Fold $\overline{BD}$ into thirds...

see AreasOfARectanglePaperFolding.gsp
(thirds folding came from
cut-the-knot.org)