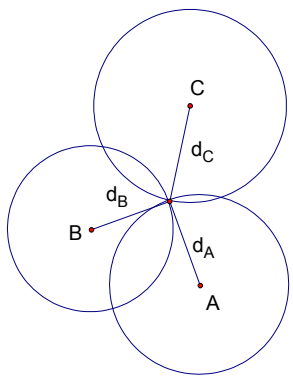
PROBLEM:

There are three listening stations, A, B, and C, which have synchronized clocks. At clock times c_A, c_B, and c_C, respectively, an explosion is heard at stations A, B, and C. Where was the explosion?

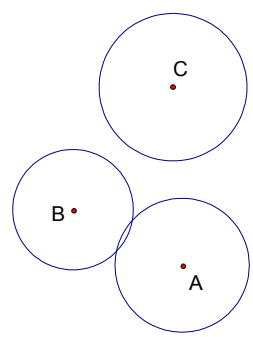


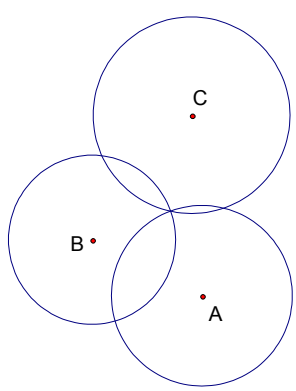
If we knew how long it took the sound to reach each of the stations, we could multiply these times by the speed of sound to calculate the distances from the explosion to each station. Say the explosion is d_A kilometers from A, d_B from B , and d_C from C. Then we can draw circles of radius d_A, d_B, and d_C around A, B, and C as shown. The point at which these three circles intersect is d_A kilometers from A, d_B from B, and d_C from C, so it is the location of the explosion.

Unfortunately, we don't know how long it took the sound to reach each of the stations, since we don't know the clock time at which the explosion itself occurred. But let's try to figure it out. Let c_A, c_B, and c_C be the clock times when the explosion was heard at each of the stations, and let t_A, t_B, and t_C be the time *between* the explosion and when they were heard at each station. If we can determine t_A, t_B, and t_C, then we can use the strategy described above to locate the explosion.

Choose any value $t_A > 0$ and fix it. Then $t_B = t_A + (c_B - c_A)$ and $t_C = t_A + (c_C - c_A)$. For instance, if the explosion was heard at time 09:40:31.9 at station A, at time 09:40:26.3 at station B, and at 09:40:38.2 at station C, and we suppose the explosion took 7.2 seconds to reach station A, then it took 7.2 + (26.3 – 31.9) = 1.6 seconds to reach station B and 7.2 + (38.2 – 31.9) = 13.5 seconds to reach station C.

Now we can use the speed of sound and graph circles as we did above. However, because we have just guessed a value of t_A, the three circles most likely will not intersect in one point. For instance, we might end up with something like the figure shown at right.





But we can adjust our guess for t_A until the three circles *do* intersect in one point, as in the figure shown at left. When the three circles intersect in one point, we know that that point is the location of the explosion.