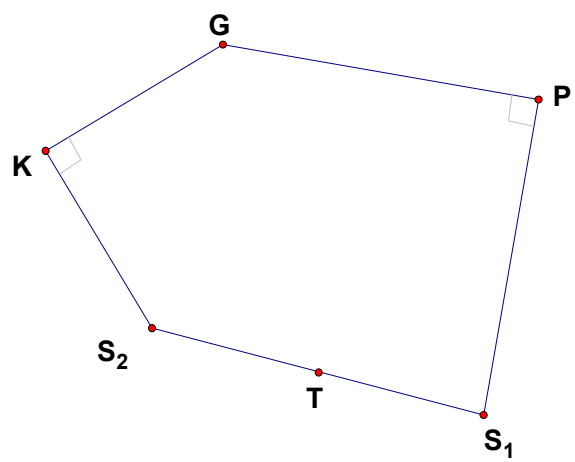
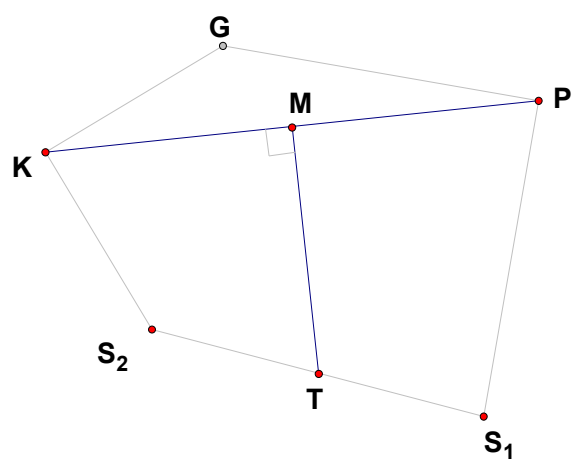
**PROBLEM:**

*A young man was going through the attic of his grandfather's house and found a paper describing the location of a buried treasure on an particular island. The note said that on the island one would find a gallows, an oak tree, and a pine tree. To locate the treasure one would begin at the gallows, walk to the pine tree, turn right 90º and walk the same number of paces away from the pine tree. A spike was to be driven at that point. Then return to the gallows, walk to the oak tree, turn left 90º and walk the same number of paces away from the oak tree. Drive a second spike in the ground. The midpoint of a string drawn between the two spikes would locate the treasure. The young man and his friends mounted an expedition to the island but found the oak tree and the pine tree but no gallows. It had eliminated years ago without a trace. Show them where to look for the treasure.*

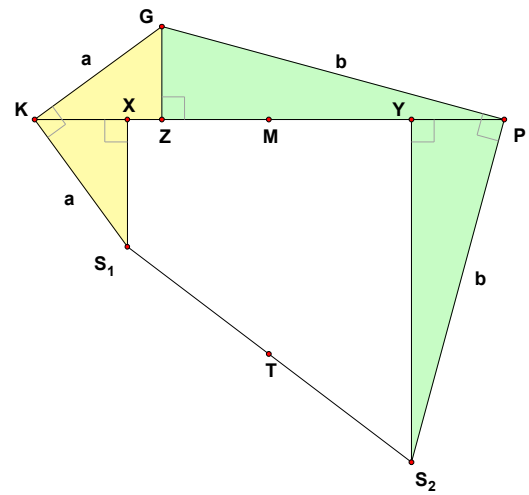
Let G be the (arbitrary) location of the gallows; P, the location of the pine tree; and K, the location of the oak tree. Follow the directions on the treasure map, and let $S_1$ and $S_2$ be the locations of the first and second spikes, respectively. Then T, the midpoint of $\overline{S_1S_2}$, is the location of the treasure.




But if G is missing, what can we do? I claim that we can find the midpoint M of $\overline{KP}$, walk from K to M, then turn right and walk the same number of paces to T.



To prove that this strategy will work, construct $\overline{KP}$ and perpendiculars as shown.




$m\angle GKZ + m\angle KGZ + m\angle KZG = 180^\circ$

So $m\angle GKZ + m\angle KGZ = 90^\circ$ (1)

And $m\angle GKZ + m\angle ZKS_1 = 90^\circ$ (2)

By (1) and (2), $m\angle KGZ = m\angle S_1KX$.

And $\angle KXS_1 \cong \angle GZK$, since both are right.

And $\overline{KG} \cong \overline{S_1K}$ by construction.

Thus, by AAS, we have that

$\Delta KZG \cong \Delta S_1XK$.

So $KZ = S_1X$ and $XK = ZG$.

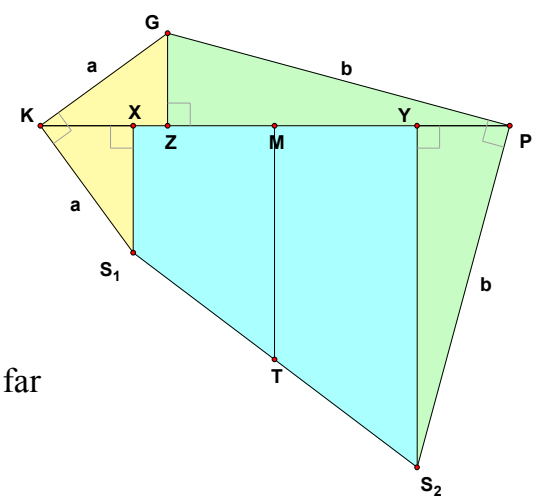
Similarly, $\Delta PZG \cong \Delta S_2YP$.

So $PZ = S_2Y$ and $XK = YP$.

$XM = KM - KX = \frac{1}{2}KP - KX = \frac{1}{2}(KP - 2KX) = \frac{1}{2}(KP - KX - PY) = \frac{1}{2}XY$

So M is also the midpoint of $\overline{XY}$.

$m\angle S_1XY + m\angle XYS_2 = 90^\circ + 90^\circ = 180^\circ$,
so quadrilateral $XYS_2S_1$ is a trapezoid.




$S_1T = TS_2$ and $XM = MY$, so

$$MT = \frac{1}{2}(S_1X + S_2Y)$$
$$= \frac{1}{2}(KZ + PZ) = \frac{1}{2}KP = KM.$$

So regardless of where G is, we know T is as far from M as M is from K and P.

We also know that $\overline{MT} \parallel \overline{XS_1}$, so $m\angle KMT = m\angle KXS_1 = 90^\circ$.
Thus, the strategy described above *will* lead us to the treasure's location.

So we can tell the young man where to look for the treasure. …Or we could go find it ourselves!