**PROBLEM:**

At the Superintendent's home the furnace used oil stored in an underground tank. It was known that the tank was installed level on its side and that it was 36 inches in diameter and 48 inches long. Using a stick dipped through the fill tube, the superintendent determined he had 10 inches of oil in the tank. How much oil did he have in the tank?

Let’s work generally. Say the tank has diameter $2r$ inches and length $L$ inches, and that the height of oil in the tank is $x$ inches.

We can look only at the cross-section of the oil, then multiply by $L$ to find the volume. So let’s find the area of the shaded portion of the figure at the right.

$$t = \arccos\left(\frac{r-x}{r}\right) \quad \text{(in radians)}.$$  

Then the area of the sector is $$\left(\frac{2t}{2\pi}\right)\pi r^2 = t \cdot r^2$$

And the area of the triangle is $$\frac{1}{2} r^2 \sin t$$

So the area filled by the oil is $$t \cdot r^2 - \frac{1}{2} r^2 \sin t = r^2 \left(t - \frac{1}{2} \sin t\right)$$ for $t$ as above.
Thus, the volume of oil in the tank is \( V = Lr^2(t - \frac{1}{2}\sin t) \).

But since \( L \) and \( r \) are both measured in inches, this result is in in\(^3\). We need to convert to gallons:

\[
1 \text{ gallon} = (1 \text{ gallon}) \left( \frac{3.785 \text{ L}}{1 \text{ gallon}} \right) \left( \frac{1000 \text{ mL}}{1 \text{ L}} \right) \left( \frac{1 \text{ cm}^3}{1 \text{ mL}} \right) \left( \frac{1 \text{ in}^3}{2.54^3 \text{ cm}^3} \right) \approx 231 \text{ in}^3
\]

So there are about 231 cubic inches in one gallon. So let’s divide our result by 231 to convert from cubic inches to gallons. The volume of oil (in gallons) is:

\[
V = \left( \frac{Lr^2}{231} \right) \left( t - \frac{1}{2}\sin t \right) \quad \text{where} \quad t = \arccos \left( \frac{r-x}{r} \right) \quad \text{(in radians)}
\]

For our specific case, we have

\[
t = \arccos \left( \frac{r-x}{r} \right) = \arccos \left( \frac{18-10}{18} \right) \approx 1.11 \text{ radians}
\]

So \( V \approx \left( \frac{(48)(18^2)}{231} \right) (1.11 - \frac{1}{2}\sin1.11) \approx 44.6 \text{ gallons}.\)

There is about 44.6 gallons of oil in the Superintendent’s tank.