**PROBLEM:**

Perfect triangles have integer sides & integer area, & their perimeter & area are numerically equal. Find all perfect triangles.

![Diagram of a triangle with an incircle](image)

**Draw in the incircle of \( \triangle ABC \). Say its radius is \( r \), and the triangle's side lengths are \( a, b, \) & \( c \).

Then Area = \( \frac{1}{2} (a)(r) + \frac{1}{2} (b)(r) + \frac{1}{2} (c)(r) = \frac{1}{2} r (a+b+c) \)

& Perim. = \( a+b+c \)

Area = Perim. \( \iff \frac{1}{2} r (a+b+c) = a+b+c \)

\( \iff \frac{1}{2} r = 1 \iff r = 2 \)

So the incircle of a perfect triangle has radius 2.

Let's start with a circle of radius 2 and draw 3 lines tangent to it.

If the bottom side is 4 units, then we have a situation like the ones below:

![Diagram of tangent lines](image)

Then the tangent lines either don't intersect (as in 0), or they intersect on the "wrong" side of the base (as in 0). So the base has to be at least 4 units long.

I created a GSP file (included) to allow me to test some locations for B & C on the number line.

I worked with the GSP file until I found five perfect triangles. They are:

- 0 5-12-13 (with B at 3 & C at -2)
- 0 6-8-10 (with B at 4 & C at -2)
- 0 6-25-29 (with B at 5 & C at -1)
- 0 7-15-20 (with B at 6 & C at -1)
- 0 9-10-17 (with B at 8 & C at -1)

If I don't use the known fact that there are exactly 5 perfect triangles, then I'm not sure how to prove that these are all the perfect triangles.