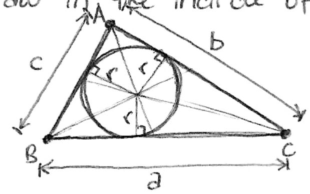
PROBLEM:

Perfect triangles have integer sides & integer area, & their perimeter & area are numerically equal. Find all perfect triangles.

Draw in the incircle of $\triangle ABC$. Say its radius is r, and the triangle's side lengths are a, b, & c.



Then $\text{Area} = \frac{1}{2}(ar) + \frac{1}{2}(br) + \frac{1}{2}(cr) = \frac{1}{2}r(a+b+c)$

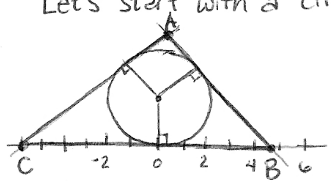
& $\text{Perim.} = a+b+c$

$\text{Area} = \text{Perim} \Leftrightarrow \frac{1}{2}r(a+b+c) = a+b+c$

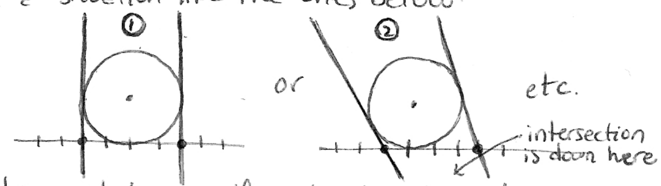
$\Leftrightarrow \frac{1}{2}r = 1 \Leftrightarrow r = 2$

So the incircle of a perfect triangle has radius 2.

Let's start with a circle of radius 2 and draw 3 lines tangent to it.



If the bottom side is 4 units, then we have a situation like the ones below:



Then the tangent lines either don't intersect (as in ①), or they intersect on the "wrong" side of the base (as in ②). So the base has to be at least 4 units long.

I created a GSP file (included) to allow me to test some locations for B & C on the number line.

I worked with the GSP file until I found five perfect triangles. They are:

① 5-12-13 (with B at 3 & C at -2)
② 6-8-10 (with B at 4 & C at -2)
③ 6-25-29 (with B at 5 & C at -1)
④ 7-15-20 (with B at 6 & C at -1)
⑤ 9-10-17 (with B at 8 & C at -1)

If I don't use the known fact that there are exactly 5 perfect triangles, then I'm not sure how to prove that these are <u>all</u> the perfect triangles.