**PROBLEM:**

Given a triangle with side lengths $a$, $b$, and $c$, and a circumcircle of radius $R$, show that the area of the triangle is given by $\frac{abc}{4R}$.

Construct a diameter of the circumcircle through point $A$, and call this diameter’s second intersection with the circumcircle point $D$. Draw in segment $BD$ as shown at right.

Construct an altitude from $A$ to side $BC$. Call its intersection with $BC$ point $E$. See diagram at left.

$\angle ABD$ subtends an arc of measure $180^\circ$ ($AD$ is a diameter), so $\angle ABD$ is right (see Circle Theorems Problem).

And $\angle AEC$ is right by the definition of an altitude.

Thus, $\angle ABD \cong \angle AEC$.

Also, $\angle ADB$ and $\angle ACB$ subtend the same segment (namely, $AB$), so they are congruent as well (see Circle Theorems Problem).

By Angle-Angle similarity, $\triangle ABD \sim \triangle AEC$. Thus,

$$\frac{AC}{AE} = \frac{AD}{AB} \rightarrow \frac{b}{AE} = \frac{2R}{c} \rightarrow AE = \frac{bc}{2R}$$

$AE$ is an altitude of $\triangle ABC$, so the area of $\triangle ABC$ is

$$\frac{1}{2} \cdot a \cdot AE = \frac{1}{2} \cdot a \left( \frac{bc}{2R} \right) = \frac{abc}{4R}$$

Q.E.D.