

Simulating Rectangles

Exploring Mathematics with Fathom Summer Institute

Materials:

Paper

Scissors

In one way this activity is about the properties and relationships you can find in a collection of rectangles whose lengths and widths have been chosen at random.

In another way, this activity is about figuring out what a graph should look like *before* you make it, and explaining why a graph looks the way it does *after* you make it.

Un-Simulating Rectangles

We re going to create rectangles with Fathom, but it will be helpful to have some real rectangles to refer to.

Try to get rectangles that are different from those you see around you.

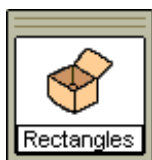
1. From a sheet of 8.5 by 11 inch sheet of paper make three cuts that make rectangles.
2. Measure each rectangle s length and width.
3. Compute each rectangle s perimeter and area.
4. Write these measurements and computations on the rectangle.

We re going to be make a *lot* of graphs with Fathom. Let s practice making one first by hand.

5. With a fresh sheet of paper, label one edge Length and a perpendicular edge Width.
6. Put a point on this graph for each rectangle with coordinates (Length , Width).

Simulating Rectangles

Now we ll make 100 rectangles with Fathom. Their lengths and widths will be randomly chosen between 0 and a maximum value set by a slider. Fathom has built-in functions for generating random numbers. We ll use one for generating a *uniform* distribution.

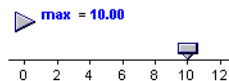


You can use either an inspector or a case table to define the attributes.

7. In a new document, make an empty collection. Change its name from Collection 1 to Rectangles.
8. Choose New Cases from the Data menu and specify that you want 100 cases.
9. Create two attributes: Length and Width.
10. Make a slider. Name it max.
11. Give each attribute the formula $\text{random}(0, \text{max})$.

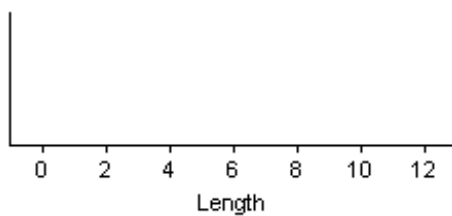
You should have something similar to what is shown at right. (You might have used an inspector instead of a case table to define the attributes.)

| Rectangles | | Length | Width | <new> |
|------------|--|--------------------------------|--------------------------------|-------|
| = | | $\text{random}(1, \text{max})$ | $\text{random}(1, \text{max})$ | |
| 1 | | 8.54075 | 4.49824 | |
| 2 | | 1.63609 | 2.45745 | |
| 3 | | 6.81322 | 2.81798 | |



Guessing the Graph

Image a stacked dot plot with Length on the x-axis. *Before* using Fathom , make a sketch of what you think the graph will look like. *Important! Draw your sketch here.*



Graphing Length and Re-randomizing

OK, so let's see what the stacked dot plot of Length looks like.

12. Make the stacked dot plot.

In what ways does it look similar to the guess you made? In what ways does it look different?

It's instructive to look at multiple examples of sets of 100 lengths.

13. With the graph selected, choose **Rerandomize** from the **Analyze** menu. Do this several times. Also try changing the dot plot to a histogram.

Describe the variation you see similarities and differences among the various plots of these random numbers.

14. Drag the slider (or animate it).

Explain the changes you see when the slider value changes.

You have been looking at a distribution of values, in this case, a *uniform* distribution. Let's move on to a two-dimensional situation.

Length Versus Width

What will a scatter plot of length versus width for the 100 random rectangles look like?

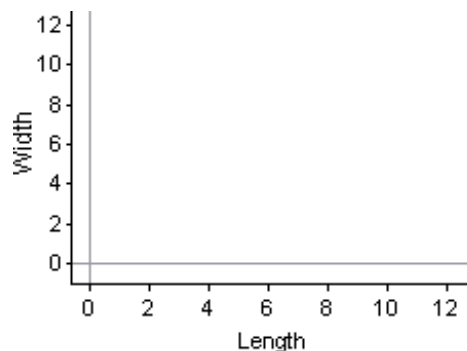
15. *Before* you make one with Fathom, make a sketch of what you expect to get. Draw it using the axes at right.

16. Make the scatter plot with Fathom.

17. Re-randomize.

18. Drag the slider.

Write about what is going on here. Why does the scatter plot look the way it does? Explain the changes produced by dragging the slider. Describe the variation from plot to plot when you re-randomize.



Perimeter

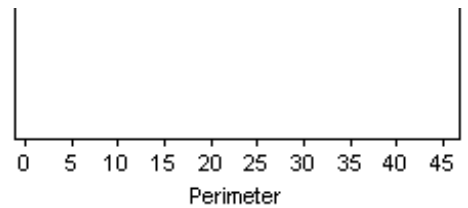
Rectangles have perimeter. We want to characterize the distribution of perimeters for these rectangles.

19. Define a new attribute, Perimeter. Give it a formula such that it will compute the perimeter of each rectangle.

The keyboard shortcut for Rerandomize, Ctrl-Y, is extremely useful.

Before you make a Fathom graph, predict what a stacked dot plot of Perimeter will look like. (Your paper rectangles will be helpful here.)

20. Make a sketch of your prediction using the axes at right.
21. Make a stacked dot plot and a histogram of Perimeter using Fathom.
22. Re-randomize. Drag the slider.



*We often speak about **features** of a plot. If a plot is mound-shaped, that is one of its features.*

Write about what you observe. In what ways does the actual distribution match your prediction and in what ways does it not? How does the distribution of perimeters differ from the distribution of lengths? Is the distribution flat or mound-shaped? Are there upper and lower limits? How do you explain its shape?

Selection

A movable line will be useful here.

Make sure you have a histogram of Perimeter and a scatter plot of Length versus Width. Select various bars in the histogram and observe the selected dots in the scatter plot. What observations can you make? Explain.

Changing the Number of Cases

One hundred cases is a good number for most situations. But, occasionally, it is worthwhile to have many more cases.

23. With the Rectangles collection selected, choose **New Cases** from the **Data** menu.
24. Add 300 cases.

Revisit the question in the preceding section on selection. Are the patterns easier or harder to see?

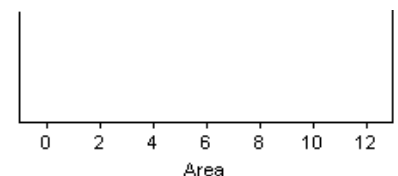
25. Use the **Undo** command to undo until you are back to having only 100 cases again.

Area

Rectangles also have area. Do you think that the areas of these rectangles will have a distribution similar to either length or perimeter?

26. Define a new attribute, Area. Give it a formula such that it will compute the area of each rectangle.

By now you should have a pretty good idea what it takes to make a prediction of what a distribution will look like and you probably appreciate the value of plotting the values from your paper rectangles.



27. Make a sketch of your prediction for the Area distribution at right. What *features* do you expect this distribution to have?
28. As with Perimeter, make a stacked dot plot and a histogram of Area. Look at a bunch of re-randomized distributions and see what happens when you vary max.

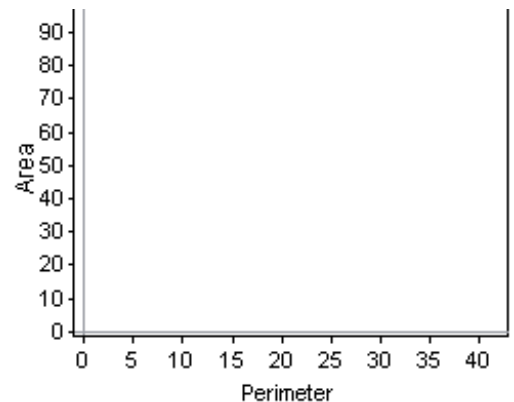
Write about what you observe. In what ways does the actual distribution match your prediction and in what ways does it not? In what ways is this distribution the same or different than the others? How do you explain its shape?

Make sure you have a histogram of Area and a scatter plot of Length versus Width. Select various bars in the histogram and observe the selected dots in the scatter plot. What observations can you make? Explain.

Perimeter Versus Area

This is your last chance at predicting what the graph will look like. Work hard on it! Don't give up on your prediction until you feel sure that you have captured in your sketch at least part of what you think will be true in the finished graph.

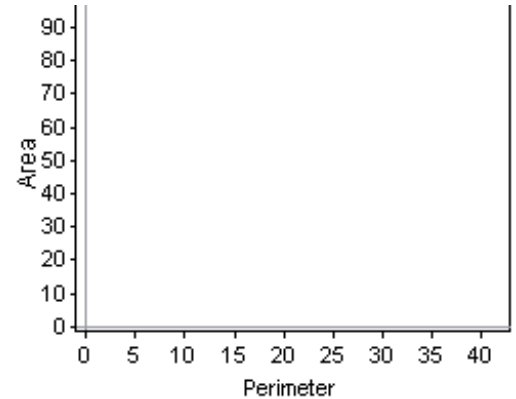
29. Predict what the scatter plot with Perimeter on the x-axis and Area on the y-axis will look like. Sketch your prediction at right.
30. Justify your prediction; i.e. *write* about *why* you think it will that way.



31. Make the scatter plot with Fathom. Do the usual re-randomization and dragging of the slider.

32. Sketch the actual scatter plot you got at right.

Describe the scatter plot in words. What features does it have? Describe how it is similar to or different from your prediction.



Boundaries

The scatter plot of Perimeter versus Area has two boundaries, one above-left, and the other below-right.

33. Find the equation of the *upper* boundary. (One or more sliders will be helpful!) Describe what you did, the equation you found, and why you think it is correct.

34. Find the equation of the *lower* boundary. (One or more sliders will be helpful!) Describe what you did, the equation you found, and why you think it is correct.

Extras

Picturing the Rectangles

Drag open the collection of rectangles. Notice that each rectangle is represented by a gold ball. It would be much more interesting to represent them as rectangles, as shown at right. Figure out how to do this using the collection's display attributes.



Beyond Two Dimensions

What are the three dimensional analogs of area and perimeter? Construct a simulation in which you investigate these and their relationships.

Negative Rectangles?

Investigate the possibilities of rectangles with either length, width, or both being negative.