Hamilton Hardison

Lesson 3

Geometrically Exploring Trigonometric Ratios as Functions

Class: Math IV or Accelerated Math III

Time: One 50-Minute Class Period (or Two 50-Minute Class Periods)

Goals:

- Students will be introduced to and understand the ratios tangent, cotangent, secant, cosecant. (Both in right triangle terminology and in terms of sine and cosine)
- Students will associate appropriate signs with magnitudes by investigating changes in values of the trigonometric ratios and by algebraic manipulation.
- Students will geometrically explore the domain of trigonometric functions
- Students will geometrically explore the range of trigonometric functions
- Students will discover angles for which certain trigonometric functions are undefined.
- Students will discover angles for which result in zeros for the trigonometric functions.
- Students will discover similarities between pairs of trigonometric functions.

GPS Standards Addressed:

- MM4A3. Students will investigate and use the graphs of the six trigonometric functions.
  a. Understand and apply the six basic trigonometric functions as functions of real numbers.
  b. Determine the characteristics of the graphs of the six basic trigonometric functions.
- MM4P4. Students will make connections among mathematical ideas and to other disciplines.
  o Recognize and use connections among mathematical ideas.
  o Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

- MM4P5. Students will represent mathematics in multiple ways.
  - Create and use representations to organize, record, and communicate mathematical ideas.
  - Select, apply, and translate among mathematical representations to solve problems.

Supplies Needed:

- Computers with Geometer's Sketchpad Software
- Provided Procedure Worksheet

Assessment: Students will be assessed on the Procedure worksheet and Class Discussion. Out of 25 points, the breakdown is as follows.

- Participation in Class Discussion – 5 pts.
- Completion of Procedure Worksheet and Previous Homework– 10 pts.
- Mathematical Reasoning Demonstrated in Procedure Worksheet – 10 pts.

Timeline:

- Introduction, Getting Started, and Previous Homework – 10 to 30 minutes
  - Discussion on right triangle terminology for tangent, cotangent, secant, and cosecant based on homework from previous activity.
  - Instruction and proofs of the four trigonometric functions in terms of sine and cosine.
  - *Note:* The procedure for the first tab in the sketch may better serve the students if done as a whole class with discussion and proof.
- Exploration Using the Procedure – 30 to 40 minutes
  - *See the above note for an optional modification to the procedure.*
  - Students will be working independently or in small groups on the assigned tasks in the procedure.
  - During this time the instructor should be walking around the room. Assistance on questions in the procedure should not be

given unless absolutely necessary; Allow the students to work with partners to arrive at solutions. Teacher assistance should be focused on issues dealing with software usage as opposed to content.

- Class Discussion – 10 to 30 minutes
    - *Remember that this activity is intended to be a bridge between geometrical representations of trig ratios and graphs of trig functions. The activity is more exploration than instruction.*
    - Ensure that proper mathematical terminology is being used and that the terminology is understood.
    - Emphasize and Discuss the interpretation of a ratio as a signed length with a unit denominator.
    - Discuss the sign changes of the functions.
    - Discuss the Domain of the functions with attention to undefined values, signed domains, and angles whose absolute value is greater than $\frac{\pi}{2}$.
    - Discuss the range of the functions
    - Discuss possible pairings of trig functions and reasons for assigned pairs.
    - Discuss any questions that come up or portions of the activity that seem to be problematic for students.
    - Discuss anything particularly interesting or unexpected that students have discovered in the exploration.