Hamilton Hardison

Activity Two Procedure

Begin by opening GSP file "A2S"

In this activity you will investigate sine and cosine of angles of less than $\frac{\pi}{2}$ from a coordinative perspective. Based on your findings, you will also examine the sine and cosine of angles greater than $\frac{\pi}{2}$ and learn a bit about reference angles.

Start this investigation in tab labeled "1". In this tab, you will find a triangle with hypotenuse of length one. The measurements of the triangle's sides have already been calculated for you. Calculate the sine and cosine of the marked angle x by using the calculate feature of GSP.

Now, using the graph menu, measure the coordinates of point B.

1. Compare the sine and cosine of the angle x to the coordinates of point B. What do you notice?

2. Move point B to change your triangle. Does your above conjecture hold? What would you call sin(x) and cos(x) in terms of the point b. Can you prove why this works? (Remember how long the hypotenuse is.)

3. For what angles is $\sin(x) = 1$, $\cos(x) = 1$, $\sin(x) = 0$, $\cos(x) = 0$, $\cos(x) = \sin(x)$?

Click on tab two to continue this exploration.

Here instead of being given a quarter circle, we have been given an entire unit circle. Until now, we have been working with all positive distances. By placing our triangle on a coordinate system, we are now allowed to have negative distances in a sense. Points in quadrants II and III have negative x values. Similarly, points in quadrants III and IV have negative y values. By moving point B to quadrants other than the first, we can examine triangles that have legs with negative distances. Move the point B to the second quadrant.

4. Using what you just did in the first tab, see if you can guess what sin(x) and cos(x) will be for your obtuse angle. Explain your conjecture below. Check your conjecture by using the calculate feature to calculate the sin(m<AxB) and cos(m<AxB). Did your conjecture hold?

5. In which quadrants must the terminal side of an angle be for sin(x) to be positive?

6. In which quadrants must the terminal side of an angle be for sin(x) to be negative?

7. In which quadrants must the terminal side of an angle be for cos(x) to be positive?

8. In which quadrants must the terminal side of an angle be for cos(x) to be negative?

Move point B around the circle to check that this always works.

We have been talking about sine and cosine in terms of right triangles; but how can we talk about sine and cosine of angles greater than $\frac{\pi}{2}$ if you cannot make a right triangle with an angle bigger than $\frac{\pi}{2}$? A nice way to visualize this is with the use of reference angles. When given an angle, x, in standard position, the reference angle, r, is the angle formed by the terminal side of angle x and the x-axis. Click the show reference angle button to see the reference angle indicated in red. By using reference angles, we can create right triangles for angles greater than $\frac{\pi}{2}$. Reference angles also make drawing angles greater than $\frac{\pi}{2}$ by hand easier (since you have an angle of reference).

Click on the next tab to learn more about reference angles.

This sketch has four angles (A, B, C, and D) which all have been constructed to have the **same reference angles**. A will always be in the first quadrant, B in the second, C in the third, and D in the fourth. Click on the show sine and cosine buttons to show the values of the ratios for these angles.

9.) What do you notice about the sine values for each angle?

10.) What do you notice about the cosine values for each angle?

11.) How can you tell when the sine of an angle will be positive? Negative?

12.) How can you tell when the cosine of an angle will be positive? Negative?

Click on the “Does this cover all angles” button to see that if we know the sine and cosine of all angles less than $\frac{\pi}{2}$, we know the sine and cosine of any angle through the use of reference angles.

Reference angles will become more important once you have started memorizing the sine and cosine of common angle measures.

Homework: Up until now we have only discussed two trigonometric ratios, but there are others. Since there are three sides of a right triangle (opposite, adjacent, and hypotenuse), and a ratio involves two quantities (a numerator and a denominator) there should be 3! permutations; meaning there should be six trig ratios total. Your homework assignment is to discover the four other trigonometric ratios (we have already done opp/hyp and adj/hyp).