Hamilton Hardison

Procedure for:
Geometrically Exploring Trigonometric Functions

The purpose of this exploration is to make our way from representing trigonometric relationships geometrically to representing these relationships as functions. We will investigate sign changes in the trig functions and verify your conjectures algebraically. We will also investigate the domains and ranges of the trigonometric functions.

Open the Sketch "A3S" in Geometer's Sketchpad.
Begin by reading the introduction and then continue to the tab labeled "Unit Circle".

The unit circle sketch has been constructed in a manner so that the values of trigonometric functions of the angle x are represented geometrically by lengths of segments. Recall from the previous activity that trig functions are ratios of sides of triangles, so how can lengths represent ratios? Due to the construction based on the unit circle, the denominator of all the ratios represented is 1, thus the lengths represent the ratios. Explore the sketch using the hide/show buttons and by dragging the point A. Notice that the segments have been colored to identify "negative lengths". The reason for we need to have negative lengths is due to the coordinate interpretation of sine and cosine explored in your previous activity and the definitions of trig functions as ratios. Your task for this sketch is to justify that the lengths actually are the values of trig functions that they claim to represent. Do not try to use measurement in your justification; perhaps looking for similar triangles will be more useful here. The trick is to find a triangle with an appropriate side of length one (the side that you want in the denominator). You have all of the triangles you need if all of the buttons are in the show position. Explain your justification below and draw sketches when necessary to illustrate which triangle you are referencing:

Sine: Since sin(x) is the ratio of the opposite side to the hypotenuse and the hypotenuse is a unit length, the ratio is just the signed length of the opposite side.
(Explored in Previous Activity)

Cosine: Since cos(x) is the ratio of the adjacent side to the hypotenuse and the hypotenuse is a unit length, the ratio is just the signed length of the adjacent side.
(Explored in Previous Activity)

Tangent:

Cotangent:

Secant:

Cosecant:

In the following tabs you will be presented with each of the trig functions represented by lengths. The appropriate segment in each sketch has been traced so that you can explore the domain, range, and sign change associated with the displayed trigonometric function. *Remember that the value of the trigonometric function evaluated at angle x is represented by the length and color of the segment being traced.* Responses to questions involving domain should be in terms of angle measures. You can manipulate the sketch by dragging the point A or by using the animation buttons.
The components of each sketch you need to explore are outlined below.

Sine:

- Positive Domain:
- Negative Domain:
- Domain:
- Range:
- Angles that are zeros:
- Angles that result in the ratio being undefined:

Cosine:

- Positive Domain:
- Negative Domain:
- Domain:
- Range:
- Angles that are zeros:
- Angles that result in the ratio being undefined:

Tangent:

- Positive Domain:
- Negative Domain:
- Domain:
- Range:
- Angles that are zeros:

- Angles that result in the ratio being undefined:

Cotangent:

- Positive Domain:
- Negative Domain:
- Domain:
- Range:
- Angles that are zeros:
- Angles that result in the ratio being undefined:

Secant:

- Positive Domain:
- Negative Domain:
- Domain:
- Range:
- Angles that are zeros:
- Angles that result in the ratio being undefined:

Cosecant:

- Positive Domain:

- Negative Domain:

- Domain:

- Range:

- Angles that are zeros:

- Angles that result in the ratio being undefined:

Which ratios have the similar coloring? Similar Ranges? Can you provide any reason for this?