Name: ____**Date:**____________________

Latitude and Longitude Explorations using Spherical Trigonometry

Directions: Answer the following questions. You may use Spherical Easel, your textbook, or allowed Internet sites.

One of the main reasons why we study spherical geometry and trigonometry is to navigate our way around the earth. Note the following figure:

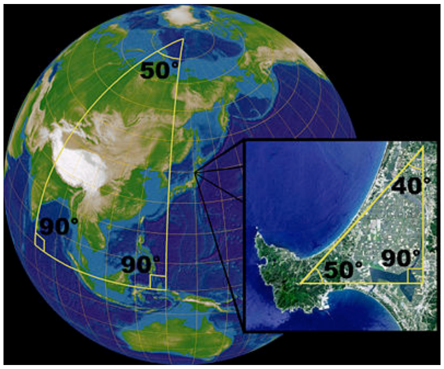

Figure from http://www.answers.com/topic/spherical-geometry

Here we see the earth and an inset showing part of the country of Japan. When we navigate our city, for example, our paths are very nearly straight, thus the rules of trigonometry follow plane trigonometry (e.g., the sum of the interior angles of a triangle is 180°). However, if we navigate our entire hemisphere, our paths are not straight, but curved. The rules we learned in plane trigonometry do not necessarily apply (e.g., the sum of the interior angles of a spherical triangle do not have to equal 180°.)

To assist in locating and navigating the globe, a coordinate system was set up. Note the following figure:

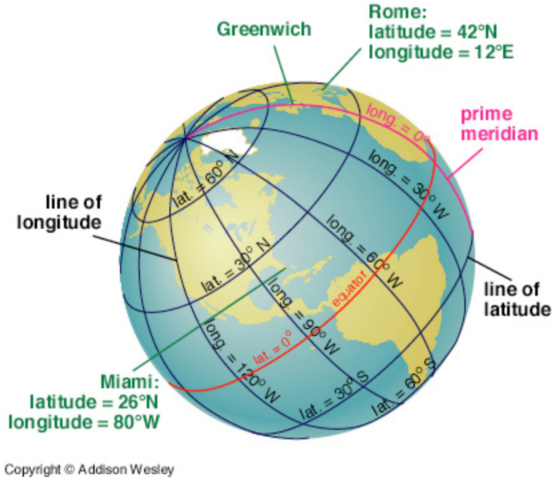

Figure from http://www.bramboroson.com/astro/jan30.html

Just like the Cartesian coordinate system, there are lines that signify zero. Because a sphere is circular, we use degrees as a unit. The earth is divided up into North, South, East, and West analogous to the divisions of Quadrants I, II, III, and IV. Lines that go north-south are called lines of longitude or meridians. Lines that go east-west are called lines of latitude. The Prime Meridian is the line that is 0° east or west. The other significant line of longitude is the International Date Line or 180° east or west. The equator is the line that is 0° north or south. The very top point on the earth, or the North Pole, is 90° north, and the South Pole is 90° south.

Notice that the figure above has given the coordinates for Miami and Rome. Naturally, you might ask, how far apart are those two cities? When flying, we want to use the shortest route to get there. We can use spherical trigonometry to find out.

1. How would you find the distance from Miami to Rome if they were in a plane? For example, say Miami had Cartesian coordinates (–80, 26) and Rome (12, 42). How far apart would they be?

2a. Using the spherical trigonometry from yesterday, find the degree distance from Miami to Rome. How did you do this?

b. Verify whether or not the way you calculated the distance in the plane works on a sphere. Why does this method work or not work in spherical geometry?

3. If 1 degree is equal to 69.2 miles, how far is Miami from Rome? How did you do this?

4. One of the powerful attributes of the Internet is that it offers tools that we can use to automate complicated calculations. One person wrote an Excel spreadsheet that will calculate any angular distance on the earth given two locations' latitude and longitude. Download the Excel spreadsheet ARC_CALC 3 from www.jqjacobs.net/astro/arc_form.html. Input the coordinates of Miami and Rome. What is the angular distance? What is the distance in miles? Are they different than what you calculated earlier? Assess the usefulness of this tool.

5. The website above advocates the following formula for finding the distance between two points on the earth when using the set up shown in Figure 1:

$$\tan\left(\frac{1}{2}c\right) = \frac{\sin\left[\frac{1}{2}(a+b)\right]\cdot\tan\left[\frac{1}{2}(a-b)\right]}{\sin\left[\frac{1}{2}(a-b)\right]}$$

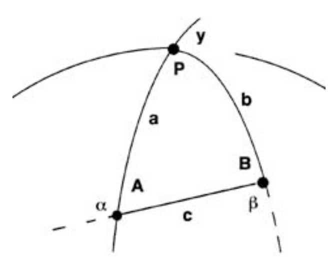

Figure 1 from www.jqjacobs.net/astro/arc_form.html

Using the coordinate given for Miami and Rome, recalculate the angular distance using this formula. How does it compare/contrast with the method you used previously? Why are there differences, if any?

6. Using the formulas given for the Law of Sines, calculate the distance from Miami to Rome. Is there any difference from the previous methods? Now use the formula for the Law of Cosines. How does this method compare? Which method was most effective?