

Cycloid

by: Joshua Wood

Problem: A cycloid is the locus of a point on a circle that rolls along a line. Write parametric equations for the cycloid and graph it.

Solution: Let's build our cycloid out of a circle of radius $a > 0$. Let's have the circle roll along the x -axis. Let's have the circle's center begin at $(0, a)$. Let $\vec{r}(t)$ be the vector that starts at the center of the circle and ends at our point on the circle which is tracing out the cycloid. Thus we can take $\vec{r}(t) = \langle a \cos t, a \sin t \rangle$.

Let $x(t)$ be the x coordinate of the center of the circle at time t . The center of the circle, progresses to the left at a constant rate, since our circle rolls counterclockwise at a constant rate. We have $x(0) = 0$ and after one revolution ($t = 2\pi$), the center has moved one circumference ($2\pi a$) to the left, i.e. $x(2\pi) = -2\pi a$. Thus $\frac{dx}{dt} = \frac{-2\pi a}{2\pi} = -a$ and therefore $x(t) = -at$.

Therefore the point tracing out the cycloid is given by the center plus the vector \vec{r} which is

$$\langle x(t), a \rangle + \vec{r}(t) = \langle -at + a \cos t, a + a \sin t \rangle$$

Below are cycloids of radius $a = 1$, and $a = 2$.

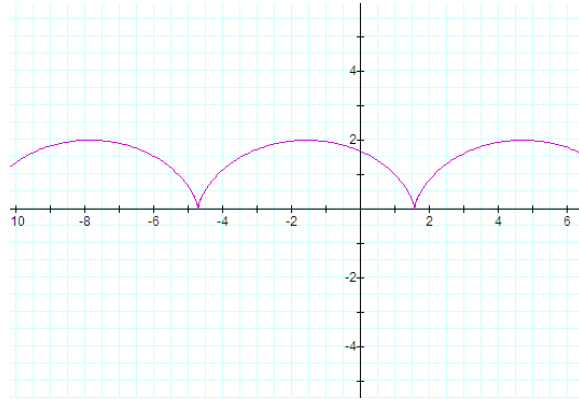


Figure 1: $a = 1$

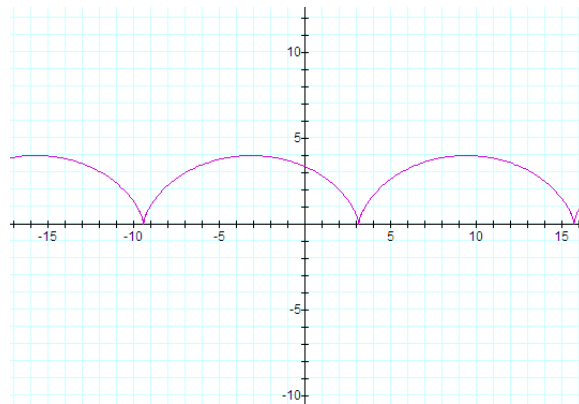


Figure 2: $a = 2$