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Some Polar Graphs

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In this document we will explore the polar equations given by $r = 2a\sin(k\theta)$, $r = 2a\cos(k\theta)$, $r = 2a\sin(k\theta) + b$, $r = 2a\cos(k\theta) + b$, and $r = \frac{c}{a\cos(k\theta)+b\sin(k\theta)}$. First let's explore $r = 2a\sin(k\theta)$. With $a = k = 1$ we get a circle.

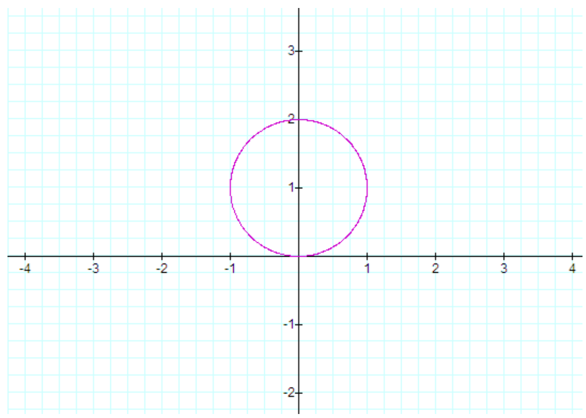

Figure 1: graph of $r = 2\sin(\theta)$

To see that this really is a circle we multiply both sides of $r = 2\sin\theta$ by r, and get $r^2 = 2r\sin\theta \Rightarrow x^2 + y^2 = 2y$. Changing a will scale the figure, and making $a < 0$ will flip the circle to the bottom of the x-axis. We illustrate these effects with $a = -2$ and $k = 1$.

Now let's explore changing k. With $a = 1$ and $k = 2$ we have a 4-leafed rose.
With $a = 1$ and $k = 4$ we have an 8-leafed rose.
Thus for positive even k, we are getting a $2k$-leafed rose. Since $\sin x$ is an odd function making k, an even negative integer will not affect the graph, as each leaf has a leaf antipodal to it. We illustrate with $a = 1$ and $k = -6$, giving us a 12-leaf rose.

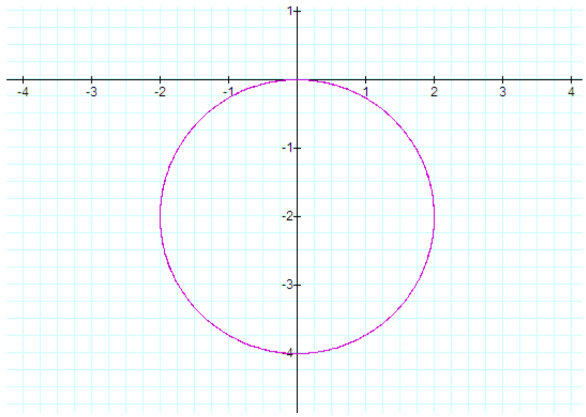

Figure 2: graph of $r = 2(-2)\sin(\theta)$

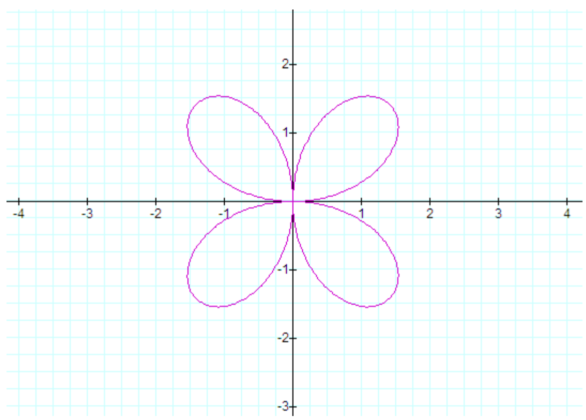

Figure 3: graph of $r = 2\sin(2\theta)$

For k an odd integer, we get k-leafed roses. We show the 3, 5, and 7 leafed roses.

For the odd-leafed roses, there is no antipodal pedal, so making k odd and negative will "flip" the rose.

We also include some neat graphs with decimal values of k.

The graphs of $r = 2a\cos(k\theta)$ are going to be similar to the graphs of $r = 2a\sin(k\theta)$. The difference for the roses is a rotational shift, as $\sin x$ has extreme values where $\cos x$ has zeros and vice versa. We illustrate with some examples. Note that the graph of $r = 2\sin(\theta/2)$ is the same as the graph of $r = 2\cos(\theta/2)$. Also note that making k negative will have no affect on the graph as $\cos x$ is an even function.

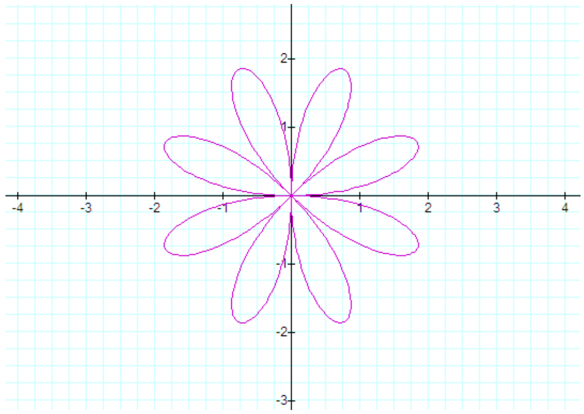

Figure 4: graph of $r = 2\sin(4\theta)$

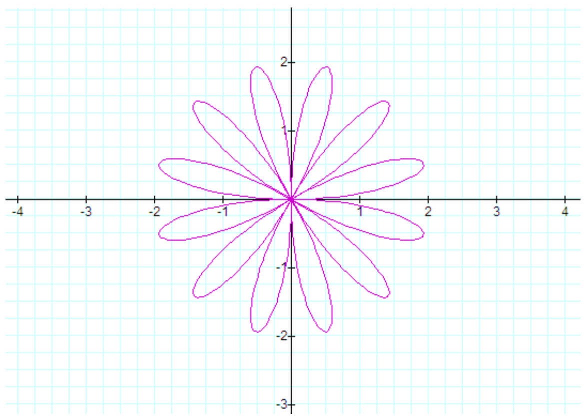

Figure 5: graph of $r = 2\sin(-6\theta)$

Next we examine $r = 2a\sin(k\theta) + b$. We see that for $a = b = 1$ and for k an integer we get roses with k big leaves and k little leaves. For k odd, the little leaves are inside the big ones, and for k even, the little leaves are outside.

We include some

Finally we explore $r = \frac{c}{a\cos(k\theta)+b\sin(k\theta)}$. First we note that c will have a minimal effect on the graphs as this parameter just scales all the radii. So we will explore the possibilities when $c = 1$. The first interesting case is with all parameters equal to 1. This gives us a straight line as we can see from the following algebraic manipulation.

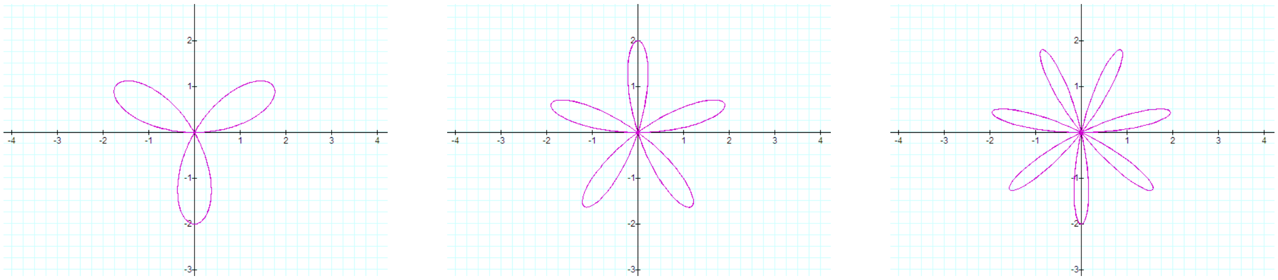

Figure 6: graphs of $r = 2\sin(k\theta)$, with $k = 3, 5, 7$

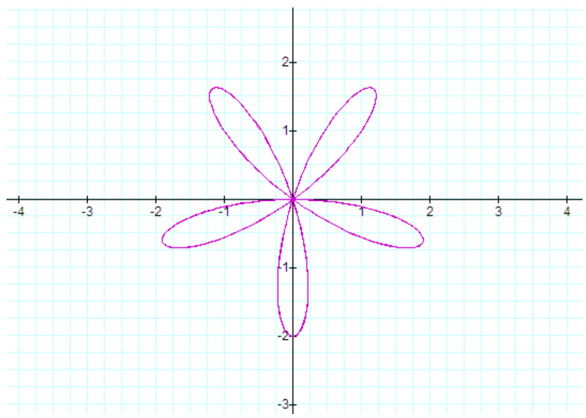

Figure 7: graph of $r = 2\sin(-5\theta)$

$$\begin{aligned} r &= \frac{1}{\cos\theta + \sin\theta} \\ r\cos\theta + r\sin\theta &= 1 \\ x + y &= 1 \end{aligned}$$

We also explored the effect of changing k, a, and b. Increasing k gave us more and more branches that go off into straight lines, but interact near the origin. Increasing a and b, both simultaneously and individually made the pictures "tighten up".

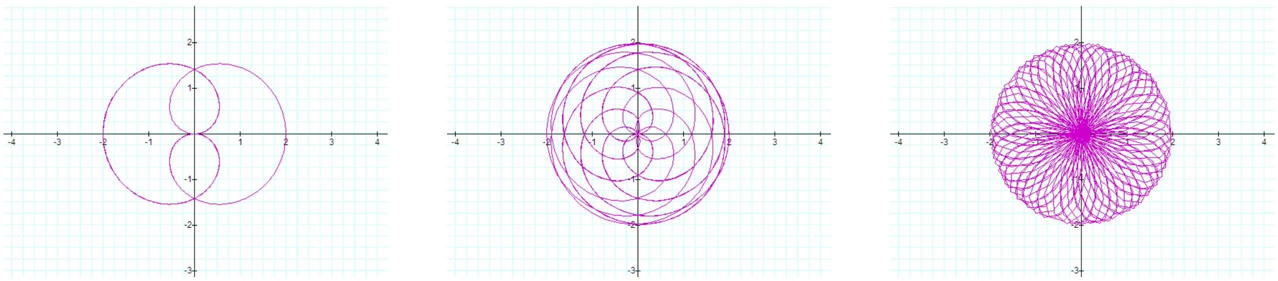

Figure 8: graphs of $r = 2\sin(k\theta)$, with $k = 0.5, 0.3, 2.7$

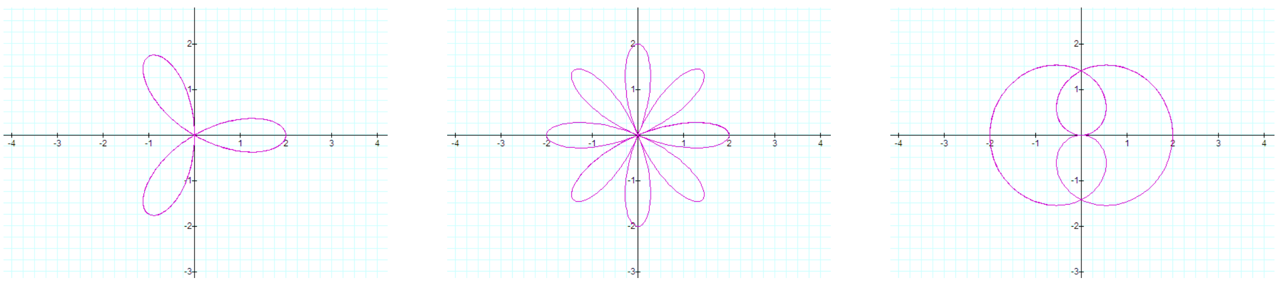

Figure 9: graphs of $r = 2\cos(k\theta)$, with $k = -3, 4, 0.5$

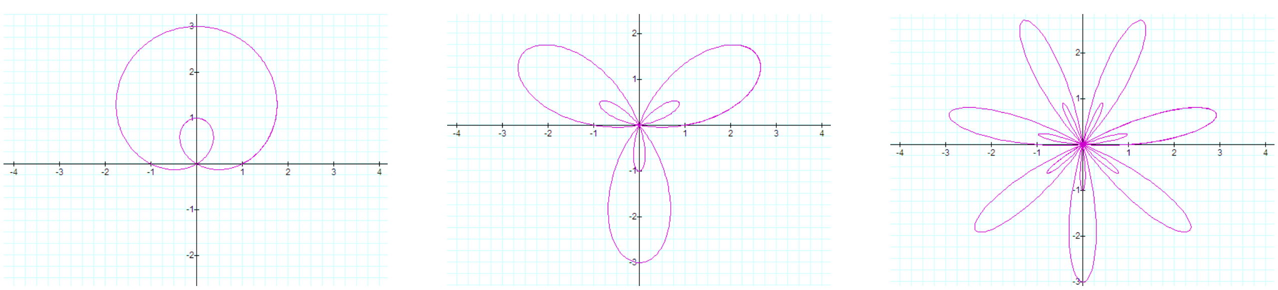

Figure 10: graphs of $r = 2\sin(k\theta) + 1$, with $k = 1, 3, 7$

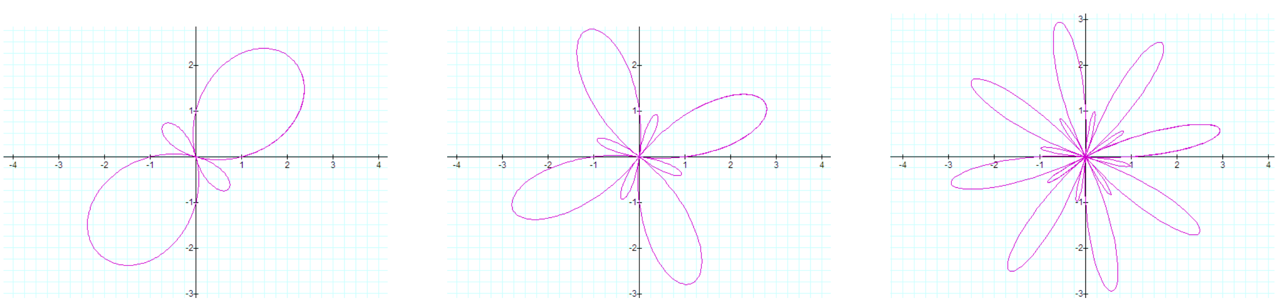

Figure 11: graphs of $r = 2\sin(k\theta) + 1$, with $k = 2, 4, 8$

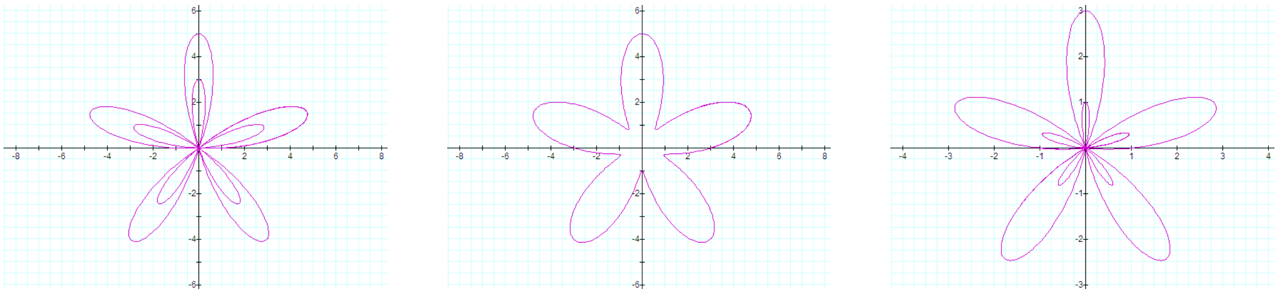

Figure 12: graphs of $r = 2a\sin(5\theta) + b$, with $a = 2, 1, 1$ and $b = 1, 3, -1$

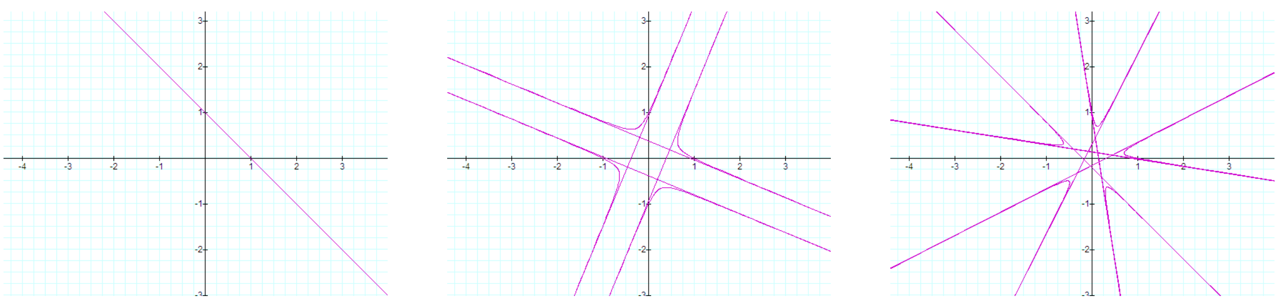

Figure 13: graphs of $r = \frac{1}{a\cos(k\theta)+b\sin(k\theta)}$, with $a = b = 1$ and $k = 1, 2, 5$

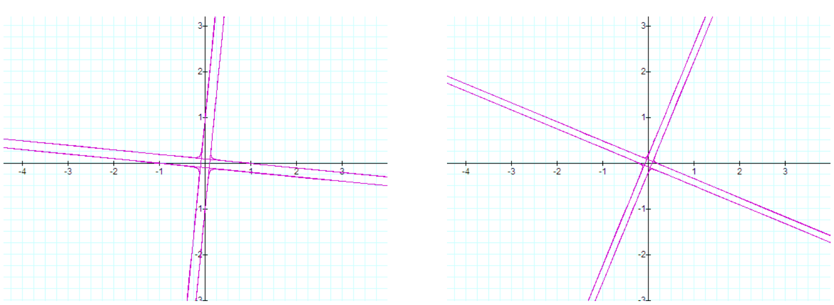

Figure 14: graphs of $r = \frac{1}{a\cos(k\theta)+b\sin(k\theta)}$, with $a = 1, 5$, $b = 5$ and $k = 2$