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Directrix Circle

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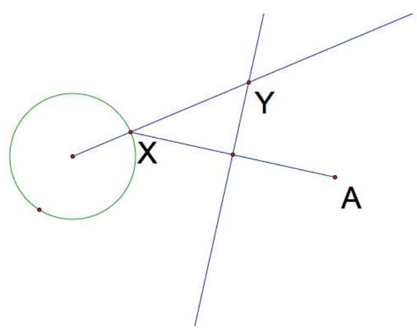
In this document we construct and explore the locus of points equidistant from a circle and a point A. First we consider the case where A is outside the circle. Let X be an arbitrary point on the circle. Let Y be the point we want to find, i.e. Y is going to be equidistant from the circle and A and the nearest point to Y on the circle is point X. Thus Y must be on the ray starting at the center of the circle and X. Since Y needs to be equidistant from X and A, Y must lie on the perpendicular bisector of AX. Thus we intersect these two lines.



Next we let X vary and trace the locus of the points Y. It appears that we get one branch of a hyperbola. If we repeat the construction but instead of taking the ray from the center to X, we take the line through the center and X, and trace the intersection of the two lines then we get the other branch of the hyperbola. Of course the other branch does not meet the requirements of the locus, as it is the locus of points equidistant from A and X, which is the point on the circle farthest from Y.

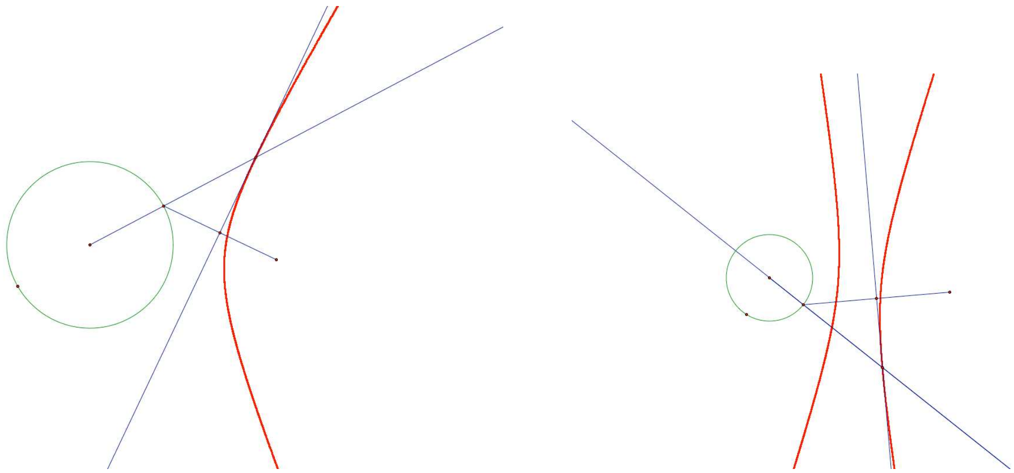
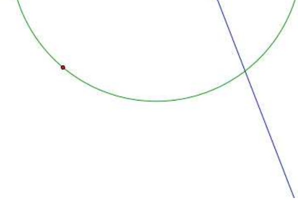


Figure 1: The locus with A outside the circle, along with the other hidden branch of the hyperbola

We now consider the case where A is inside the circle. Again we let X be an arbitrary point on the circle and find a point Y that is equidistant from X and A, with the closest point of the circle to Y being X. Thus, Y is on the segment connecting X with the center of the circle. Also, the equidistant requirement means that Y needs to be on the perpendicular bisector of AX. Thus we intersect the line with the segment to find Y. As we vary X we get an ellipse for the locus of the points Y.



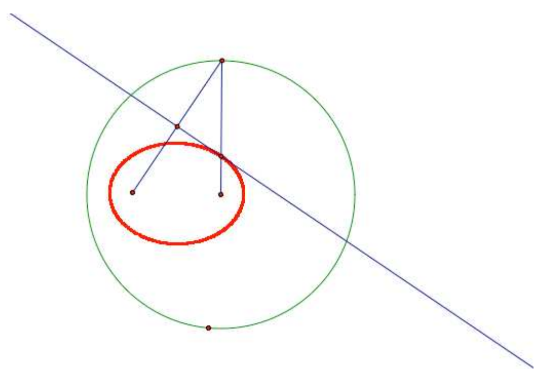


Figure 2: The locus with A inside the circle