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Shifting Parabolas

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First we will graph the parabola $y = 2x^2 + 3x - 4$.

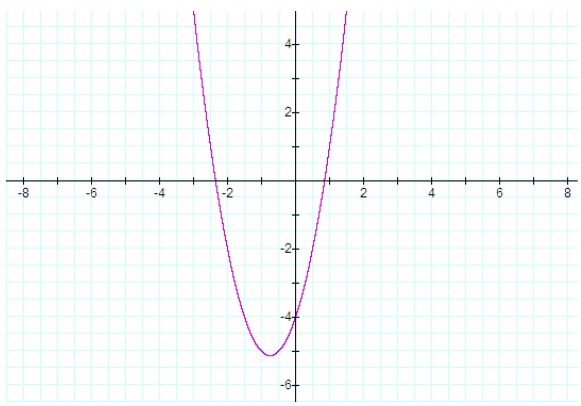

Figure 1: graph of $y = 2x^2 + 3x - 4$

Next we overlay the parabola (shown in red) obtained by replacing x with $x - 4$.
As we should expect this new parabola is shifted to the right 4 units. Now suppose we want to shift the parabola so that its vertex is in the second quadrant. We see that the original parabola has its vertex in the fourth quadrant with its y-coordinate about -5. So adding say 7 to the right hand side of the equation should give us the desired result. We graph $y = 2x^2 + 3x - 4 + 7 = 2x^2 + 3x + 3$ in blue below.

Now suppose that we want to produce a parabola that shares the same vertex but is concave down. We will first complete the square to write our parabola in a form where we can easily "flip it over."

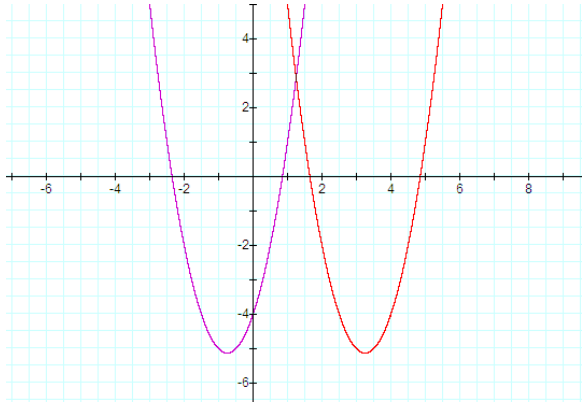

Figure 2: graph of $y = 2(x-4)^2 + 3(x-4) - 4$

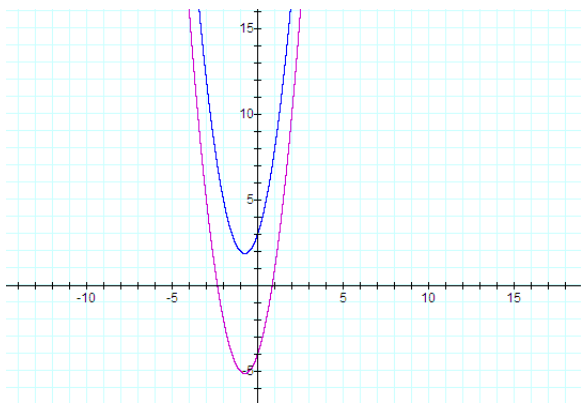

Figure 3: graph of $y = 2x^2 + 3x + 3$

$$
\begin{aligned}
y &= 2x^2 + 3x + 3 \\
\frac{y}{2} &= x^2 + \frac{3}{2}x + \frac{3}{2} \\
&= x^2 + \frac{3}{2}x + \frac{9}{16} - \frac{9}{16} + \frac{3}{2} \\
&= \left(x + \frac{3}{4}\right)^2 + \frac{15}{16} \\
y &= 2\left(x + \frac{3}{4}\right)^2 + \frac{15}{8}
\end{aligned}
$$

Since the smallest $2\left(x+\frac{3}{4}\right)^2$ can be is 0, which happens at $x=-3/4$, we know our vertex is at $(-3/4, 15/8)$. Furthermore if we replace the 2 by -2, i.e. make a new parabola with equation $y=-2\left(x+\frac{3}{4}\right)^2+\frac{15}{8}$, then since $-2\left(x+\frac{3}{4}\right)^2 \leq 0$ we will have a new parabola with vertex $(-3/4, 15/8)$ opening downward. Note also that our new parabola (graphed in green below) is symmetric to our original parabola over the line $y=15/8$.

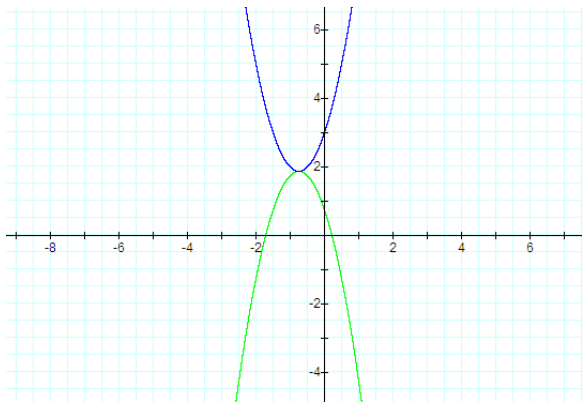

Figure 4: graph of $y=-2(x+3/4)^2+15/8$

Now suppose we want to place a parabola in the plane with its vertex at (a,b). We can make an upwards facing parabola by plotting $y=(x-a)^2+b$ or a downwards facing parabola by plotting $y=-(x-a)^2+b$. We can adjust the "fatness" of the parabola by introducing a parameter c and plotting $y=c(x-a)^2+b$.