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Maximum Picture Viewing Angle

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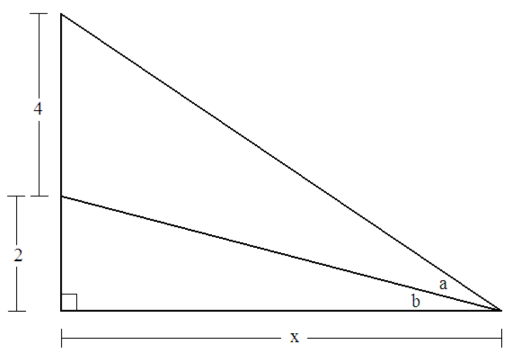
Problem: A 4 by 4 picture hangs on a wall such that its bottom edge is 2 ft above your eye level. How far back from the picture should you stand, directly in front of the picture, in order to view the picture under the maximum angle?

Solution: Let the distance x and angles a and b be as in the figure below.



Thus $\tan(a+b) = \frac{6}{x} \Rightarrow a + b = \tan^{-1}\frac{6}{x}$. Also we have $\tan b = \frac{2}{x} \Rightarrow b = \tan^{-1}\frac{2}{x}$. So

$$a = \tan^{-1}\frac{6}{x} - b = \tan^{-1}\frac{6}{x} - \tan^{-1}\frac{2}{x}$$

Therefore

$$\begin{aligned}
\frac{da}{dx} &= \frac{1}{1+\frac{36}{x^2}} \cdot \frac{-6}{x^2} - \frac{1}{1+\frac{4}{x^2}} \cdot \frac{-2}{x^2} \\
&= \frac{-6}{x^2+36} + \frac{2}{x^2+4}
\end{aligned}$$

Setting $da/dx = 0$ gives us

$$\begin{aligned}
\frac{6}{x^2+36} &= \frac{2}{x^2+4} \\
\frac{3}{x^2+36} &= \frac{1}{x^2+4} \\
3x^2+12 &= x^2+36 \\
2x^2 &= 24 \\
x^2 &= 12 \\
x &= 2\sqrt{3}
\end{aligned}$$

Now we just need to establish that this value of x gives us a maximum angle a. $a = a(x)$ is a continuously differentiable function on $(0, \infty)$. Note that

$$\begin{aligned}
\lim_{x\to\infty} a &= \lim_{x\to\infty} (\tan^{-1}\frac{6}{x} - \tan^{-1}\frac{2}{x}) \\
&= \tan^{-1} 0 - \tan^{-1} 0 \\
&= 0 \\
&= \frac{\pi}{2} - \frac{\pi}{2} \\
&= \lim_{x\to 0^+} (\tan^{-1}\frac{6}{x} - \tan^{-1}\frac{2}{x}) \\
&= \lim_{x\to 0^+} a
\end{aligned}$$

Also, by the monotonicity of arctangent we have that for all $x \in (0, \infty)$, $a(x) > 0$, since

$$\frac{6}{x} > \frac{2}{x} \Rightarrow \tan^{-1}\frac{6}{x} > \tan^{-1}\frac{2}{x}$$

(this is apparent from the physical setup of the problem as well). Thus we have a positive, continuously differentiable function on $(0, \infty)$ that limits to zero on both ends with one critical value. Therefore, the critical value $x = 2\sqrt{3}$ ft, gives us a maximum viewing angle.