Unit Plan - Lesson #3

- **Topic:** Hyperbolas
- **Grade Level:** 11 (Math 3)
- **GPS Standards:**
    - **Content Standards:**

      MM3G2. Students will recognize, analyze, and graph the equations of the conic sections (parabolas, circles, ellipses, and hyperbolas).
      b. Graph conic sections, identifying fundamental characteristics.

    - **Process Standards:**

      MM3P3. Students will communicate mathematically.
      a. Organize and consolidate their mathematical thinking through communication.
      b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Mathematical Goals: At the end of this lesson students will be able to construct a hyperbola and understand its graphical representation. They will also be able to understand a hyperbola from its definition: as the set of all points such that the difference of the distances from a point P to two fixed points is constant. Understanding of the graphical representation and the definition of a hyperbola will be evaluated by analyzing the answers students provide to the questions in each activity.

Materials to be used:

- Two computer desktops with Geometer's Sketchpad (GSP) installed
- Sixteen sheets of paper with two sets of concentric circles printed on them
- Sixteen pencils

Prior Knowledge Needed:

- Basic Proficiency in Navigating Geometer's Sketchpad (including use of menus and the tools palette)

- Properties of Segments (including definition, midpoint, perpendicular segments)
- Properties of Lines (including definition, distance to a line)
- Properties and Construction of Ellipses (from the first lesson)

Out of the four conic sections, the hyperbola is the shape to which students probably have the least amount of exposure. Up to this point in the Integrated GPS curriculum, the closest students have come to working with a hyperbolic shape is in the graphing of rational functions. This task builds upon students' previous exposure to and experience with constructing points and lines in the coordinate plane. Although students have previously worked with these two objects in the coordinate plane separately, they have not yet used them prior to this unit to find other points on the plane. The definition of a hyperbola, the set of all points such that the difference of the distances from a point P to two fixed points is constant, is the basis for the development of the understanding of hyperbolas. This task will also build on students' prior exposure to ellipses in the first lesson and compare the construction of ellipses to the construction of hyperbolas. These connections are made in the task and in the closing discussion.

Potential Challenges to Struggling Students or IEP/ELL Students

Although much consideration has taken place to provide a thorough yet efficient lesson, there are potential challenges that might be met by students who struggle with basic concepts or by students who are either designated as English Language Learners (ELL) or have other Special Education needs. The primary challenge that will be met will be in the usage of mathematical terminology and vocabulary to describe the ideas and concepts covered in the lesson. This issue can be addressed by restating mathematical terminology and vocabulary with simpler language. For example, if a student has difficulty understanding the meaning of the term "equidistant", then alternate terminology, such as "equal distance" or "same distance", can be utilized. When alternate terminology is employed along with drawings and other representations, this challenge can be minimized.

There are also a multitude of potential challenges that can face students with Specific Learning Disabilities (LD). The best method to pursue to address challenges to these students begins with the teachers in the Special Education program. Collaboration with these professionals will lead to strategies to address specific student concerns in the lesson.

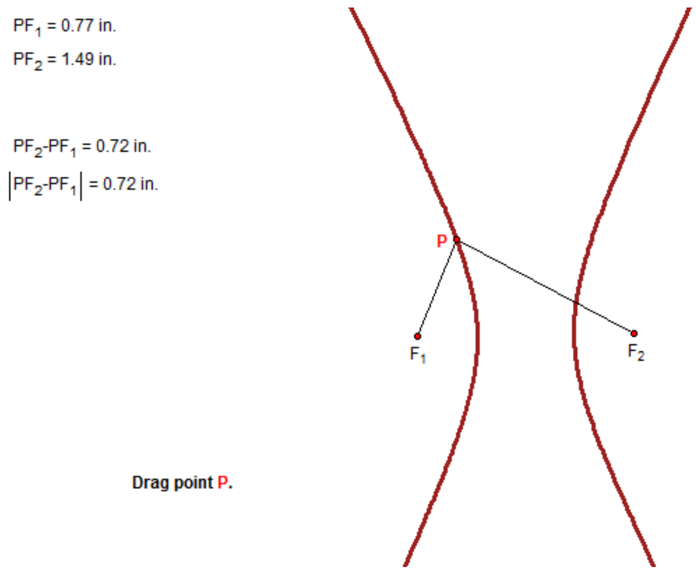
Classroom Setup:

The classroom period is 50 minutes. After an introduction that provides the definition of a parabola and connections to other academic subjects and everyday life, the students will be split into three groups that will rotate through three stations. Each rotation will last one class period. One station will have the worksheets and pencils for constructing a hyperbola using concentric circles. The other two stations will each consist of a desktop computer with Geometer's Sketchpad installed for dynamic constructions. Since there are sixteen students in each class, two groups will have five students and the third group will have six students. Each student will be engaged in one of five roles utilized at each station: instruction reader, instruction follower, second instruction follower, question reader, and question answerer (writes answers to questions on supplied worksheets). The extra student in the third group will share duties with another group member. Students will switch roles as they rotate from station to station and will fill each role once.

Lesson Structure

I. Introduction

To introduce the lesson, the basic properties of a hyperbola are presented. The graph of a rational function is reviewed focusing on the different parts of the graph, including the asymptotes. Students are told that a hyperbola can be seen as two parabolas facing (or opening) in opposite directions. On the Promethian Board, a GSP Sketch of a hyperbola is opened.



The parts of the hyperbola are introduced, and the definition is provided to the students. This definition is compared to the definition of an ellipse, where the <u>sum</u> of the distances is constant. The following questions are then posed: "How is the hyperbola similar to the ellipse? How are they different?" Students are given an opportunity to make conjectures that are written on the board. The students are instructed to keep these ideas in mind as they progress through the lesson tasks. The tasks will be introduced along with the steps required to complete the tasks. Each group will be handed a packet of three pages, each page having the steps and questions to be answered at the end of each task.

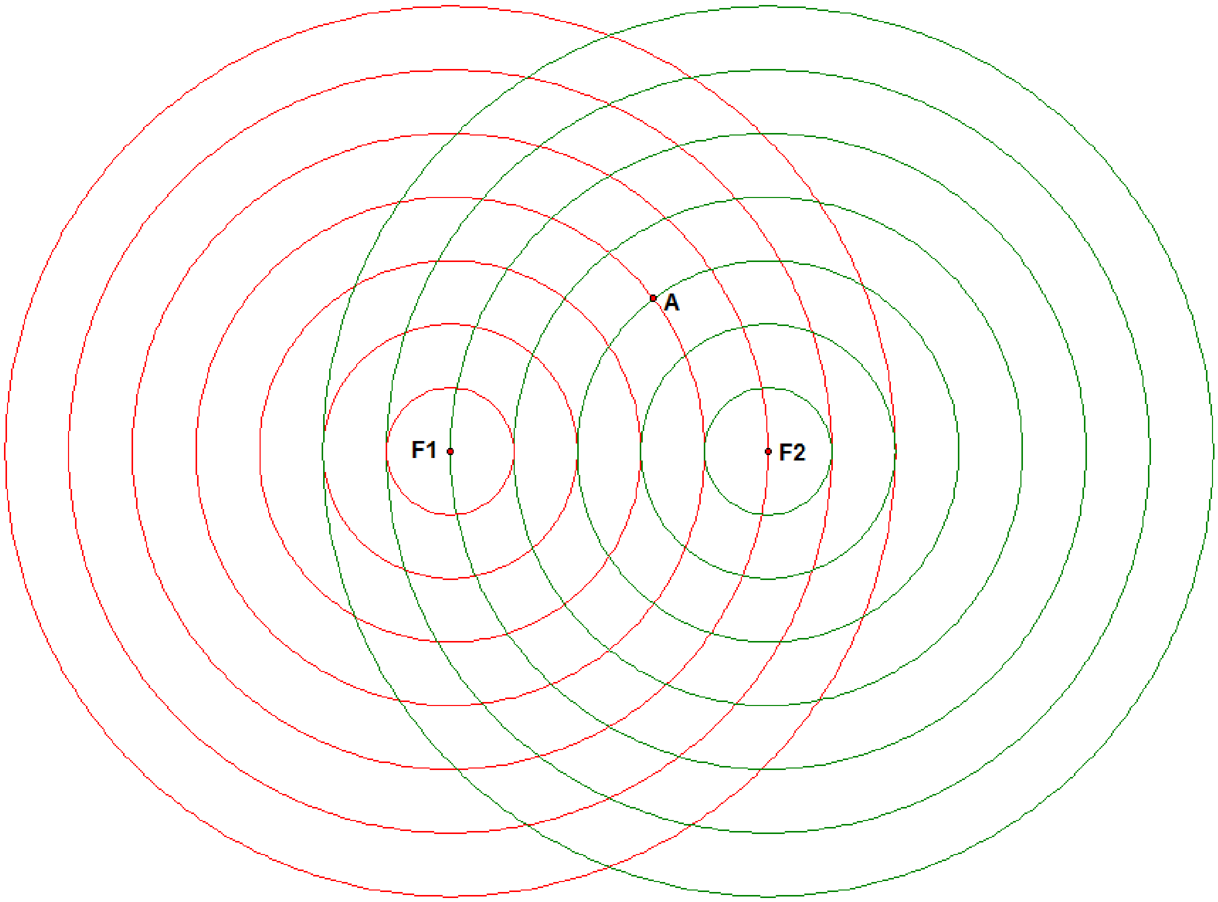
The review of the basic properties of lines, and points ensures access to all students, since the students, by the time they have reached Math III, have had multiple exposures to these topics in Math I and Math II. Cognitive demand is maintained throughout the task by focusing students' attention on procedures that are utilized to develop a better understanding of parabolas. By following the step-by-step procedures and by answering the questions that follow them, connections are established between students prior knowledge and the underlying concepts involved with understanding parabolas. The tasks result in graphical representations of hyperbolas that, in later discussions, will lead to algebraic representations that are compared and contrasted to the equations of circles, parabolas, and ellipses, with greater awareness of the connection between all four shapes. The questions that follow the procedures force students to engage with the ideas and concepts so that the task can be completed successfully and understanding is created.

II. Investigation

During the investigation, each group goes to the assigned starting station. The students in the group are allowed to pick the role they will assume in their starting station, with the understanding that each group member must fill a different role at each station until each member has filled all five roles. This change of roles will help to keep students engaged and minimize frustration with the task. Whenever possible, the teacher will have extra steps to follow to extend the activity for students who finish the activity quickly. During the investigation the teacher will monitor group work, only joining the groups and posing questions when needed, as well as making notes to select student responses for whole class discussion. Since the students are following "*procedures with connections*" tasks, steps are followed during the investigation phase that lead to solution strategies which are shared and discussed during the Whole Class Discussion Phase.

The tasks for each station are as follows:

<u>Station 1</u> uses the sheets of paper with concentric circles printed on them.



There are two groups of seven concentric circles, one group centered at F1 and the other group centered at point F2. The radii of the circles each increase by one unit all the way up to seven units. Points F1 and F2 are the foci of an infinite number of hyperbolas, but in this activity we are only interested in constructing the hyperbola that passes through point A. The following questions/steps are asked at Station 1:

- How many units apart are points A and F1? How many units apart are points A and F2? What is the numerical value for the difference between these two distances?
- Locate and mark at least 16 points that sit on either branch of the hyperbola that passes through point A. How did you find them?

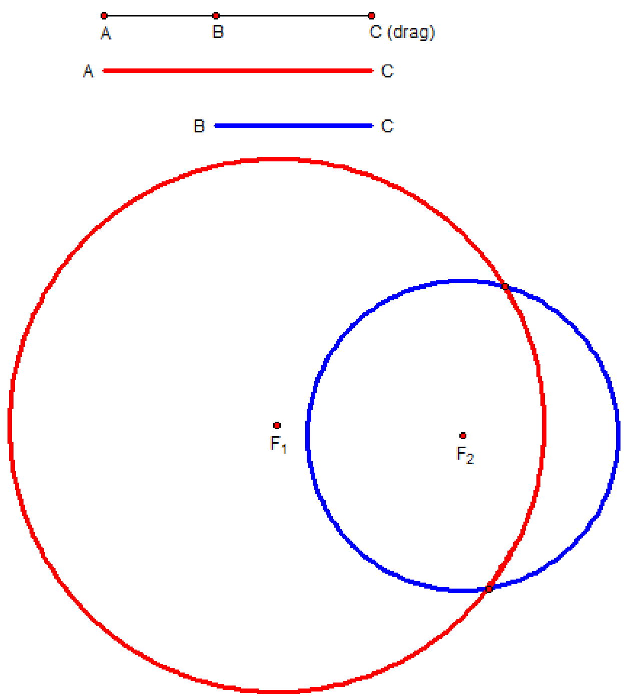
- Using the points you found as guidelines, sketch both branches of the hyperbola.

<u>Possible Solution Strategies for Station 1</u>

Students must take care to use the circles as representations of distance from the foci to point A. As long as the students can find the distance from F1 to point A and from F2 to point A and find the numerical difference between those two distances, the student should be able to use the circles to find other distances that can be subtracted to get the same numerical difference. As long as they realize that the intersection points that yield the same difference in distance are on the hyperbola, then the points should be relatively easy to find.

There is a possible difficulty that may arise with students who have visual perception problems. These visual perception problems can cause students to count the incorrect number of circles, resulting in the students choosing incorrect intersection points. This problem is alleviated by displaying each group of concentric circles in a different color.

<u>Station 2</u> uses a desktop workstation with Geometer's Sketchpad (GSP) installed on it to model the concentric circle exercise performed at Station 1. This activity will show how the hyperbola changes shape as its parts are relocated. A premade GSP sketch is opened that shows two circles, one red and one blue.



The sizes of the two circles are controlled by the segments at the top of the screen. The red circle has radius AC and the blue circle has radius BC. To operate this model point C is dragged and the radii adjust to remain equal to AC and BC. The intersections of the two circles are also traced as the circles change shape.

The following questions are asked at Station 2:

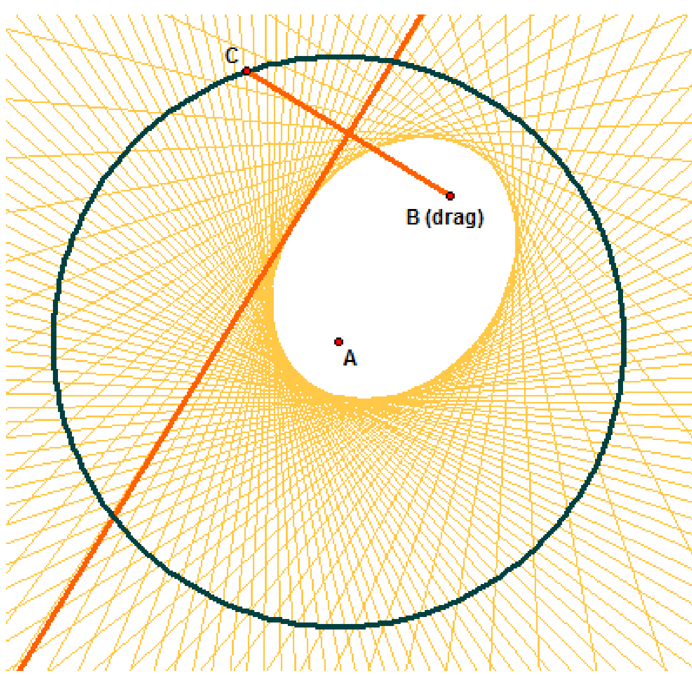
- As point C is dragged, changing the radii of both circles, there is a relationship that exists between the two radii whenever C is not between A and B. What is that relationship?
- Why do the circle intersection points trace a hyperbola?

- Select the two circles and choose Trace Circles from the Display menu, then drag point C. What similarities exist between this GSP construction and the Concentric Circles activity from Station 1?

<u>Possible Solution Strategies for Station 2</u>

By observing the segment at the top of the screen, students can see that whenever point C is not between A and B the difference between AC and BC is always AB. For this reason, since the difference is constant, the intersections of the two circles will trace a hyperbola. When the two circles are traced, the two circles create two groups of concentric circles, just like the activity in Station 1.

<u>Station 3</u> uses a desktop workstation with Geometer's Sketchpad (GSP) installed on it to extend the paper folding activity in Station 3 from the lesson on ellipses. In that construction, paper folds were created on circular pieces of paper so that the circumference of the paper lied on a point inside the circle, point B. In Station 4 of the same lesson on ellipses, a GSP sketch was created that modeled that paper folding activity.



The following questions are asked at Station 3:

- What happens to the ellipse when the focal point B is moved outside the circle?
- To find the tangent point of the ellipse we constructed a line through points A and C, then found the intersection of this line with the crease line. Will this intersection point still be tangent when B is moved outside the circle?

Possible Solution Strategies for Station 3

Simply by dragging point B outside the circle, the locus of crease lines now forms a hyperbola. Even though point B is moved outside the circle and the shape changes from ellipse to hyperbola, the intersection of line AC and the crease line is still tangent to the hyperbola. This can be seen as point C is dragged around the circumference of the circle.

III. Whole Class Discussion of Investigation

Once all four stations have been visited by all four groups, the students will return to their own assigned seats in the class for a discussion of the findings. A representative from one of the three groups (the group is preselected during teacher's monitoring of the activities) will come to the overhead (ELMO document camera) and demonstrate the group findings. The teacher will ask the selected group for the reasoning behind their findings. The teacher will also ask the other groups if their findings were similar or the same and will ask them to explain the reasoning behind different outcomes. This process will repeat, beginning with Station 1 and ending with Station 3.

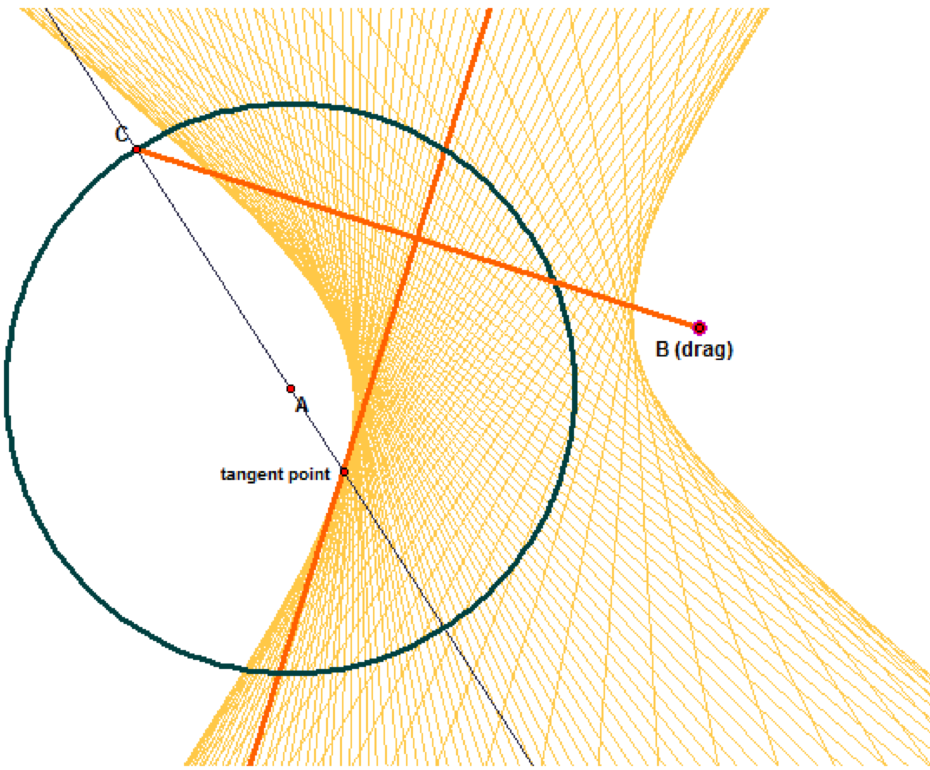
After the findings from each activity are discussed, questions are posed to make sense of the findings and connect them to the properties of hyperbolas, expand on the ideas suggested from the findings, and begin to form generalizations from the patterns that are observed.

The following questions from the entire activity were posed:

- How was the concentric circles activity for parabolas similar to the one for hyperbolas? How was it different?
- If the ellipse can be seen as two parabolic shapes that connect, how can the hyperbola be described?

IV. Closure and Extension

To wrap up discussion and close the lesson, the key properties of a hyperbolas are summarized through analysis of conjectures posed during the introduction, observations and applications of the hyperbola are discussed, including light projected on a wall from a cylindrical lamp and the shape of mirrors in reflecting telescopes, and an assessment is used which is based on a final class activity. We use the Promethian Board and open the GSP sketch from Station 3 with line AC and its intersection with the crease line.



The following question is asked as a Ticket out of the Door:

- When we move point B outside of the circle, how can we use triangle congruency to prove that the intersection point is tangent to the hyperbola?

References

Scher, D. (2002). *Exploring conic sections with the geometer's sketchpad*. (pp. 3-13). Emeryville, CA: Key Curriculum Press.

Conics & sketchpad . (2011, February 15). Retrieved from http://www.youtube.com/watch?v=X3HhpS3m0Cw