The University of Georgia

Mathematics Education Program

J. Wilson, EMAT 6600

7-11 Problem

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Goal: Identify the prices of 4 items that which sum and product is $7.11

Problem

A guy walks into a 7-11 store and selects four items to buy. The clerk at the counter informs the gentleman that the total cost of the four items is $7.11. He was completely surprised that the cost was the same as the name of the store. The clerk informed the man that he simply multiplied the cost of each item and arrived at the total. The customer calmly informed the clerk that the items should be added and not multiplied. The clerk then added the items together and informed the customer that the total was still exactly $7.11.

What are the exact costs of each item?

7-11 Solution.

Let w, x, y and z be the prices then if we make the prices times 100 (to work without decimals at first) then

$w + x + y + z = 711$ and $wxyz = 711000000$

Now factoring 711000000, gives $2^6\,3^2\,5^6\,79$ so we know that one of the amounts must be a multiple of 79 (let that amount be w) so we set up a system with all the multiples of 79 that up to 711 we get the following table:

			w	x+y+z	w*y*z
	PRODUCT	SUM	MULTIPLES OF 79	possible sum of other 3	Possible product of other 3
1	711000000	711	79	632	9000000
2	711000000	711	158	553	4500000
3	711000000	711	237	474	3000000
4	711000000	711	316	395	2250000
5	711000000	711	395	316	1800000
6	711000000	711	474	237	1500000
7	711000000	711	553	158	1285714.286
8	711000000	711	632	79	1125000
9	711000000	711	711	0	1000000

Now we can rule out the multiples of 79 at 7 and 9 because at 7 the 158 does not divide evenly into

71100000 and at 9 the remainder is 0, which leaves nothing for the value of the other items.

			w	x+y+z	w*y*z
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1	711000000	711	79	632	9000000
2	711000000	711	158	553	4500000
3	711000000	711	237	474	3000000
4	711000000	711	316	395	2250000
5	711000000	711	395	316	1800000
6	711000000	711	474	237	1500000
7	711000000	711	553	158	1285714.286
8	711000000	711	632	79	1125000
9	711000000	711	711	0	1000000

The lines beginning at 5, 6 and 8 can be ruled out because the products of x, y and z exceed the maximum the they should be if they (x, y, and z) were all equal, that is, exceeds $(x+y+z)^3/27$. Recall that the product of 3 positive numbers is a maximum when they are all equal.

			w	x+y+z	w*y*z
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7	711000000	711	553	158	1285714.286
8	711000000	711	632	79	1125000
9	711000000	711	711	0	1000000

The lines beginning with 1, 2, and 3 can be ruled out because if sum x + y + z products is not a multiple of 5 then one of x, y, or z is not a multiple of 5. Choose z. Then xy is a multiple of 5^6 . Neither x nor y can be a multiple of 625 because this is too much a large portion of the 711 total. As such if x or y were 625 then only 7 cents would remain for the other two item in the first case (711 – 625 - 79) and nothing in the second and third cases (711 – 625 – 158 and 711 – 625

– 237). Thus x and y are multiples of 125. Thus z can be obtained by subtracting some multiple of 125 from the possible of 632, 553 and or 474. We have the following possible values.

possible x+y	multiple of 125	difference
632	250	382
632	375	257
632	500	132
553	250	303
553	375	178
553	500	53
474	250	224
474	375	99

The eight possible values 382, 257, 132, 303, 178, 53, 224, and 99 are all unacceptable because they have prime factors other that 2 or 3.

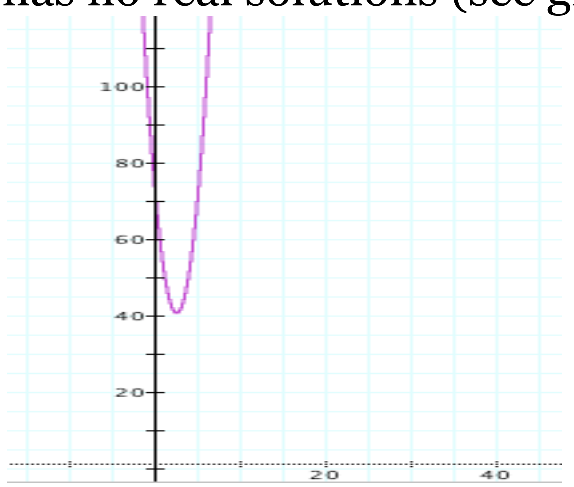
This leaves only the case of line beginning with 4 in the original table. Thus w = 316 = **$3.16**, x + y + z = 395, xyz = 2250000

			w	x+y+z	w*y*z
	PRODUCT	SUM	MULTIPLES OF 79	possible sum of other 3	Possible product of other 3
1	711000000	711	79	632	9000000
2	711000000	711	158	553	4500000
3	711000000	711	237	474	3000000
4	**711000000**	**711**	**316**	**395**	**2250000**
5	711000000	711	395	316	1800000
6	711000000	711	474	237	1500000
7	711000000	711	553	158	1285714.286
8	711000000	711	632	79	1125000
9	711000000	711	711	0	1000000

So the value of x= $3.16. At least one of x, y or z is a multiple of 5. If only one of these was a multiple of 5, it implies that it is 15625, which is larger than the sum 395. So let x, y and z be all multiples of 5 and so let x = 5x', y = 5y' and z = 5z', then x'+ y' + z' = 79 (i.e. 395/5) and x'y'z' = 18000 (i.e. 2250000/125)

The sum of the three new variables is not divisible by 5 so the three new variables are not all divisible by 5. At least one of the variables is divisible by 5 but it cannot be a multiple of 5^3 =125 because this exceeds 79 so it must be a multiple of 25. Thus one variable can be a multiple of 25 and one variable can be a multiple of 5. Let this variable be x' = 25x''. Let y' = 5y". So 25x'' + 5y" + z' = 79 and x"y"z'=144 (i.e. 18000/125).

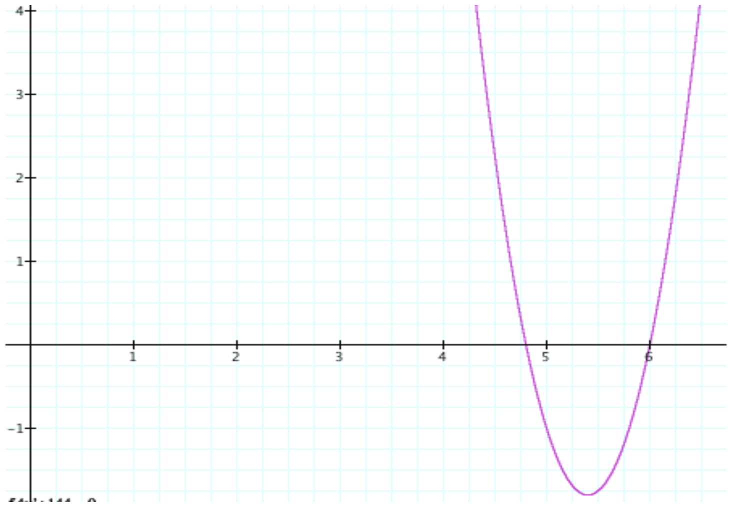
This leaves x" to be either 1 or 2. The value of 2 is ruled out because if we substitute x" = 2 then 5y" + z' = 29 and y"z' = 72 gives the $5y''^2-29y''+72=0$, which has no real solutions (see graph) .



Thus x" = 1 and

x = 5 (25x") = 125 **= $1.25**

We are now left with 2 variables to solve for. Because x" = 1, then 5y" + z' = 54 and y"z' = 144. Thus $5y''^2-54y'+144=0$



There are two solutions but we use the integer solution of y" = 6. . Therefore y = 5(5)(6) =150=**$1.50**.

For z we know that z' = 144/y" = 24 which means z =5z'=120=**$1.20**.

So the solution to the problem is for the prices of the 4 items :

$3.16, $1.25, $1.50, and $1.20

Check:

w	x	y	z	sum	product
3.16	1.25	1.5	1.2	7.11	7.11
