The University of Georgia

Mathematics Education Program

J. Wilson, EMAT 6600

Tangled Tale

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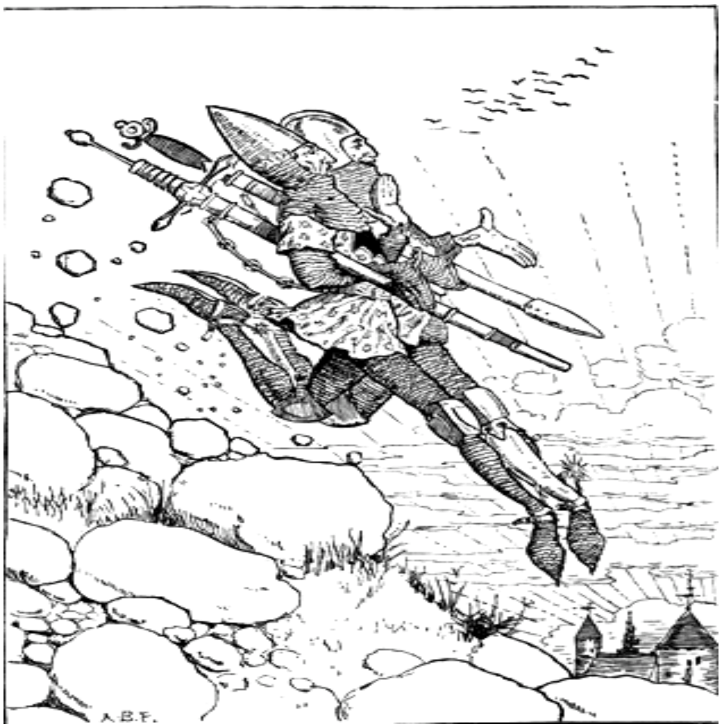
Goal: To calculate distance given different rates

A Tangled Tale Problem

A problem from Lewis Carroll --

A man walked for 5 hours, first along a level road, then up a hill, and then he turned around and walked back to the starting point along the same path. He walks 4 mph on the level, 3 mph uphill, and 6 mph downhill. Find the distance he walked.



Solution

Rates:

Level ground forward = 4 mph

Uphill forward = 3 mph

Downhill backwards = 6 mph

Level ground backwards = 4 mph

Solution 1

Average rate on level ground: 4 mph

Average rate on hill:

Let d = distance

When rate = 3, time = t_1 ; when rate = 6, time = t_2

$d = rt = 3\,t_1 = 6t_2$

Thus $3t_1 = 6t_2 \Rightarrow t_1 = 2t_2$

$$r = \frac{d}{t} = \frac{3t_1 + 6t_2}{t_1 + t_2} = \frac{6t_2 + 6t_2}{2t_2 + t_2} = \frac{12t_2}{3t_2} = 4$$

Thus average rate is 4 miles per hour over the entire distance on level ground and on the hill.

The total distance that the man travels at a rate of 4 miles per hour for 5 hours

Distance = rate * time = 4 * 5 = 20 miles

Solution 2 Using the Harmonic Mean

From the elaboration provided, the same distance d is traveled at two different rates r_1 and r_2. Denote by t_1 and t_2, times travelled at the two rates.

Total time $t = t_1 + t_2$

Let $d = r_1t_1$, and $d = r_2t_2$ be the distance travelled forwards and backwards

Then

$2d = rt = r\,(t_1 + t_2)$

$\quad = r\left(\frac{d}{r_1} + \frac{d}{r_2}\right)$

$\Rightarrow 2 = r\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$ [dividing by d]

$\Rightarrow r = \frac{2r_1r_2}{r_1 + r_2}$ [dividing by $\frac{1}{r_1} + \frac{1}{r_2}$]

Now substituting the values for r_1 and r_2 as 3 and 6 respectively gives:

$r = \frac{(2)(3)(6)}{3 + 6} = 4$

so the harmonic mean of the rate on the hill is the same as the rate on the level ground = 4 miles per hour. Thus the average rate over the entire distance is 4 mph.

The total distance traveled is 4 mph * 5 h = 20 miles

Discussion

This problem works out neatly because the numbers 3 and 6 were carefully chosen so the mean rate would be the same on level ground and on the hill.

If different rates for going uphill and downhill were chosen such that the harmonic mean does not equal the rate on level ground then the question would be impossible to solve. For example if rates of 2 mph uphill and 5 mph downhill were chosen then the harmonic mean rate, $r = \frac{(2)(2)(5)}{2 + 5} = \frac{20}{7} = 2.857$ is not equal to 4, the rate on level ground, so it is not possible to calculate the distance in that case.