The University of Georgia

Mathematics Education Program

J. Wilson, EMAT 6600

Distance Survey

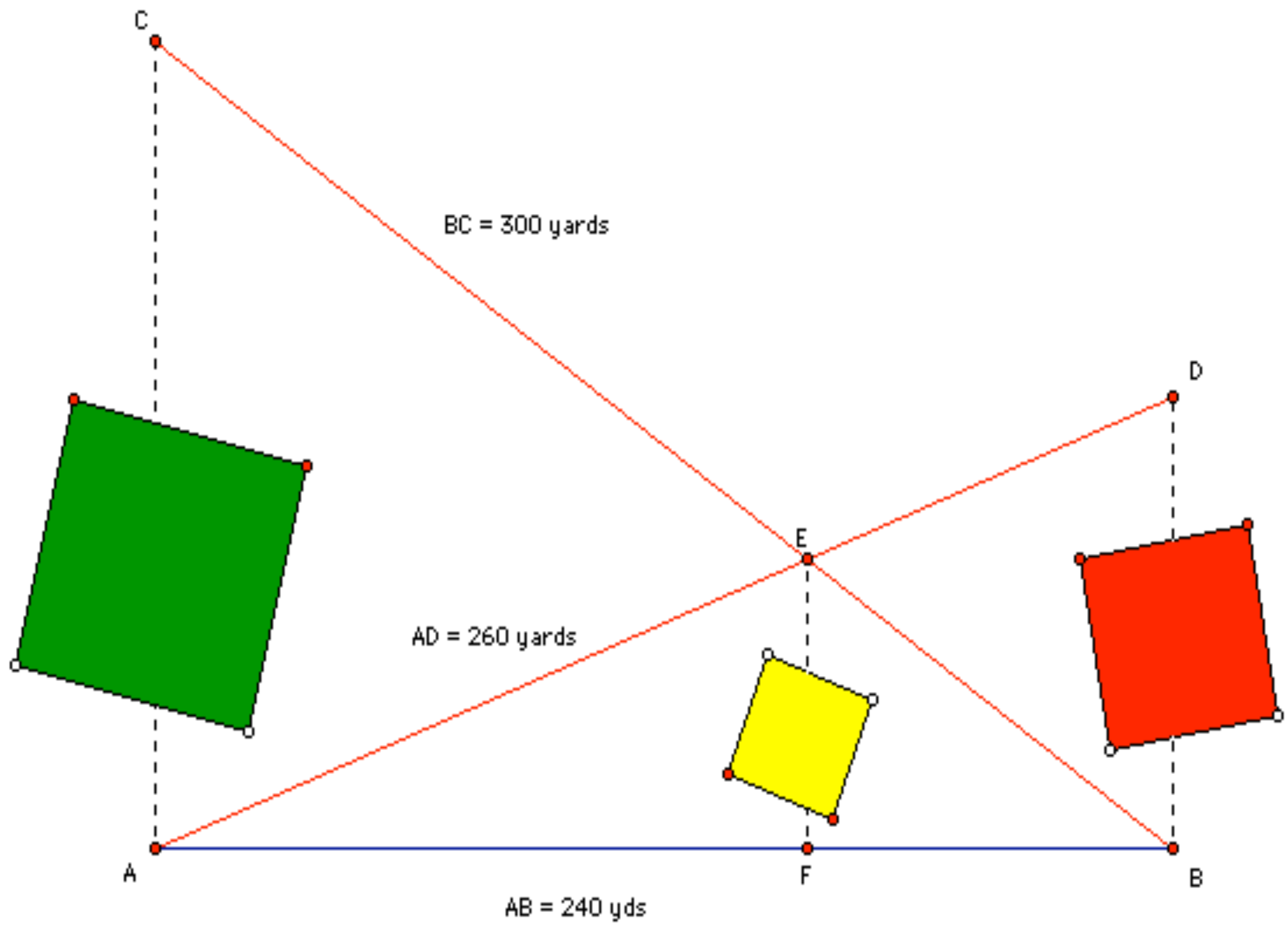
By Leighton McIntyre

Goal: to calculate distance given obstructions in path

Problem

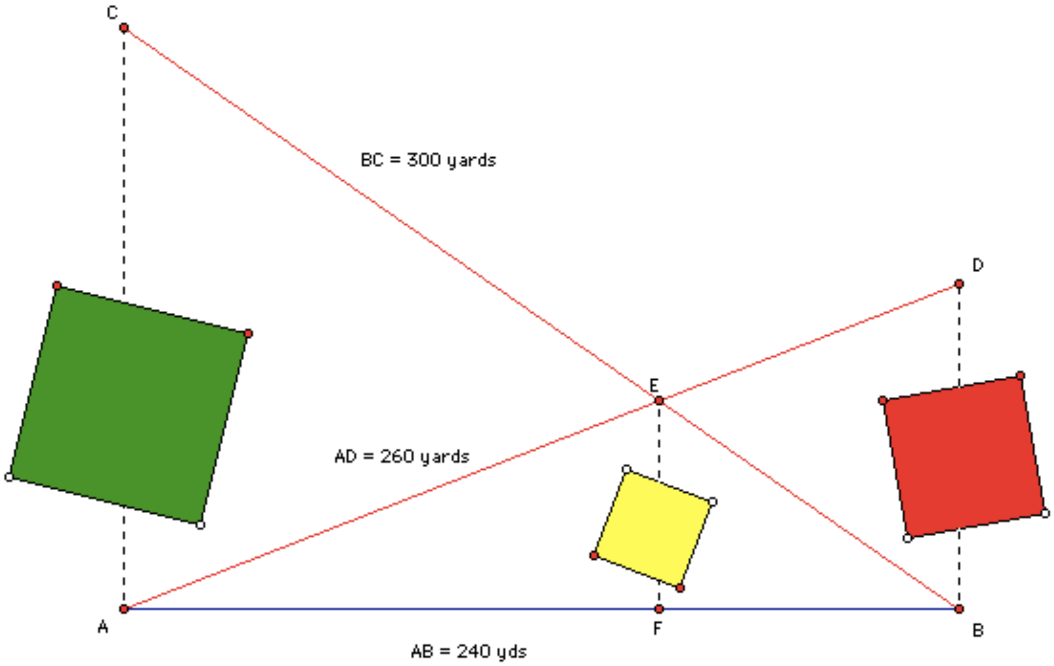
In conducting a land survey, the following problem arose. There were two points A and B along a road and points C and D off the road along the respective perpendiculars to the road at A and B. There were buildings on the property that prevented direct measurement of the distances along BD and AC. Measurements, however, could be made for AD, BC, and AB as follows:

AB = 240 yards
AD = 260 yards
BC = 300 yards



A light pole is to be installed at point E, the intersection of CB and AD. How far will the pole be from the road? That is, what is the distance EF? Again, an existing building blocks direct measurement.

Solution 1



AC ⊥ AB , BD ⊥ AB, ⇒ AC //BD

then BC and AC are both transversals to AC and BD. The following angle congruencies hold:

<ACE ≈ <BED , <ACB ≈ <CBD, <CEA ≈ <DEB

$\Delta ACE \sim \Delta BED$

$AC^2 = CB^2 - AB^2 = 300^2 - 240^2 = 32400$

hence AC = 180

$BD^2 = AD^2 - AB^2 = 260^2 - 240^2 = 10000$

hence BD = 100

Ratio of sides of Δ ACE to Δ BED 180:100 or 9:5

CE: BE = 9:5

$CE = \frac{9}{5}BE$ and CE +BE = BC

Hence BE = $\frac{5}{14}$BC = $\frac{5}{14}$*300 = $\frac{750}{7}$

AE = $\frac{9}{5}$DE and AE + DE = AD

Hence AE = $\frac{9}{14}$AD = $\frac{9}{14}$*260 = $\frac{1170}{7}$

Now using Herons Formula we can calculate the area of triangle ABE because we have the three sides and then find height EF

Area $_{\Delta ABE}$ = $\sqrt{s(s-a)(s-b)(s-c)}$

Where a = 240, b = $\frac{1170}{7}$, c = $\frac{750}{7}$, s = $\frac{a+b+c}{2} = \frac{1800}{7}$

Area $_{\Delta ABE}$ = $\sqrt{\frac{1800}{7}(\frac{1800}{7}-240)(\frac{1800}{7}-\frac{1170}{7})(\frac{1800}{7}-\frac{750}{7})} = \frac{378000}{49}$

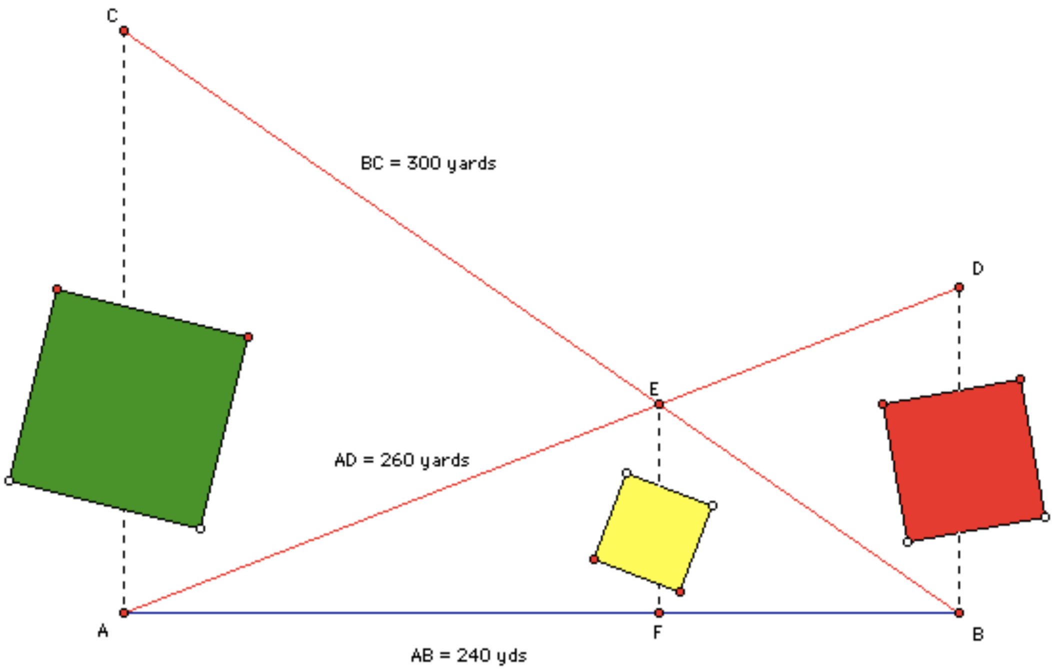
Now we know using the base of the triangle as 240 and EF as height in formula

Area $_{\Delta ABE}$ = $\frac{1}{2}$base*height

$\frac{378000}{49} = \frac{1}{2}$(240) EF

EF = $\frac{378000}{49*120} = \frac{3150}{49}$ = 64.29

Solution 2



AC ⊥ AB , BD ⊥ AB, EF ⊥ AB ⇒ AC //BD// EF

From solution 1, we calculated that AC = 180 and BD = 100

Denote by x the distance EF. Denote by y the distance FB

Consider Δ ABC and Δ FBE are similar by AA similarity.

That is <CAB ≈ <EFB, by right angles; <FBE ≈ <FBE, by reflexive property.

Consider Δ ABD and Δ AFE are similar by AA similarity.

That is <ABD ≈ <AFE, by right angles; <FAE ≈ <FAE, by reflexive property.

We have the following proportions

$$\frac{CA}{AB} = \frac{EF}{FB} \Rightarrow \frac{180}{240} = \frac{x}{y} \Rightarrow \frac{3}{4} = \frac{x}{y} \Rightarrow y = \frac{4}{3}x$$

$$\frac{DA}{AB} = \frac{EF}{FA} \Rightarrow \frac{100}{240} = \frac{x}{240-y} \Rightarrow \frac{5}{12} = \frac{x}{240-y} \Rightarrow 12x = 5(240 - (\frac{4}{3}x))$$

$$\Rightarrow 12x = 1200 - (\frac{20}{3}x))$$

$$\Rightarrow 12x + (\frac{20}{3}x)) = 1200$$

$$\Rightarrow \frac{56}{3}x = 1200$$

$$\Rightarrow x = 64.29$$
