The University of Georgia

Mathematics Education Program

J. Wilson, EMAT 6600

100 Degree Isosceles Triangle

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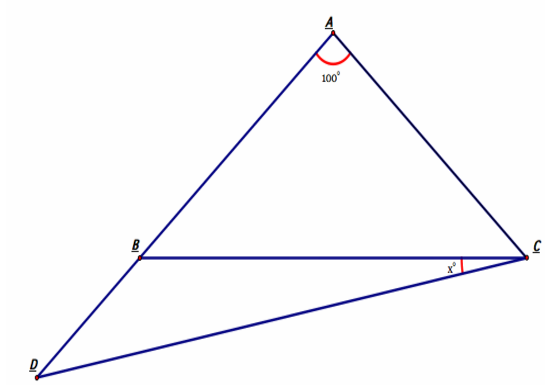
Goal: To find the missing angle.

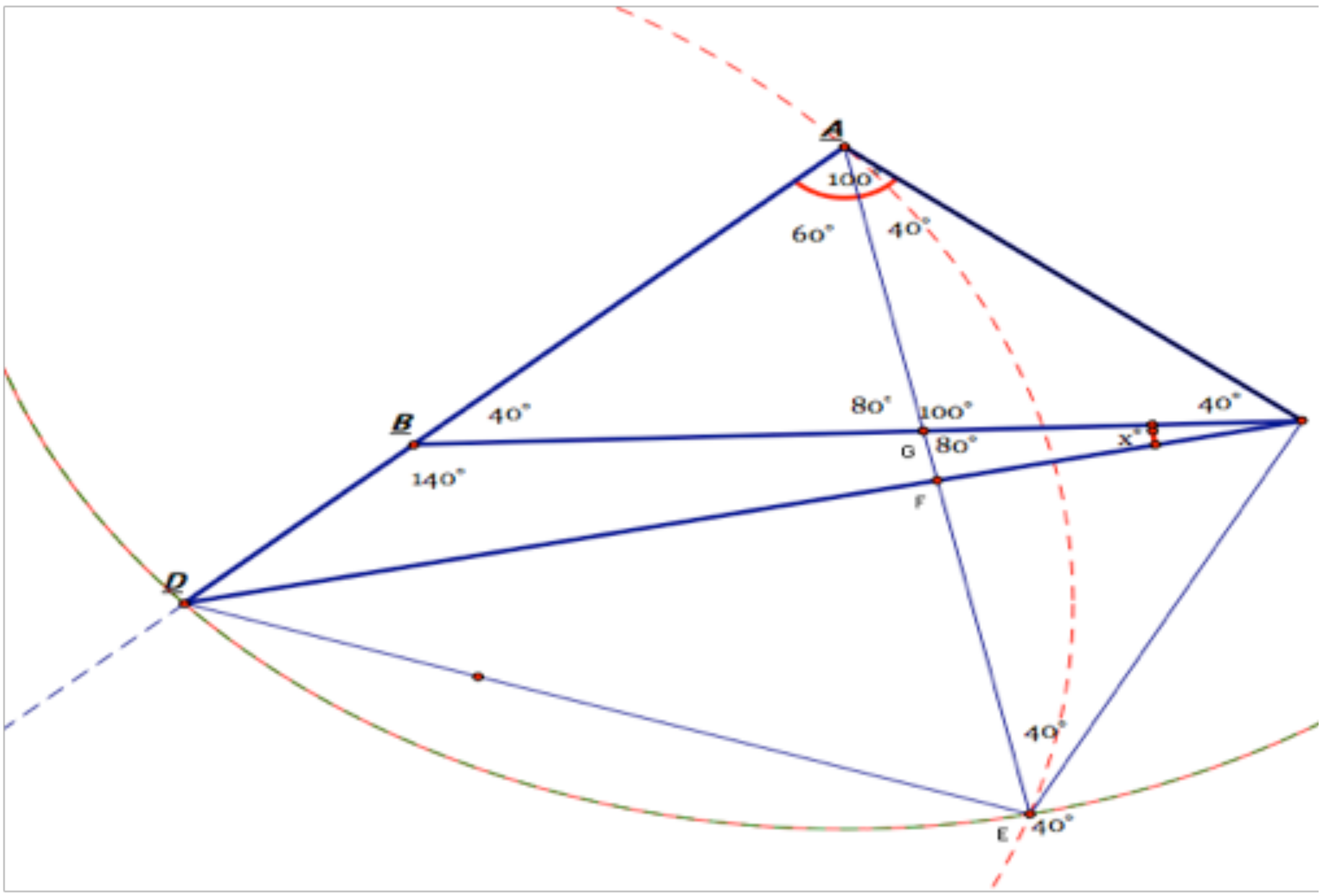
Problem

Given an isosceles triangle ABC with AB = AC and the measure of angle BAC = 100 degrees. Extend AB to point D such that AD = BC. Now draw segment CD. What is the measure of angle BCD? Prove your results by geometric reasoning, rather than measuring.



Solution 1: The Equilateral Triangle Strategy

By constructing an equilateral triangle using AD as one side and a third vertex E below the line BC we can join the vertex C to E and mark the angles as follows:



Recall that <BAC = 100°. Because <BAC = <DBE + < CBE, and <DBE = 60° then < CBE = 40°. Now by the construction we know that AD ≈ DE ≈ AE. We can also show that BC ≈ AE, AC ≈ AC and < CAE ≈ < ACB.

Therefore ΔBAC ≈ ΔCAE by SAS congruence. Hence, < ACE = < BAC = 100° .

Consider triangles ADC and DEC. Now AC ≈ CE, AD ≈ DE, DC ≈ DC, thus ΔADC ≈ ΔDEC by SSS congruence. Now because DC is the common side to the two congruent triangles then the angles of the triangles on opposites sides of DC will be congruent. Now <DCE = 100°. This is composed of two equal angles of triangles ADC and DEC, hence the two angles ACD and ECD = 50°.

Consider <ACD = <ACB + <BCD.

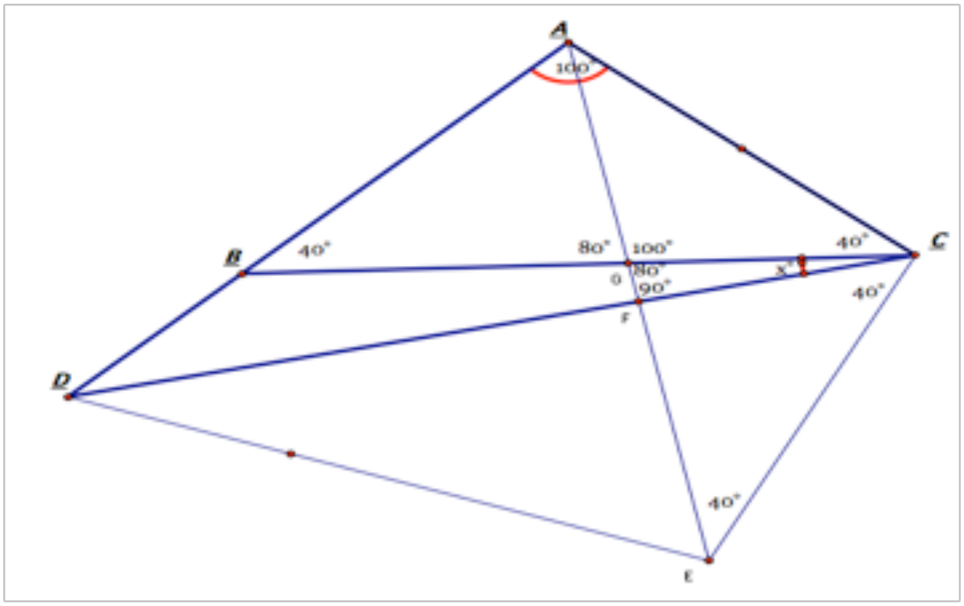
50° = <ACB + 40°

Hence, <BCD= 10°

Solution 2: The Reflection Strategy

Following the hints given in the problem the side BC is used as the side through which the shape is reflected and a new point E is made is the image of A after the reflection. A segment is constructed from A to E and the resulting angles are marked off as shown.

This discussion follows on from the solution 1 above, where the equilateral triangle ADE is also constructed.



We now have AD = DE, AC = CE, and the segment AE is perpendicular to DC. Hence < AFC = 90°. Consider Triangles ABC and ACE are congruent. Consider angles AGB and CGF are vertical angles = 80°.

Consider triangle CFG. <CFG + <CGF + <GFC = 180°.

90° + 80°+ <GFC = 180°.

Hence <GFC = 10°= <BCD