



The University of Georgia

Mathematics Education Program

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Areas of Lunes 1

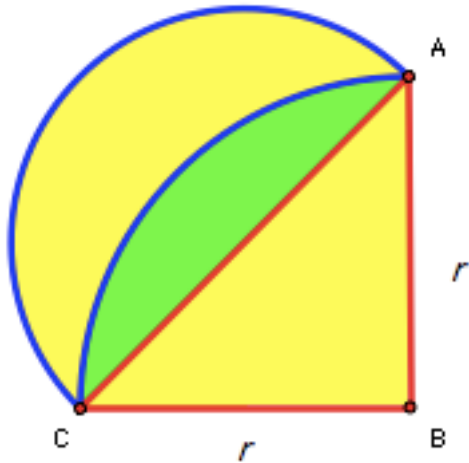
By Leighton McIntyre

Goal: To prove that area of lune on hypotenuse of isosceles right triangle is equal to the area of the triangle

Problem

Given a right triangle ABC . Construct a lune with vertices at A and C with the center of the inner arc being at C and the center of the outer arc being at the midpoint of segment AB .

Prove that the area of the lune so constructed is the same as the area of the triangle ABC .



$$\text{Area of } \triangle ABC = \frac{1}{2} \text{base} * \text{height} = \frac{1}{2} r^2$$

$$\text{Area of quarter circle} = \frac{1}{4} \pi r^2$$

Area of green sector is Area of quarter circle minus area $\triangle ABC = \frac{1}{4} \pi r^2 - \frac{1}{2} r^2$

$$= \left(\frac{1}{4} \pi - \frac{1}{2} \right) r^2$$

$$\text{Area of half circle (radius } \frac{r}{\sqrt{2}}) = \frac{1}{2} \pi \left(\frac{r}{\sqrt{2}} \right)^2 = \frac{1}{2} \pi \left(\frac{r^2}{2} \right) = \frac{1}{4} \pi r^2$$

Area of yellow lune = Area of half circle (radius $\frac{r}{\sqrt{2}}$)-

Area of green sector = $\frac{1}{4} \pi r^2 - \left(\frac{1}{4} \pi r^2 - \frac{1}{2} r^2 \right) = \frac{1}{2} r^2 = \text{Area of } \triangle ABC .$
