The University of Georgia

Mathematics Education Program

J. Wilson, EMAT 6600

Areas of Lunes 2

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Goal to show that the area of the two lunes on the legs of the right triangle is equal to the area of the right triangle.

Problem

On a Right Triangle

Given a right △ABC with hypotenuse AB.

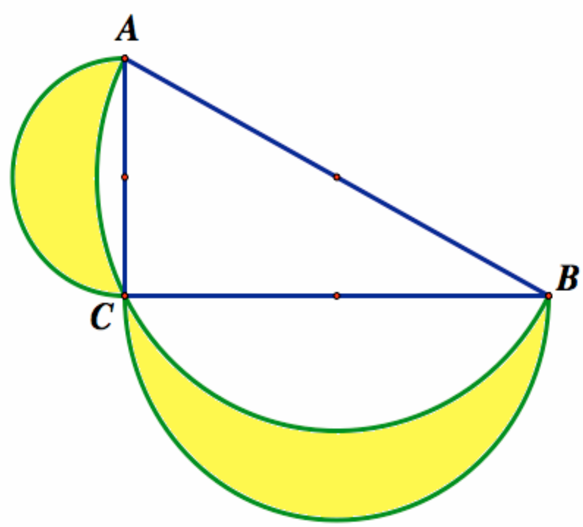
Construct a semicircle through A, B, and C with diameter AB.

Construct a semicircle with diameter AC so that the semicircle does not overlap the triangle.

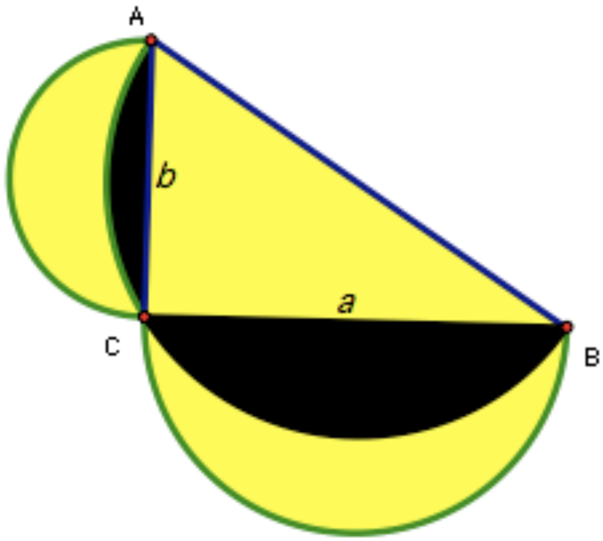
Construct a semicircle with diameter BC so that the semicircle does not overlap the triangle.

The result is two lunes spanning the legs AC and BC of the right triangle.

Prove that the sum of the area of these two lunes is equal to the area of △ABC.



Solution



Area of Δ ABC = $\frac{1}{2} base * height$ = $\frac{1}{2}$ab

Area of half circle diameter AB = $\frac{1}{2}\pi(\frac{\sqrt{a^2+b^2}}{2})^2$ =

$\frac{1}{8}\pi(a^2+b^2)$

Area of black sectors combined is Area of half circle (diameter AB) minus area Δ ABC = $\frac{1}{8}\pi(a^2+b^2)$ - $\frac{1}{2}$ab

Area of half circle (diameter BC) = $\frac{1}{2}\pi(\frac{a}{2})^2$ = $\frac{1}{8}\pi a^2$

Area of half circle (diameter AC) = $\frac{1}{2}\pi(\frac{b}{2})^2$ = $\frac{1}{8}\pi b^2$

Total Area of yellow lunes = Area of half circle (diameter BC) + Area of half circle (diameter AC) -

Area of half circle (radius $\frac{r}{\sqrt{2}}$)- Area of black sectors combined

= $\frac{1}{8}\pi a^2$ + $\frac{1}{8}\pi b^2$ - ($\frac{1}{8}\pi(a^2+b^2)$ -$\frac{1}{2}$ab) = $\frac{1}{8}\pi(a^2+b^2)$ - ($\frac{1}{8}\pi(a^2+b^2)$ +$\frac{1}{2}$ab = $\frac{1}{2}$ab = Area of Δ ABC . ♦