The University of Georgia

Mathematics Education Program

J. Wilson, EMAT 6600

Inscribed Quadrilateral

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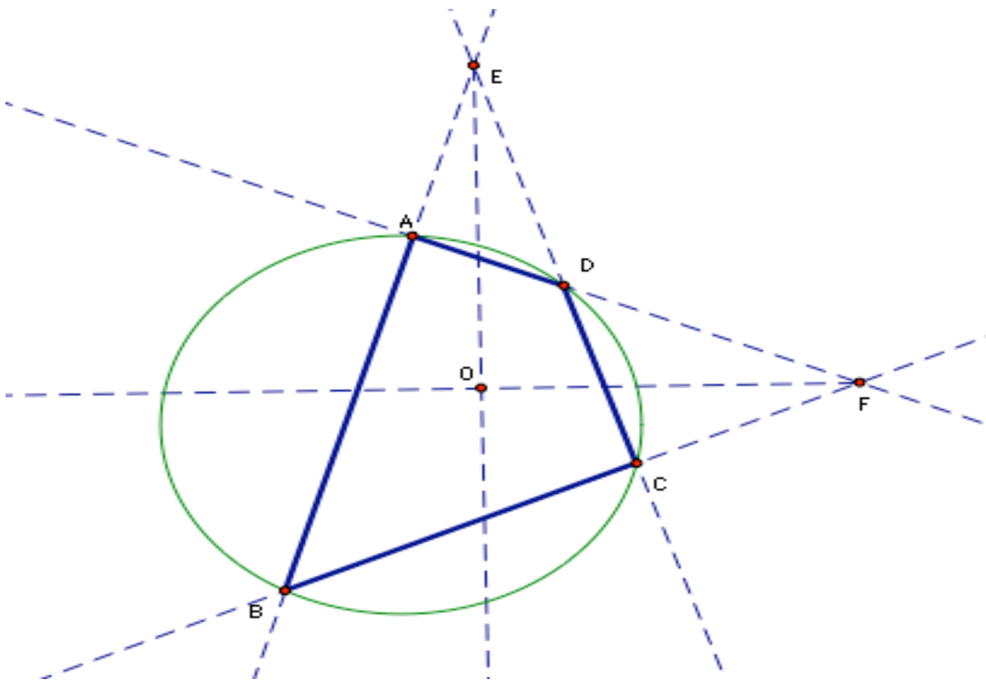
Goal: To prove that the angle bisectors of the opposite sides of an inscribed quadrilateral meet at right angles.

Problem

Claim: The angle bisectors of the opposite sides of an inscribed quadrilateral meet at right angles.

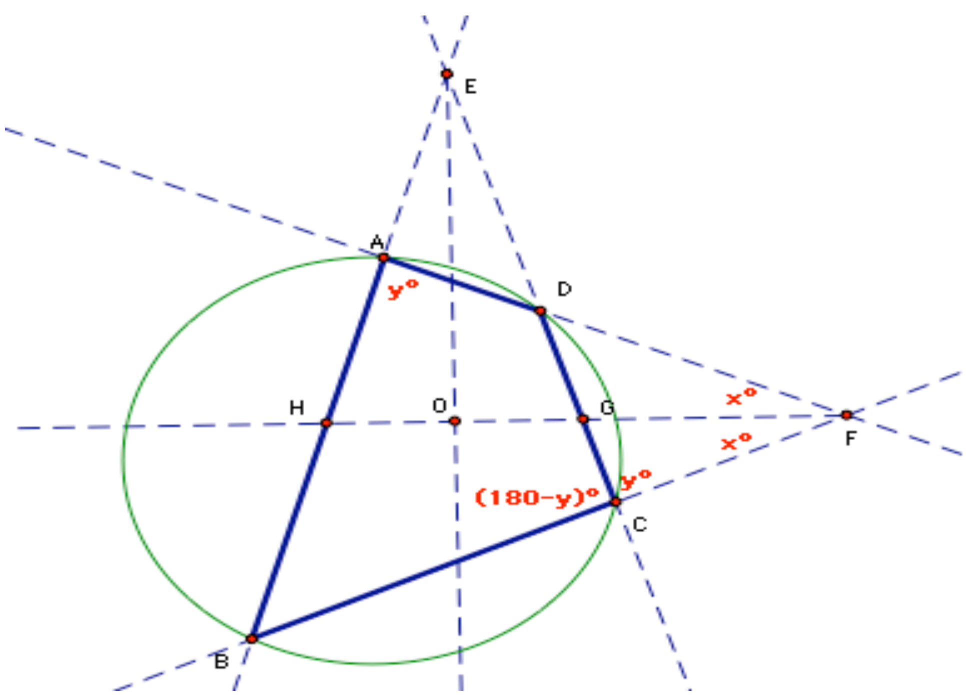
Proof

Consider Cyclic Quadrilateral ABCB. Let the extensions to the pair of opposite sides such that they intersect at points E and F. Then let the angle bisectors of AED and DFC intersect at point O.



The opposite angles of a cyclic quadrilateral are supplementary that is they sum to 180°.

Thus in the above diagram <BAD + <BCD = 180°



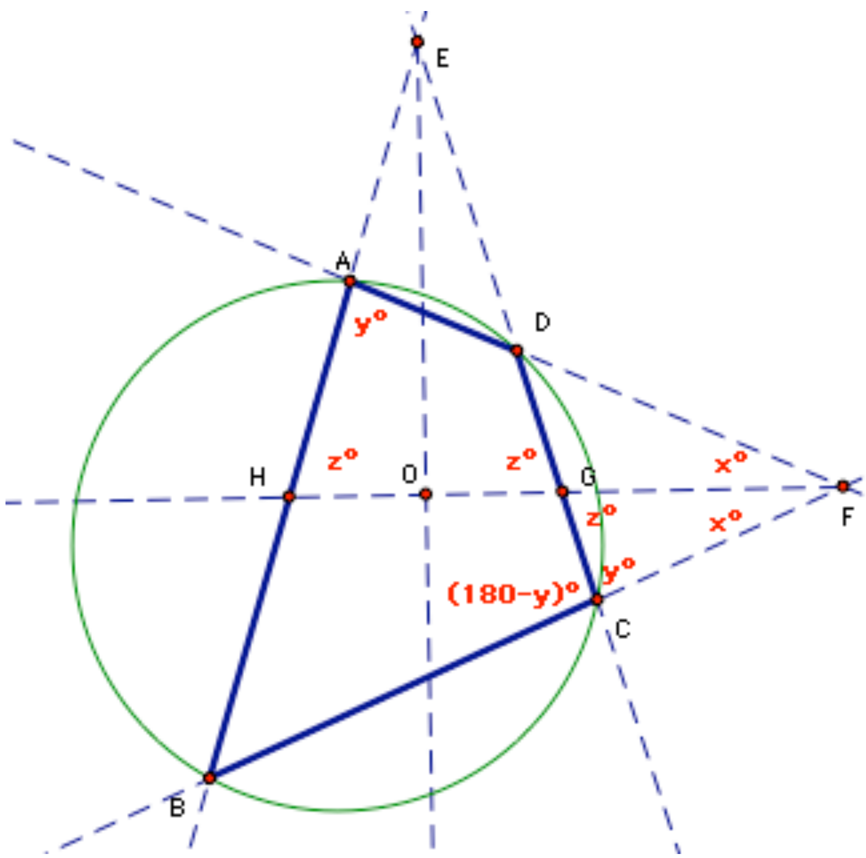
Let the angle bisector of <DFC intersect the cyclic quadrilateral at points G and H respectively.

Let <BAD of the cyclic quadrilateral be y^o and thus <BCD = $(180 - y)^o$. Consider <BCF is straight angle, so < GCF = y^o.

Let <CFG = <GFD because of angle bisector.

Consider Δ FGC and Δ FAH are similar by AA similarity.

Then < FGC ≈ FAH



Denote by z^o the angles FGC and FAH. Note < FGC ≈ <DGH by vertical angles.

Consider Δ EHG is isosceles with < EHG ≈ <EGH = z^o

Now EO is a bisector of the vertex angle of the isosceles triangle thus EO will intersect the base HG at right angles. Thus <EOG = 90^{o}. The base HG is part of the angle bisector of the other pair of opposite sides of the cyclic quadrilateral.

Hence the angle bisectors of the opposite sides of an inscribed quadrilateral meet at right angles.