The University of Georgia

Mathematics Education Program

J. Wilson, EMAT 6600

Island Treasure

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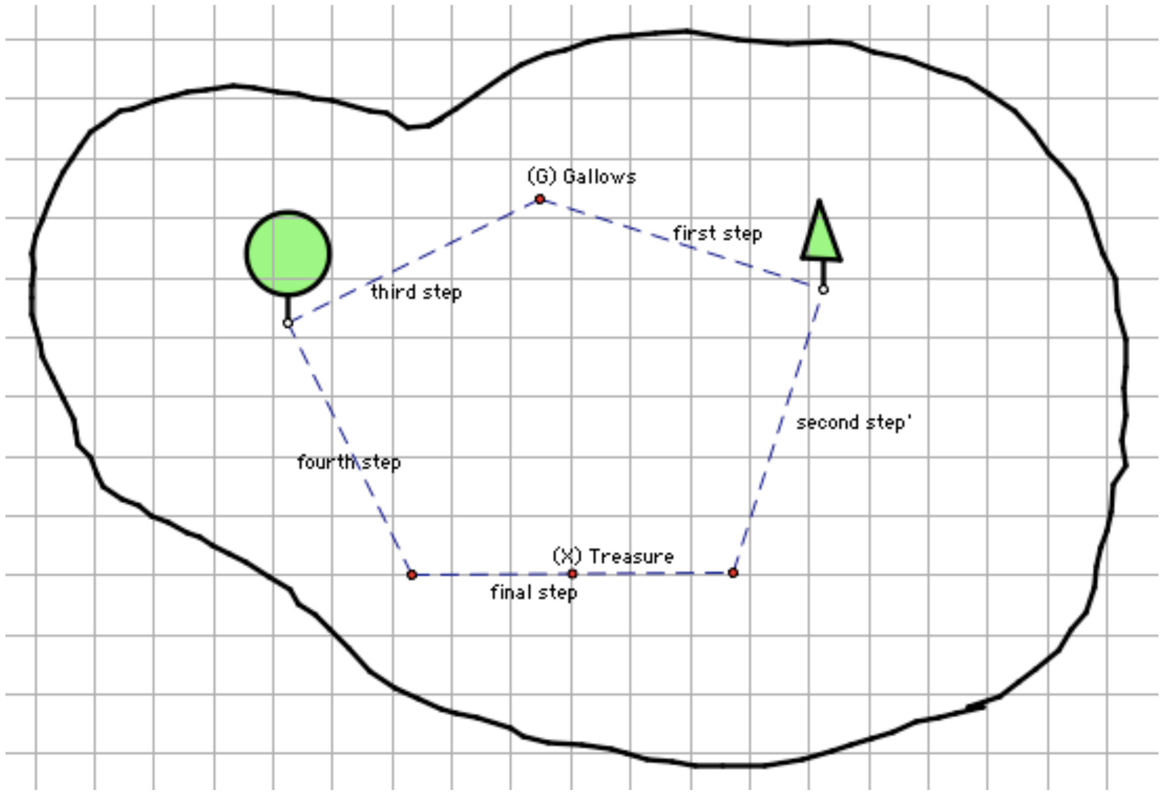
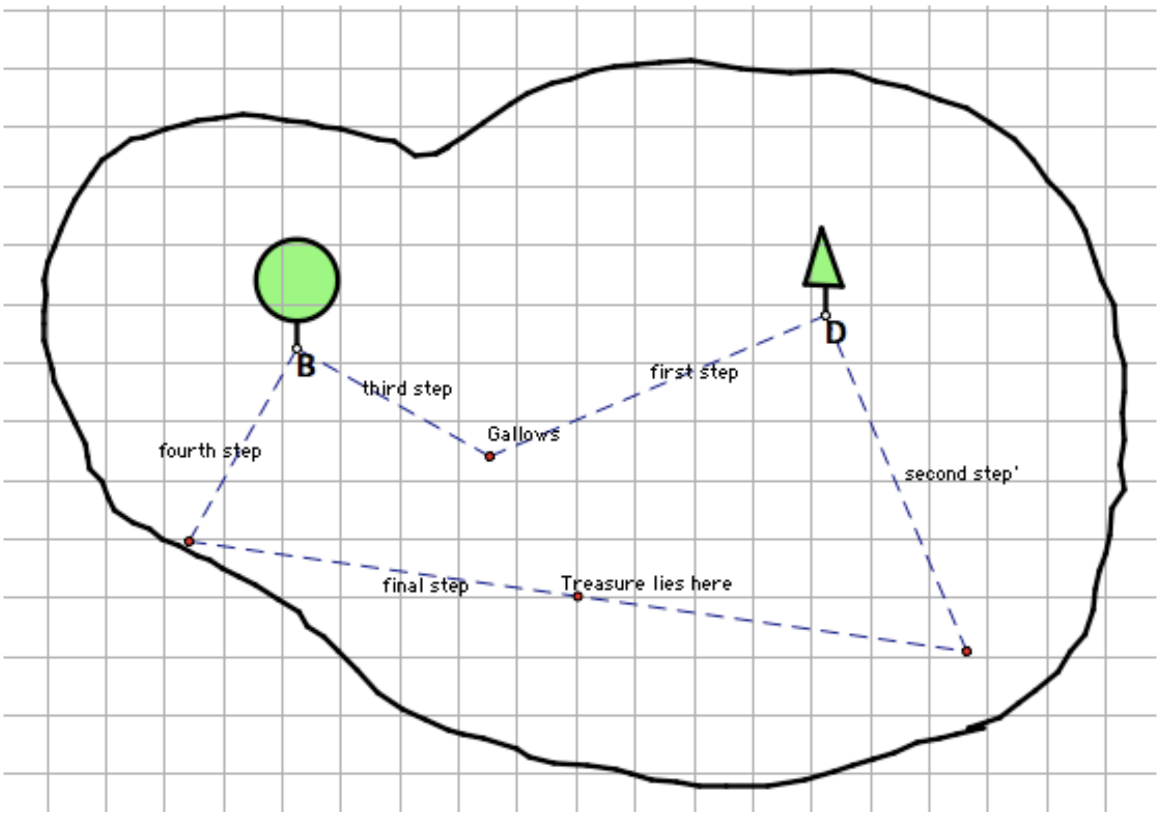
Goal: to locate the treasure on an island given limited information

Problem

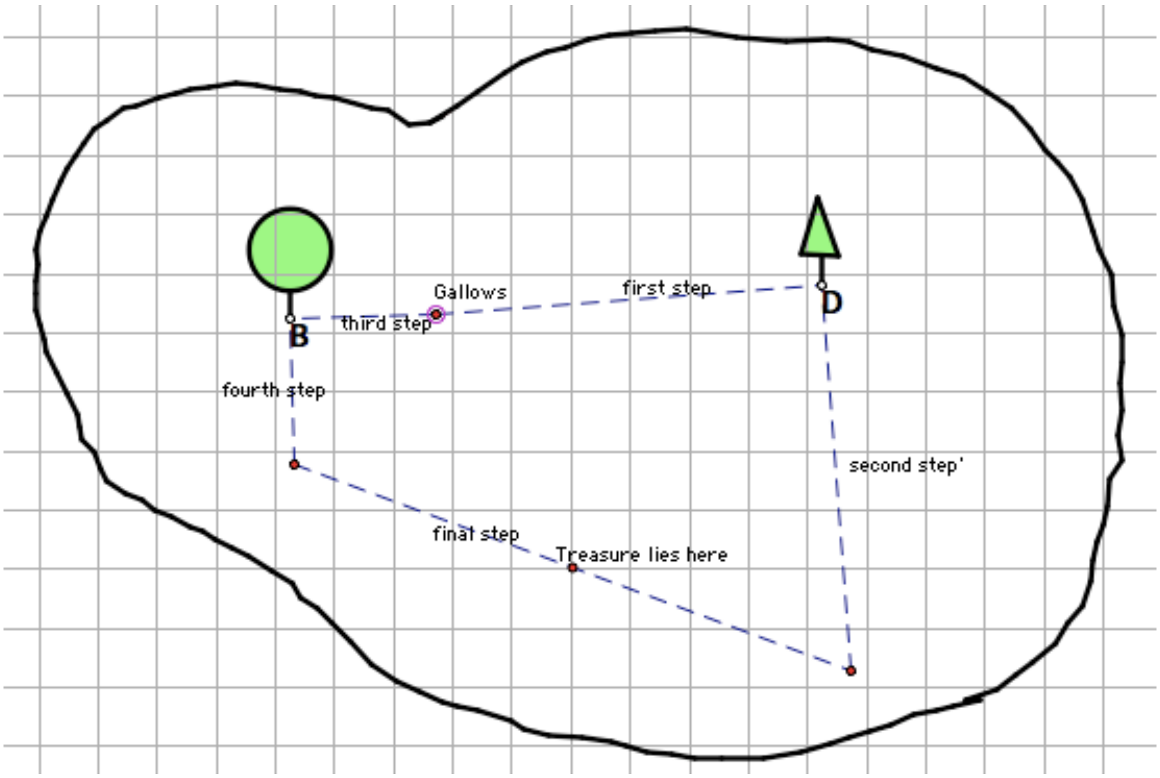
Consider an arbitrary starting point of the gallows (G) somewhere north of the both trees then the exact position of the treasure will be at the location X on the map after following the directions on the paper.

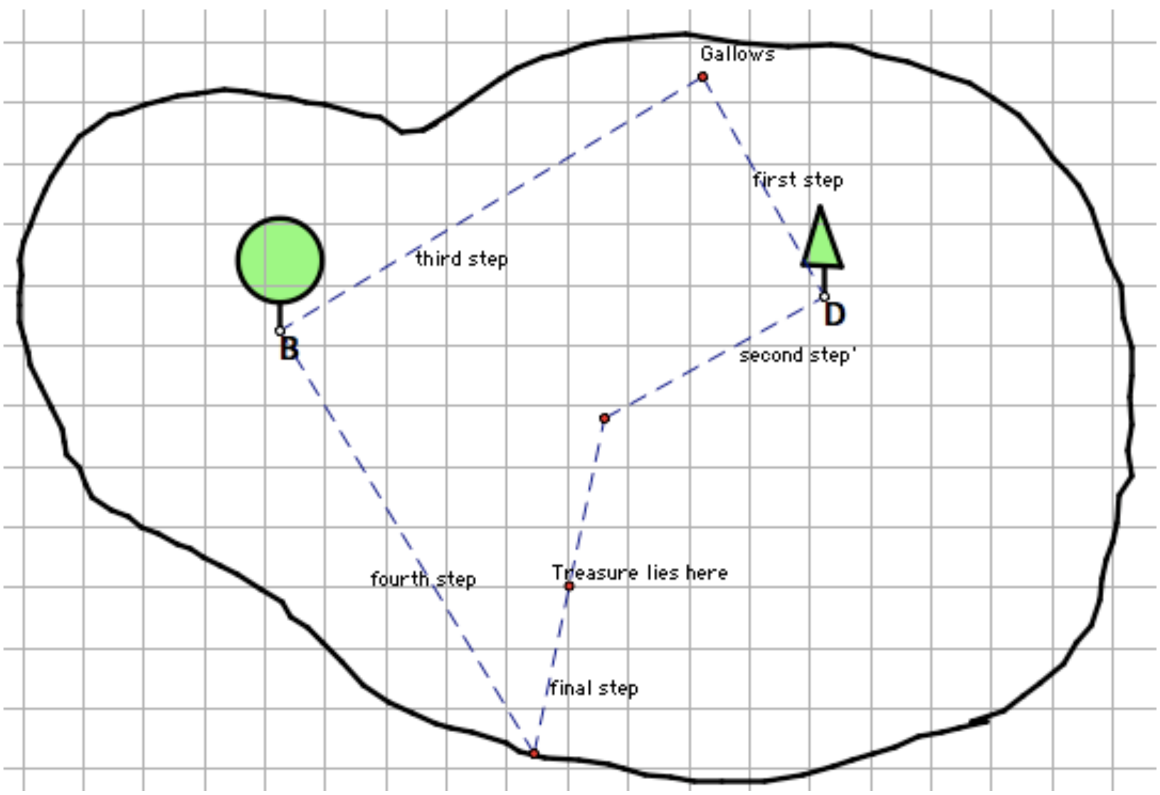


Consider a starting point slightly south of both trees then the exact position of the treasure will be at the SAME location X on the map after following the directions on the paper.



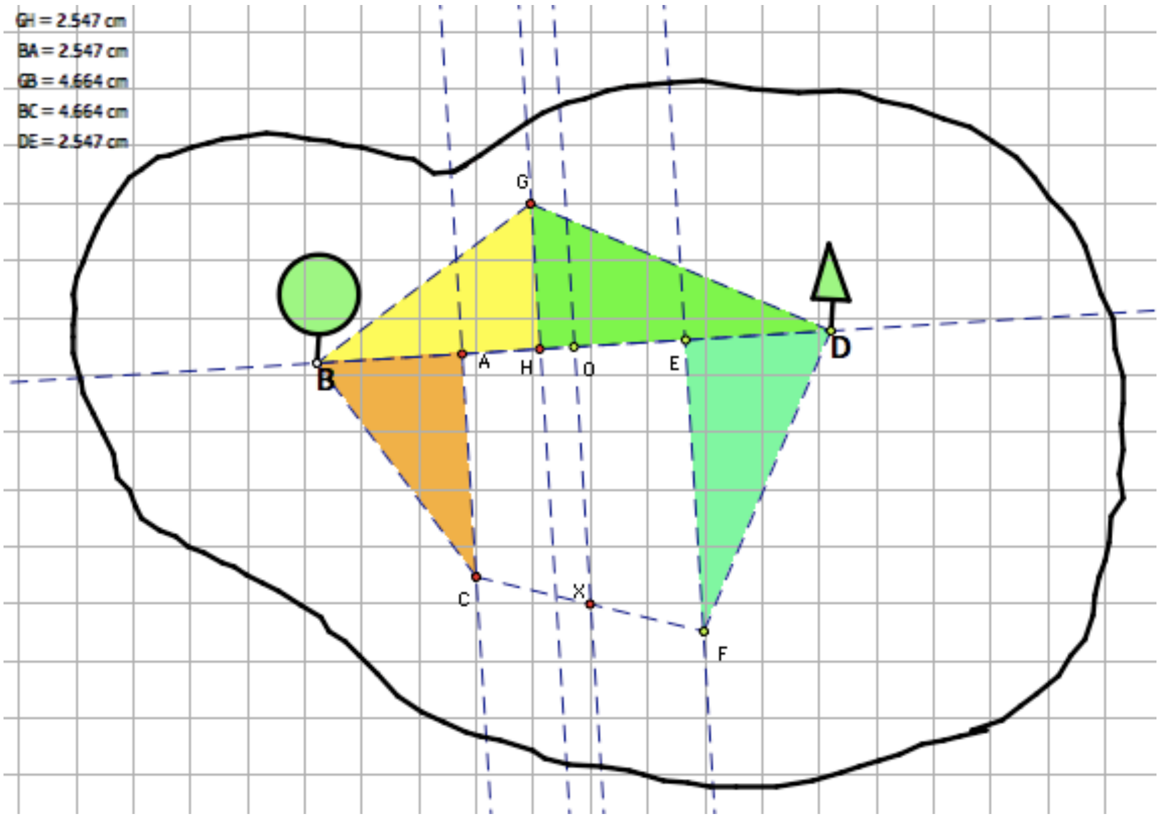
After animating the positions of the gallows, we find that the location of X always remains the SAME in the GSP sketch.


Gallows
first step
third step
B
D
fourth step
second step'
final step
Treasure lies here


Gallows
first step
third step
D
B
second step'
Treasure lies here
fourth step
final step

Geometric Proof

One proof of the location of the treasures can be the use of congruent triangles as in the diagram below.



In the diagram, perpendicular segments are constructed to the line through points B and D (the locations of the oak tree and pine tree). This forms triangles GBH, ABC, GHD, and DEF. Consider triangles GBH and ABC are congruent because they have the same length hypotenuse are both similar to the small triangle formed by AB and the point of intersection of GB and line CA.

Hence AB ≈ GH

Consider triangles GHD and DEF are congruent because they have the same length hypotenuse are both similar to the small triangle formed by DE and the point of intersection of GD and line FE.

Hence DE ≈ GH

consider also that AB $\approx$ DE by transitive property.

Note that AB and DE lie at equal distances from the two trees and that the spikes driven at C and F will be exactly below A and E on lines perpendicular to BD. Now because the treasure is at the midpoint of segment CF, then it is also below the midpoint of AE on the line perpendicular to BD.

Not that the quadrilateral AECF takes the shape of a right trapezoid. One property of this is that the midsegment is a fixed length with respect to the bases.

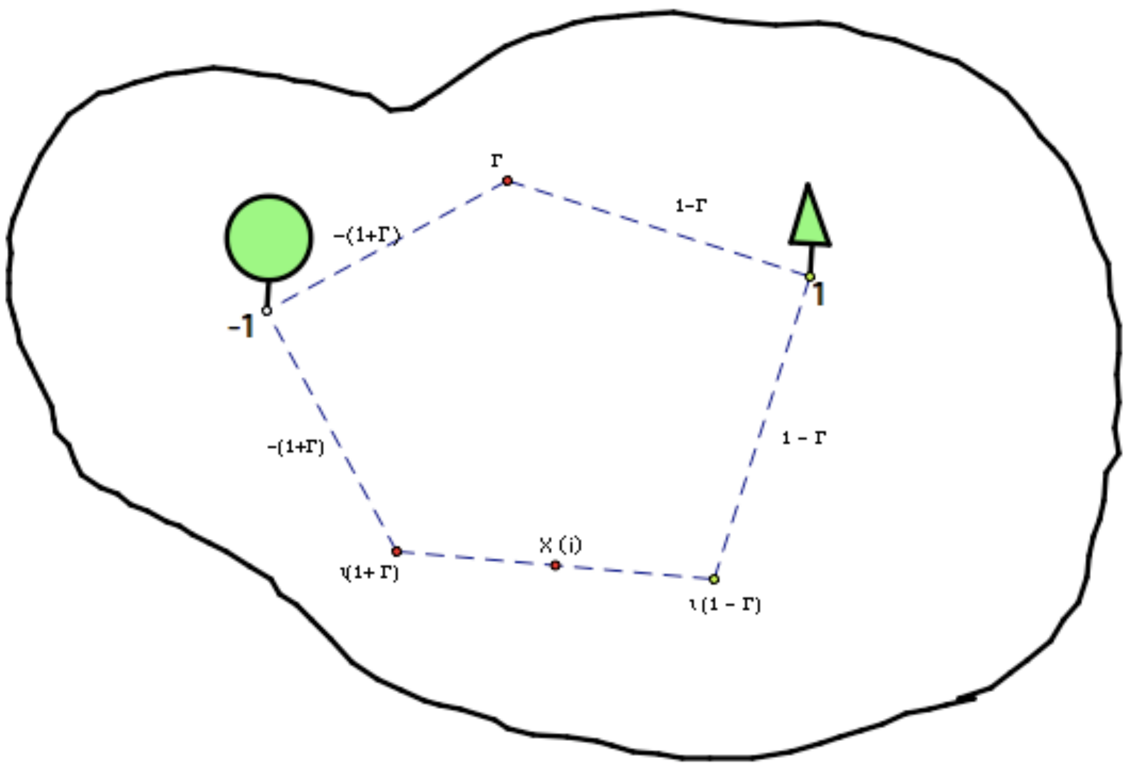
Thus the exact point of the treasure will be at (EF + AC)/2 along the perpendicular to the midpoint of the segment BD. It will always be at point X.

Complex Number Proof

Let the Position of the gallows be represented by any complex number Γ of the form a + bi where a is the real part and bi is the imaginary part. For this problem the Γ will be used during the computations.

Assume that the trees lie on the real axis where the midpoint between the trees represent the origin and the pine tree is one unit to the right, that is at 1 + 0i (or 1), while the oak tree is one unit to the left of the origin that is -1 + 0i or (-1). Thus B is at -1 and D is at 1.

We can now calculate the distance between the gallows, Γ, and pine tree, D , as 1- Γ. Similarly, the distance between the gallows, Γ, and the oak tree, B, as – 1 - Γ or –(1 + Γ).



In order to make a clockwise 90° turn from point D by the same magnitude as 1- Γ., multiply by i, so we do i(1- Γ) to get to the new position

i(1- Γ) .

In order to make a counterclockwise clockwise 90° turn from point B by the same magnitude as –(1 + Γ), multiply by -i, so we do -i(–(1 + Γ)), to get to the new position

$i(1 + \Gamma)$.

To find the position of the treasure take the sum of the two points above and divide by two.

$[i(1- \Gamma) + i(1 + \Gamma)]/2$

$= [i(1- \Gamma+1 + \Gamma)]/2$

$= 2i/2$

$=i$.

So regardless of the position of the gallows that is chosen the treasure will always be at the point i. This is marked as X in the diagram.
