The University of Georgia

Mathematics Education Program

J. Wilson, EMAT 6600

Minimum Perimeter of an Inscribed Triangle

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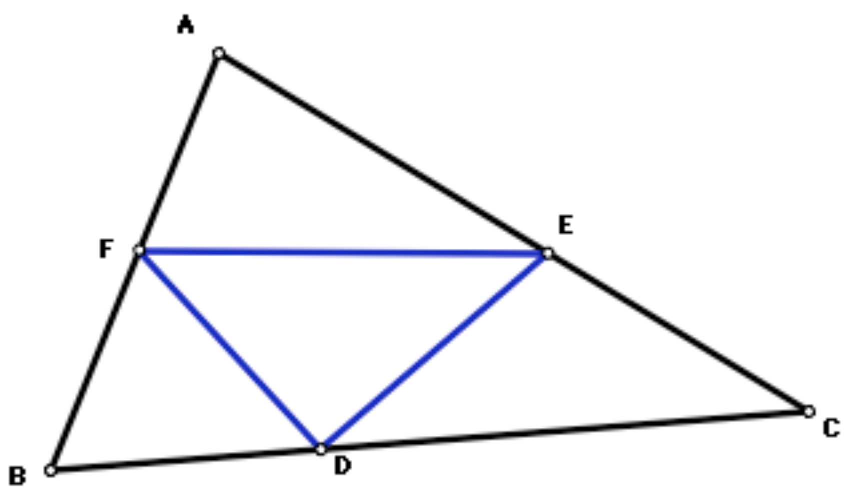
Goal: To investigate angles, triangles and concurrency in incircles

Problem

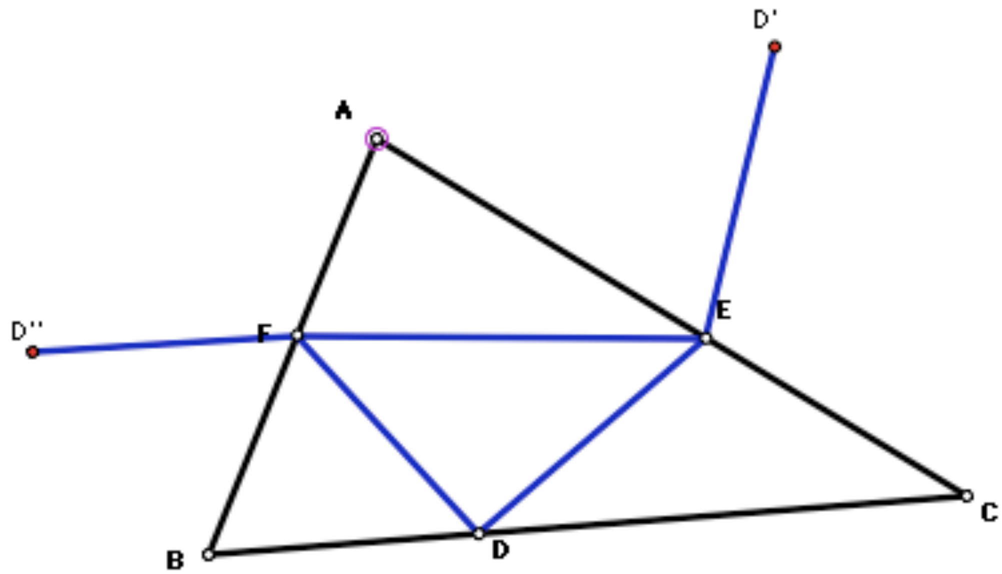
Triangle ABC is constructed with points D, E, and F on sides BC, AC and AB respectively as shown:



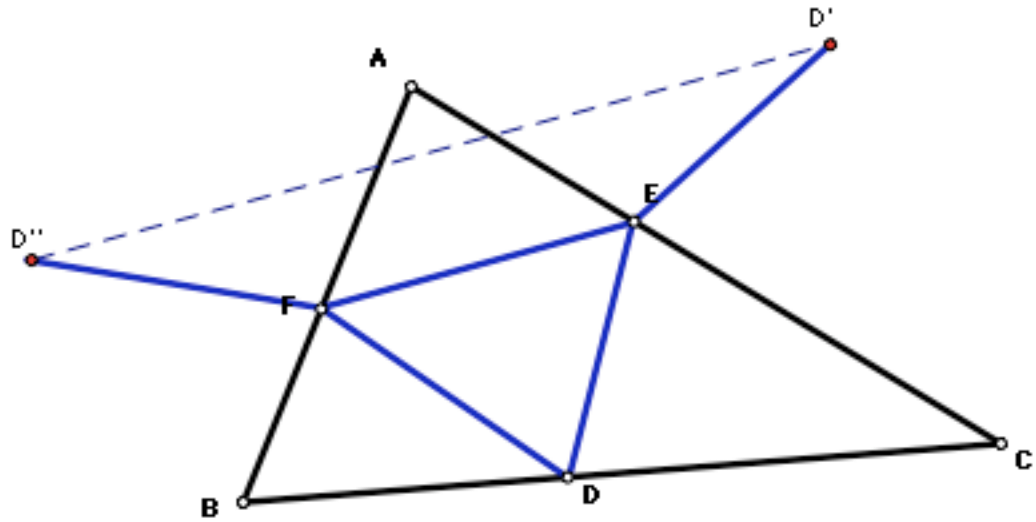
The minimum perimeter of the inscribed triangle can be found by the following series of steps:

mark AB as mirror, and reflect segment DE of triangle DEF. Label the endpoint of the reflected segment D'

mark AB as mirror and reflect segment DF through AB. Label the endpoint of the reflected segment D''.

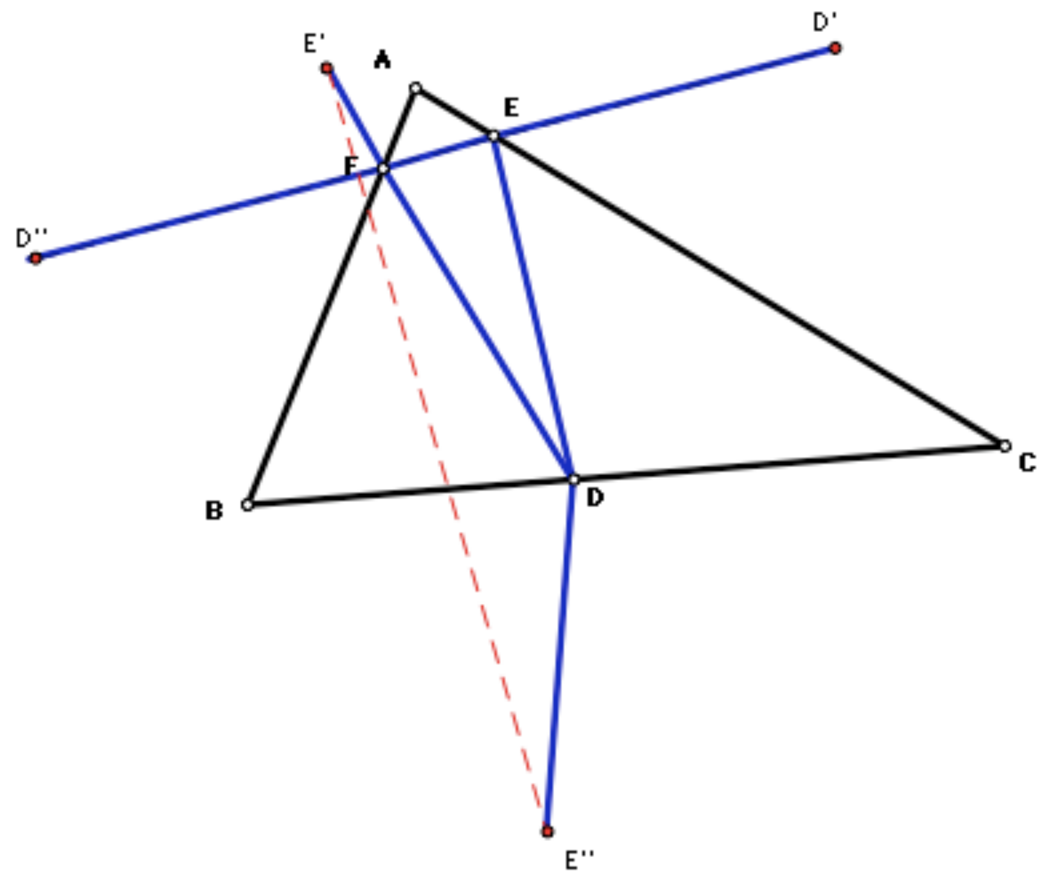


Note that the combined distance D'E + EF + ED" is equal to the perimeter of triangle DEF. the MINIMUM perimeter of the triangle will be equal to the distance ece D'E + EF + ED" is shortest. The distance D'E + EF + ED" is shortest when there is a straight line connecting D' to D" through points E and F.



In GSP we can drag the E and F until D'E + EF + ED" becomes a straight line.

We will need another set of line segments line through segment DF or through DE that will be equal to the perimeter of Triangle DEF.

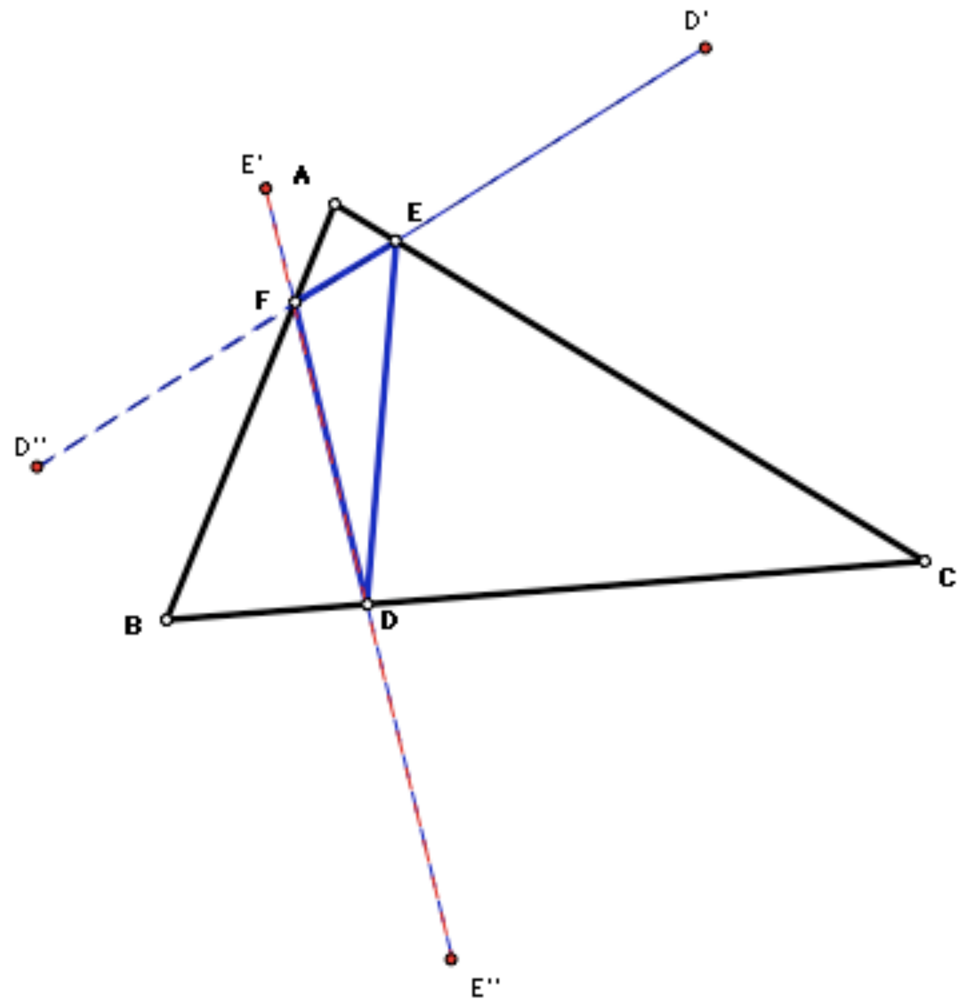


using AB as mirror reflect segment FE and label the end point of the reflected segment E'.

Using BC as mirror, reflect segment ED and label the reflected endpoint E''.

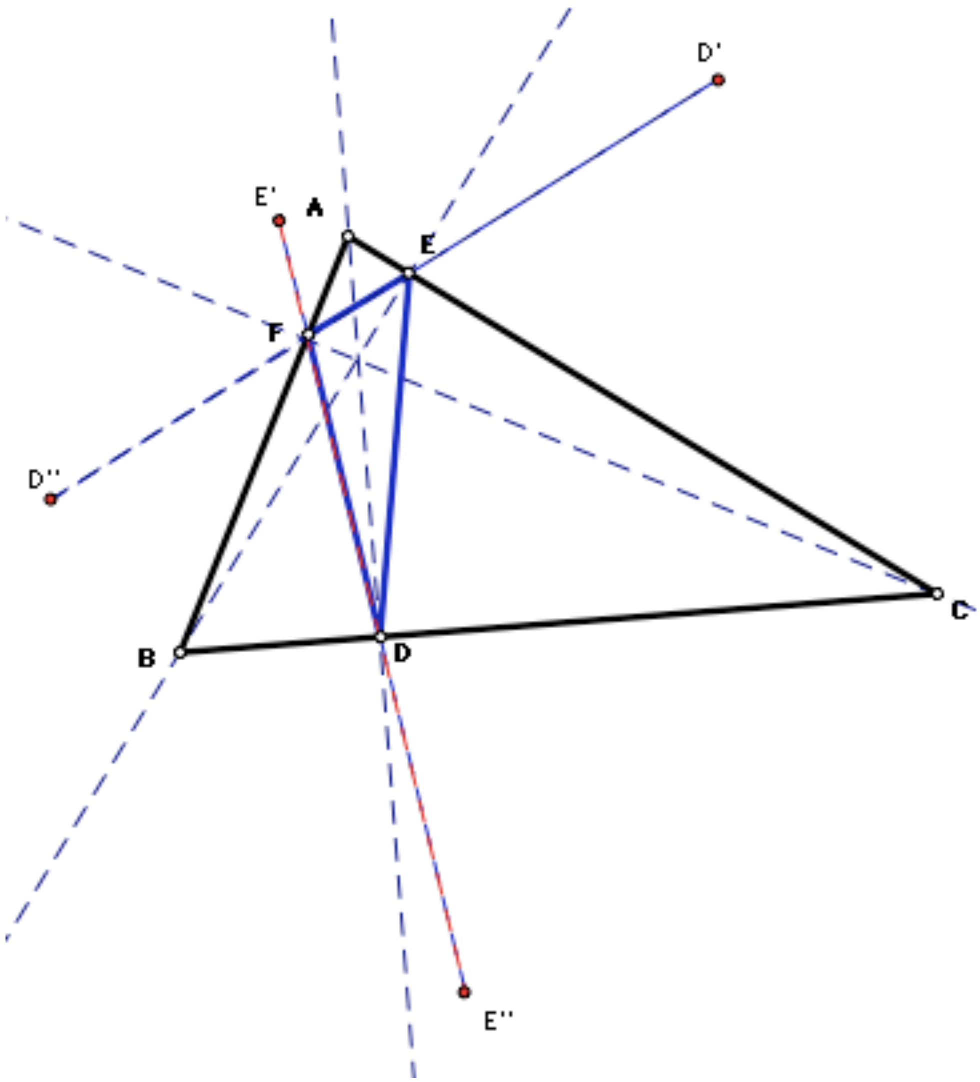
Connect E' and E'' with a segment and then Drag points D and F to line up with the segment connecting E' and E''.

Now the combined distance E'D + DF + FE'' equals the perimeter of the triangle ABC and perimeter of the triangle ABC is smallest when it is equal to both D'E + EF + ED'' and E'D + DF + FE''.



Is there a unique inscribed triangle with the minimum perimeter?

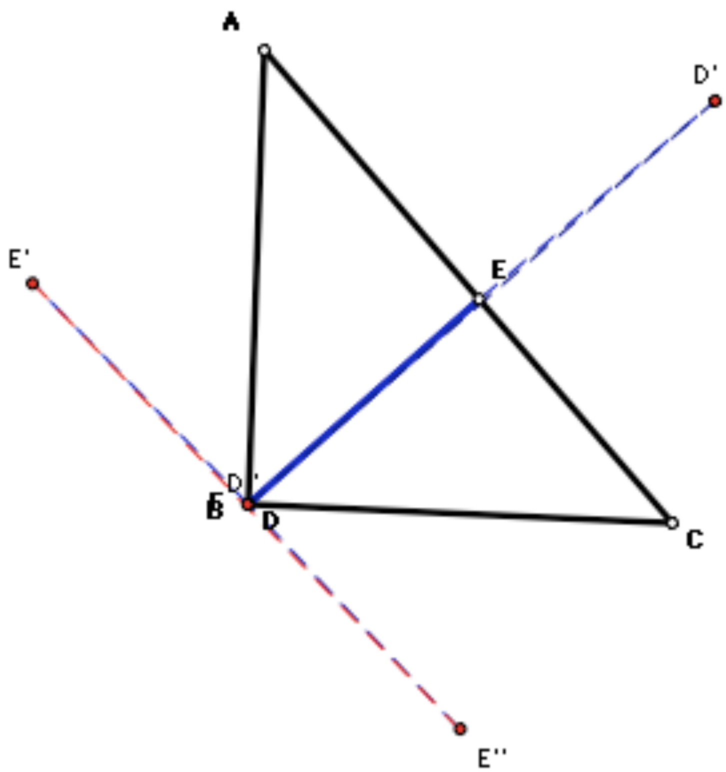
Yes, the triangle with the minimum perimeter is the orthic triangle. That is the inscribed triangle with its vertices as the foot of the altitudes of the vertices of triangle ABC.



This is demonstrated by constructing the perpendiculars from the vertices of triangles ABC to the opposite sides BC, AC and AB.

What if the triangle ABC was a right triangle?

If triangle ABC was a right triangle the triangles with the least perimeter would be a straight line (degenerate).



What if the triangle ABC was an obtuse triangle?

The triangle with the smallest perimeter cannot be inscribed