



The University of Georgia

Mathematics Education Program

J. Wilson, EMAT 6600

## Three Adjacent Squares

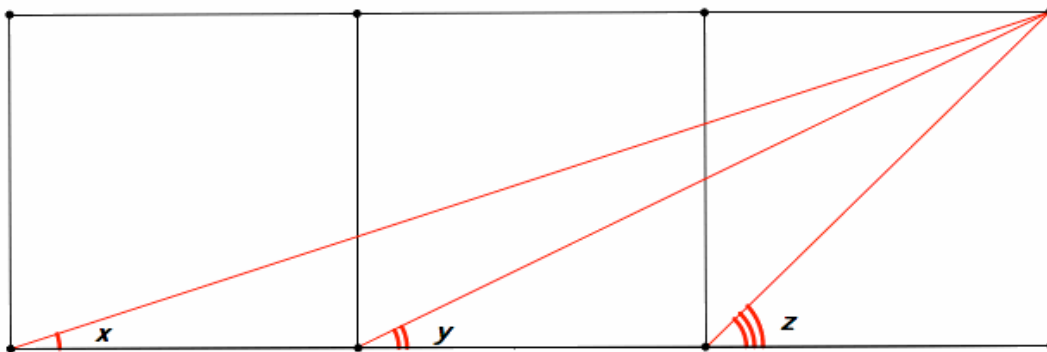
By Leighton McIntyre

Goal: To prove that three angles  $x$ ,  $y$  and  $z$  formed from connecting some vertices of three adjacent squares, sum to 90 degrees.

### Problem

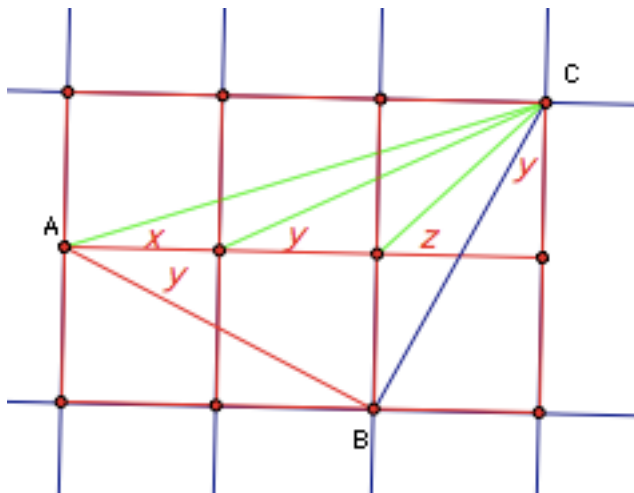
Three Adjacent Squares form a rectangle. Let  $x$ ,  $y$ , and  $z$  be the degree measures of the angles indicated.

Prove  $x + y + z = 90$  degrees.



## Solution

Construct another three squares below the original three or alternatively, reflect the original three squares in the horizontal axis. Connect the points ABC as shown and note that an isosceles triangle is formed.



Denote by  $a$ , each side of a single square.

$$AB = \sqrt{(2a)^2 + a^2} = \sqrt{5} a$$

$$BC = \sqrt{(2a)^2 + a^2} = \sqrt{5} a$$

$$AC = \sqrt{(3a)^2 + a^2} = \sqrt{10} a$$

We show that ABC is right triangle by if

$$AC^2 = AB^2 + BC^2$$

$$(\sqrt{10} a)^2 = (\sqrt{5} a)^2 + (\sqrt{5} a)^2$$

$$10a^2 = 5a^2 + 5a^2$$

$$10a^2 = 10a^2$$

So ABC is a right triangle

It is right isosceles because  $AB = BC$

So  $\angle BAC = 45$  degrees

So  $x + y = 45$  degrees

Recall  $z$  is also 45 degrees because it comes from an isosceles right triangle with congruent legs =  $a$

So  $x + y + z = 90$  degrees

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