The University of Georgia

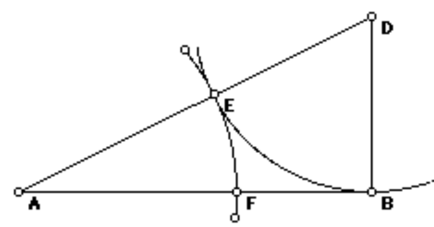
Mathematics Education Program

J. Wilson, EMAT 6600

Ratio on a Line Segment

By Leighton McIntyre

Goal : To show that equality of ratios on a line segment- the golden ratio.



Show that

$$\frac{AF + FB}{AF} = \frac{AF}{FB}$$

Solution:

Let DB = y = DB , AD = x , AB = 2y

Then Given triangle ABD is right triangle, by Pythagorean theorem

$y^2 + (2y)^2 = x^2$

$y^2 + 4y^2 = x^2$

$5y^2 = x^2$

$\sqrt{(5y^2)} = x$

thus AE = AF = $\sqrt{(5y^2)} - y$ and FB = $2y - (\sqrt{(5y^2)} - y)$ = $3y-\sqrt{(5y^2}$

We aim to show that

$$\frac{AF+FB}{AF} = \frac{AF}{FB}$$

$$\frac{2y}{\sqrt{5y\text{^}2} - y} = \frac{\sqrt{5y\text{^}2} - y}{3y - \sqrt{5y\text{^}2}}$$

$2y(3y-\sqrt{(5y^2)}) = (\sqrt{(5y^2)}-y)(\sqrt{(5y^2)}-y)$

$6y^2-2y\sqrt{(5y^2)} = (5y^2)- 2y\sqrt{(5y^2)}+y^2$

$6y^2-2y\sqrt{(5y^2)} = 6y^2- 2y\sqrt{(5y^2)}$

$6y^2-6y^2 = 2y\sqrt{(5y^2)} - 2y\sqrt{(5y^2)}$

$0 = 0$ QED. true

thus

$$\frac{AF+FB}{AF} = \frac{AF}{FB}$$

Now we can relate this to the golden ratio $\frac{a+b}{a} = \frac{a}{b}$ where AF = a, and FB = b

And thus we have the golden ration constructed on this line.