



The University of Georgia

Mathematics Education Program

J. Wilson, EMAT 6600

## Combining Rates

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Goal: To find average rate

### Problem

Danielsville is 20 miles from Jim's home in Athens. He Drives 60 mph going to a meeting (he is almost late) but coming home the weather is bad and drives 30 mph. What is his combined speed for the time he is on the road?

A. Discuss why 45 mph is not correct.

Going to D'ville took  $\frac{1}{3}$  of an hour. Coming home took  $\frac{2}{3}$  of an hour. So the total 40 miles took one hour. The speed is 40 mph.

B. Use

$$d = r t$$

to verify and develop an understanding of how to combine rates

## Solution

Denote by  $r_1$  the rate going to Danielsville and  $r_2$  the rate returning from Danielsville. Denote by  $t_1$  the time it takes to drive to Danielsville at rate 1 and by  $t_2$  the time it takes to drive back from Danielsville at rate 2.

Recall distance = rate \* time

Denote by  $d$  the distance to Danielsville

Then

$$d = r_1 t_1 \quad \text{so } t_1 = d / r_1$$

$$d = r_2 t_2 \quad \text{so } t_2 = d / r_2$$

Now combining to get the total distance travelled

$$2d = r_1 t_1 + r_2 t_2$$

Recall that the total time traveled is  $t_1 + t_2$

Now let  $2d = rt$  where  $t = t_1 + t_2$

And  $r$  is the average rate

$$2d = r(t_1 + t_2)$$

$$2d = r(d / r_1 + d / r_2)$$

factoring and dividing through by  $d$  gives

$$2 = r(1 / r_1 + 1 / r_2)$$

$$\text{thus } 1/r = [(1 / r_1 + 1 / r_2)] / 2$$

$$r = \frac{2r_1r_2}{r_1 + r_2}$$

Plugging in  $r_1 = 60$  and  $r_2 = 30$

$$\begin{aligned} r &= \frac{2(60)(30)}{60 + 30} \\ &= \frac{3600}{90} \\ &= 40 \end{aligned}$$

So the average rate is 40 miles per hour.

Take a look at the picture step by step. For the first segment of the route, he drives 20 miles at 60 miles per hour thus taking thus taking 20 minutes; and for the second segment, he drives 20 miles at 30 miles per hour thus taking 40 minutes.

So the total distance he drives is 40 miles and the total time he takes is one hour (20 minutes plus 40 minutes), Thus the average rate is 40 miles per hour.

A simple arithmetic average or  $(60+30)/2 = 45$  does not work here because one has to take the time it takes to travel the distance into consideration and not just the numbers for the two rates.

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