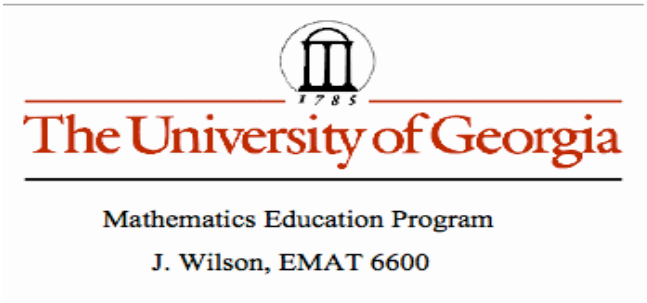

Cost of Fencing for a Field

By Leighton McIntyre

Goal: To find the minimum cost of fencing given different costs

Problem

A farmer wants to fence in 60,000 square feet of land in a rectangular plot along a straight highway. The fence he plans to use along the highway costs $2 per foot, while the fence for the other three sides costs $1 per foot. How much of each type of fence will he have to buy in order to keep expenses to a minimum? What is the minimum expense?

Use the AM-GM Inequality to find a solution. No calculus needed.

AM GM solution

Let side of fence facing road (say south) be x and so north is x, east and west sides are y

Side x to the road costs 2 per foot, side y costs 1 per foot.

$2x + x + y + y =$ cost $\quad 3x+2y =$ cost [1]

$x * y = 60{,}000$ so $y = 60000/x$ [2]

Substitute [1] into [2]

$3x + 120000/x$ is cost function

Graph

$Y = 3x + 2\left(\frac{6000}{x}\right)$

Compare to

$Y = 2\sqrt{(3x)(2)\frac{6000}{x}}$

$2\sqrt{(3^2)(2^2)(100^2)}$

$= 2*3*2*100$

$= 1200$

AM = GM

iff $3x = 2(6000/x)$

$X^2 = 40000$

$X=200$

Now if one side is 200 we show the other side is 300 = 60000/200

Minimum Cost = 3(200) + 2 (60000/200) = 600 + 600 = $1200

Solution using Calculus

Using x as the side facing the road as in the above we have

3x + 120000/x is cost function

Derivative is 3 – 120000/ x^2

Set = 0 to minimize

3 x^2 = 120000

x^2 = 40000

x = 200

Cost = 3x + 120000/x = 3(200) + 120000/200 = 1200

The fencing uses more of the cheaper material to offset to costs of the more expensive side..
