The University of Georgia

Mathematics Education Program

J. Wilson, EMAT 6600

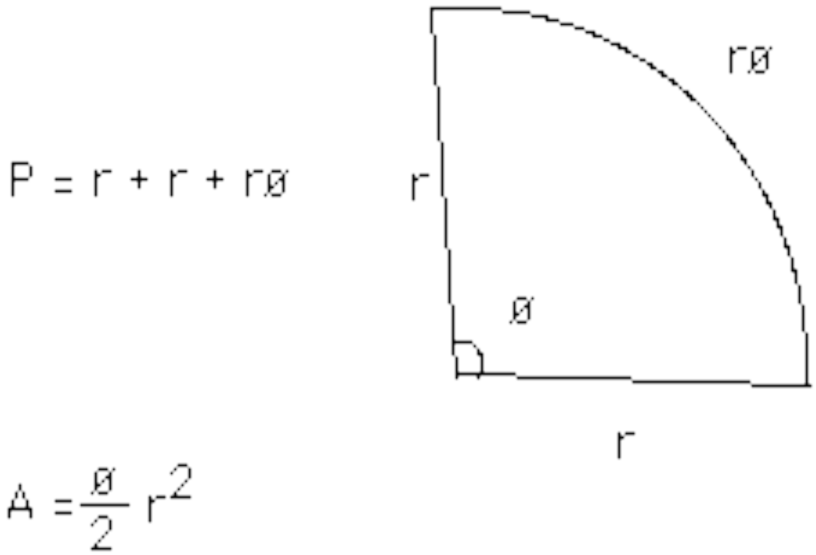
Maximum Area of a Sector of a Circle - Fixed Perimeter

By Leighton McIntyre

Goal: To find the maximum area of a sector given that perimeter is fixed.

Problem

Use the Arithmetic Mean -- Geometric Mean Inequality to find the maximum area of a circular sector with a **fixed** perimeter.



Solution

For the maximum area of a sector of fixed perimeter let us take a look at the AM-GM inequality

What fraction gives the maximum area?

Given the diagram above

C = 2π

fraction OF CIRCLE = θ/2π

$A_{sector} = \frac{\theta}{2} r\text{^}2$

Let Perimeter = 100 then

$100 = 2r + \theta r$

$100 - 2r = \theta r$

$\theta = (100 - 2r)/r$

So $A_{sector} = \frac{1}{2}(\frac{100-2r}{r})r\text{^}2$

$= (50 - r)r$

Using AM – GM inequality

$(50 - r)r \leq (\frac{50 - r + r}{2})\text{^}2 = 25^2$ (constant)

Thus the maximum area is a square so this means that is when

$(50 - r) = r$

$r = 25$

So we plug this in to get the sixe of the sector and show

$(\frac{100-50}{25}) = 2$ radians