Goal: To find the maximum area of a sector given that perimeter is fixed.

Problem

Use the Arithmetic Mean -- Geometric Mean Inequality to find the maximum area of a circular sector with a fixed perimeter.
Solution

For the maximum area of a sector of fixed perimeter let us take a look at the AM-GM inequality

What fraction gives the maximum area?

Given the diagram above

\[ C = 2\pi \]

fraction OF CIRCLE = \( \theta / 2\pi \)

\[ A_{\text{sector}} = \frac{\theta}{2} r^2 \]

Let Perimeter = 100 then
\[ 100 = 2r + \theta r \]
\[ 100 - 2r = \theta r \]
\[ \theta = \frac{(100 - 2r)}{r} \]

So \[ A_{\text{sector}} = \frac{1}{2} \left( \frac{100 - 2r}{r} \right) r^2 \]
\[ = (50 - r)r \]

Using AM – GM inequality
\[ (50 - r)r \leq \left( \frac{50 - r + r}{2} \right)^2 = 25^2 \] (constant)

Thus the maximum area is a square so this means that is when
\[ (50 - r) = r \]
\[ r = 25 \]

So we plug this in to get the size of the sector and show
\[ \left( \frac{100 - 50}{25} \right) = 2 \text{ radians} \]