



The University of Georgia

Mathematics Education Program

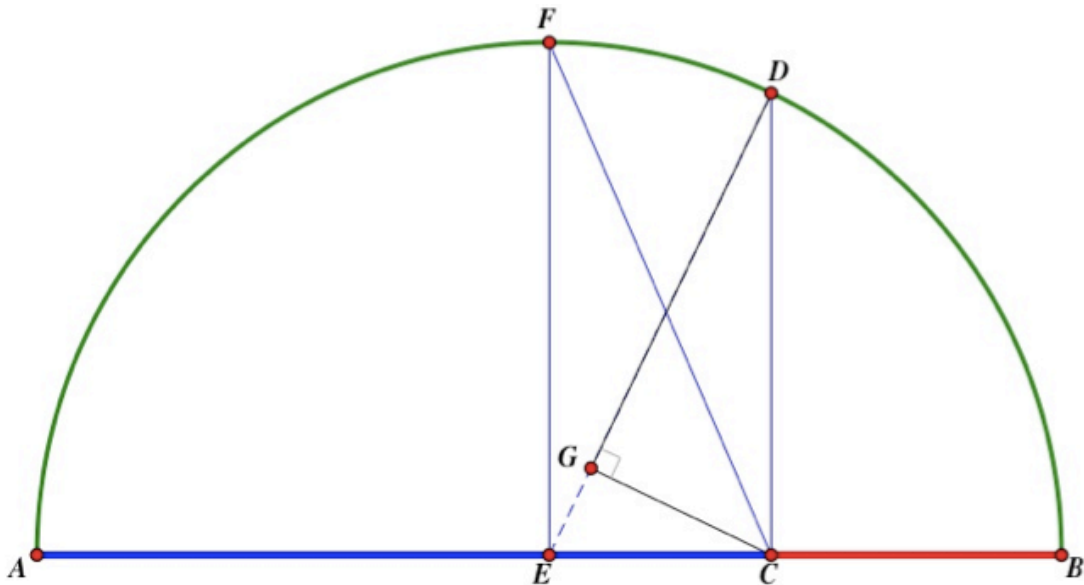
J. Wilson, EMAT 6600

## Segments in a Circle

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Goal: Identify how special segments in a circle relate to the means

### Problem



Let  $AC = a$ ,  $BC = b$

Find the following in terms of  $a$  and  $b$

$EF =$

$CD =$

$CF =$

$GD =$

Given also that the EF and DC are both perpendicular to AB

EF (is a radius of the circle) =  $(a+b) / 2$ ; the arithmetic mean

CD =  $\sqrt{ab}$  the geometric mean

$$CF^2 = EF^2 + CE^2$$

$$CE^2 = [(a+b) / 2] - b = (a-b)/2$$

$$\text{Then } CF^2 = ((a+b) / 2)^2 + ((a-b)/2)^2$$

$$= (a^2+b^2) / 2$$

$$CF = \sqrt{[(a^2+b^2) / 2]}$$

Consider  $\Delta DGC$  is similar to  $\Delta DCE$ , then the following proportions hold

$$\frac{DG}{DC} = \frac{DC}{DE}$$

$$DG = \frac{DC^2}{DE}$$

$$DG = \frac{(\sqrt{ab})^2}{(a+b)/2} = \frac{2ab}{a+b} ; \text{ the harmonic mean.}$$