The University of Georgia
Mathematics Education Program
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Magic Squares

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Goal: To arrange numbers in 3x3 and 4x4, addition and product magic squares

Magic Squares

Given the integers 1 through 9, we know that 1 + 2+ 3 + ...+ 9 = 45

Since there are 3 rows or 3 columns then 45/3 = 15 so each set of three numbers should sum to 15 in the magic square.

Now let the numbers denoted by a,b,c,d,e,f,g,h,i be inserted into the 15 slots as follows

a	d	g	15
b	e	h	15
c	f	i	15
15	15	15	

Then a+d+g = 15, a+e + I = 15, a+b+c = 15

e + a+ i = 15, e + c + g = 15, e + b + h = 15, e + d + f = 15

Because e is common to all equations it means that the pairs of numbers must be equal thus a+ i = c + g = b + h = d + f

Choose e to be any number between one and 9 inclusive , say 4, then the other pairs should sum to 11, so 6 + 5, 7 + ? (we need a 4 here) , 8 + 3, 9+ 2, ?(we need a 10 here) + 1. So e = 4 does not work

Then let e = 5, so the other pairs should sum to 10. Se we have 6 + 4, 7 + 3, 8+ 2, and 9 + 1. So e =5 checks out. Next we need to place the pairs of numbers in there respective places.

Let us look at the *a* position. There are two pairs of even numbers and two pairs of odd numbers. For each row or each column there must be either three odd numbers of one odd number. If an odd number is in the a position there will needs be another odd number in the first row or first column which will not make the row or could be 15, so the a position has to be an even number. Thus the pairs of even numbers must be placed on the diagonals.

The possible placements can be as follows:

8	1	6	15

3	5	7	15
4	9	2	15
15	15	15	

This solution is unique because the other combinations are only found by rotation the square.

Alternative Strategy

An alternative strategy to begin this problem would be to note that the there are 5 odd numbers and 4 even numbers in the set of integers 1 through 9. Notice that the sum of 15 should consist of either one odd or 3 odd numbers. Thus an odd number has to be in the center. The set up of the pattern would be as follows:

O	E	O	15
E	O	E	15
O	E	O	15
15	15	15	

We could then follow steps to fill out the odd number patterns that sum to 15 and then fill in the even numbers. The patterns would lead to the choice of 5 as the middle number.

Other 3X3 Magic Squares.

Other 3 x3 magic square can be found using the numbers 2 – 10 in each of the 9 squares. The solution to this magic square is as follows: in this square each of the rows and columns sum to 18.

9	2	7	18
4	6	8	18
5	10	3	18
18	18	18	

It is possible to have a 3 x 3 magic square with 21 in the center and other number being different around by considering the relationship between 21, and the original center 5. Now 21 equals 5 + 16 so we can try to make a magic square by adding 16 to each of the original entries, the new square is as follows:

24	17	22	63
19	21	23	63
20	25	18	63
63	63	63	

Four by Four Magic Square

A four by four Magic Square can be completed if the with some degree of effort, a guess and check method was used to obtain the following correct solution:

1	2	15	16	34

13	14	3	4	34
12	7	10	5	34
8	11	6	9	34
34	34	34	34	

And another solution…

1	15	14	4	34
12	6	7	9	34
8	10	11	5	34
13	3	2	16	34
34	34	34	34	

A 3 x 3 magic square where the operation is **multiplication** rather than addition and the entries are 9 different numbers can be found as follows:

Let ***a*** be any base and raise it to each of the numbers in the original 3 x 3 magic square as exponents.

a^8	a^1	a^6	a^{15}
a^3	a^5	a^7	a^{15}
a^4	a^9	a_2	a^{15}
a^{15}	a^{15}	a^{15}	

Let a = 2, then the resulting magic square is as follows:

256	2	64	32768
8	32	128	32768
16	512	4	32768
32768	32768	32768	
