The University of Georgia

Mathematics Education Program

J. Wilson, EMAT 6600

Bridge Expansion

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Goal: To find the change in height cause by bridge expansion

Problem

1. **Suppose a bridge could be built with rigid material so that expansion is absorbed by a hinge action in the middle of the bridge. The ends are firmly anchored at A and C. Let the bridge be one mile (5280 ft) long. If the coefficient of expansion is (1/2640) the bridge surface would increase in length by 2 ft. Discussion: What is the intuition? Would a person 6 ft tall still be able to see a person 6 ft tall at the other end of the bridge at maximum expansion?**

My intuition tells me that a person who is six ft tall would not be able to see a person six ft tall at the other end of the bridge. If the bridge surface increases by 2 ft using the expansion factor of 1/2640, then if you look at the situation such that it adds one foot to each end but the ends are fixed, then the bridge would rise in the middle by a height that that is definitely more than 2ft.

Let's explore that a bit further. Letting h be the height we have a right triangle with legs of length h and 2640 and hypotenuse of 2641.

$(2640)^2 + h^2 = (2640)^2$

$\rightarrow (2641)^2 - (2640)^2 = h^2$

$\rightarrow (2641 + 2640)(2641 - 2640) = h^2$

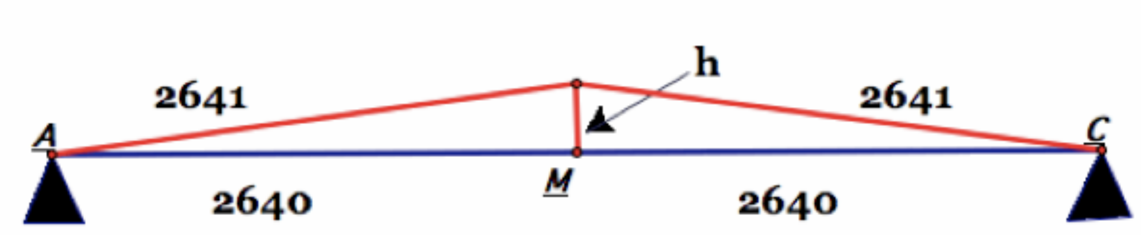
$\rightarrow 5281 = h^2$

Now if we recall the squares of 70 is 4900 and 80 is 6400 so h must lie between 70 and 80.

Now our six-foot friend measures 72 inches so we have the idea that we might be able to see our six-foot friend at the other end.

Using the Pythagorean relation, we find that the height is between 70 and 80 feet!

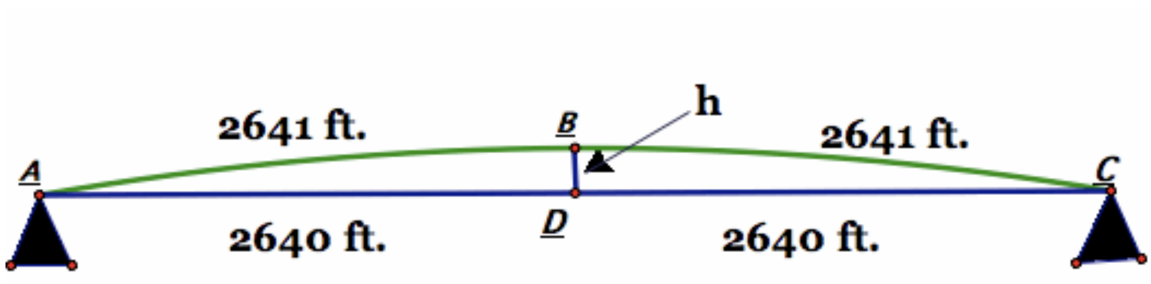
Does this seem reasonable? Yes it would be quite reasonable for the height to be between 70 and 80 feet. This is because with the given coefficient of expansion, the change caused to the height of the bridge is quite likely.



2. *A mile-long bridge expands by 2 feet due to the heat. It bows up in an arc as shown.*

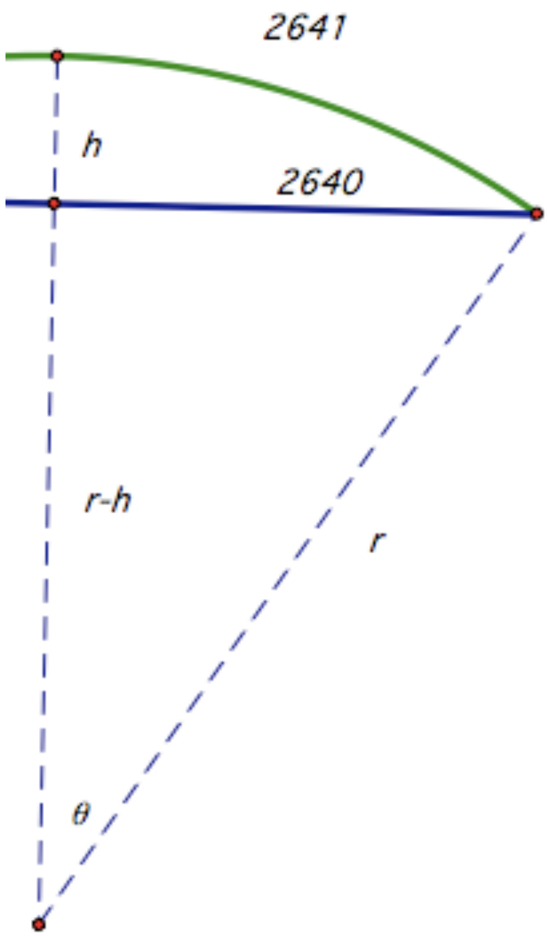
How big is h? If you were standing on one end of the bridge, could you see your

friend standing on the other end?



Trigonometric solution with iterations

Extend the diagram and consider one half of the bridge with radii going towards the center of a circle and we can set up the following equations form looking at the diagram.



So we have two equations in

$2641 = r\theta \quad \Rightarrow \quad \theta = \frac{2641}{r}$

$2640 = r\sin\theta \quad \Rightarrow \quad r = \frac{2640}{\sin\theta}$

Then plugging (1) into (2) gives: $r = \dfrac{2640}{\sin\dfrac{2641}{r}}$

Now we can set up a sequence such that $R_{n+1} = \dfrac{2640}{\sin\left(\dfrac{2641}{R_n}\right)}$

and try to identify the point where r is stable, or where R_{n+1} converges to R_n

.

Using an Excel spreadsheet we can set arbitrary starter values for r and observed what happens as we make 20 iterations in Excel using the formula 2640/(SIN(2641/A1)), where A1 represents the input cell of the starter value. The cells following will use this formula to calculate a value based on the previous value.

Now given that we know that the chord has length 5280ft, and we made a sketch of the circle was the radius was say about 7 to 12 times as long as the chord we can start with an estimate of 40,000 ft and place in A1. The excel output looks as follows:

40000

40013.92

40027.82461

40041.71386

40055.58776

40069.44633

40083.28962

40097.11762

40110.93039

40124.72792

40138.51025

40152.27741

40166.02941

40179.76628

40193.48804

40207.19472

40220.88634

40234.56292

40248.22449

40261.87106

40275.50267

Notice that this is increasing so next start with 60,000 and got the following results:

60000

59996.65304

59993.30844

59989.96618

59986.62626

59983.28869

59979.95347

59976.62058

59973.29003

59969.96182

59966.63595

59963.31241

59959.9912

59956.67233

59953.35578

59950.04157

59946.72968

59943.42012

59940.11288

59936.80796

59933.50537

After trying and noting results for 50,000 (increasing), 55000(increasing), 56,000 (decreasing), and various numbers between 55,000 and 56,000 it was found that the number which is the best approximation was 55,405.5. The following is the data:

55405.5

55405.49994

55405.49989

55405.49983

55405.49978

55405.49972

55405.49967

55405.49961

55405.49956

55405.4995

55405.49945

55405.4994

55405.49934

55405.49929

55405.49923

55405.49918

55405.49912

55405.49907

55405.49901

55405.49896

55405.4989

Thus the radius of the is approximately 55405.

Plugging this value of r into equation (1)

$2641 = r\theta$

$\Rightarrow$ $2641 = 55405*\theta$

$\theta = 2641/55405 = 0.048$

Using Pythagorean Theorem

$(r - h)^2 + 2640^2 = r^2$

$\rightarrow (55405 - h)^2 + 2640^2 \approx 55405^2$

$\rightarrow (55405 - h)^2 \approx 55405^2 - 2640^2 = 3062744425$

$\rightarrow 55405 - h \approx 55342.067$

$\rightarrow h \approx 62.93$ feet

If our friend is 6ft tall (or 72 inches) then it would be possible for us to see our friend at the other end of the bridge because the middle of the bridge is taller than out friend