The University of Georgia

Mathematics Education Program

J. Wilson, EMAT 6600

Trig Segments

By Leighton McIntyre

Goal: To derive trigonometric ratios from given segments

Problem

In the figure to the right, $\odot O$ is a unit circle with A being a point on the circle. Give

$OA = 1$ unit

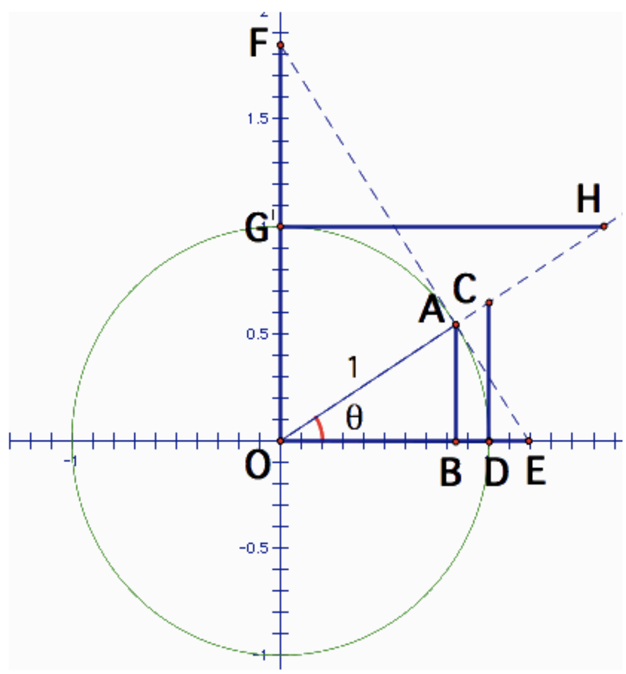
Frlom the definitions of trigonometric ratios, the lenths of $\overline{AB}$ and $\overline{OB}$ are

$AB = \sin\theta$
$OB = \cos\theta$

Construct $\overline{CD}$ tangent to $\odot O$ at D, C is on $\overrightarrow{OA}$
$\overline{GH}$ tangent to $\odot O$ at G, H is on $\overrightarrow{OA}$
$\overline{EG}$ tangent to $\odot O$ at A, with E of the x-axis and F on the y-axis

Find:

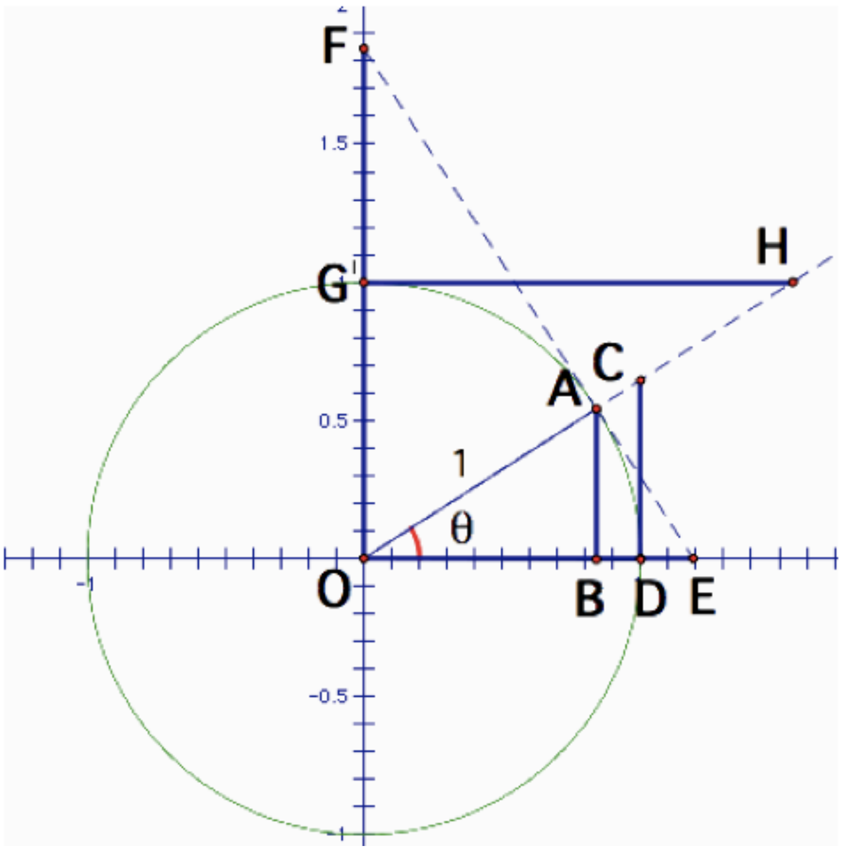
CD =
OE =
OF =
GH =
AE =
AF =
OC =
OH =



Solution

Circle O is a unit circle

Length OA = 1



The following ratios exist

AB = $\frac{AB}{OA}$ = sinθ

OB = $\frac{OB}{OA}$ = cosθ

CD = $\frac{CD}{OD}$ = $\frac{AB}{OB}$ = tanθ

OE = $\frac{OC}{OA}$ = $\frac{1}{\cos\theta}$ = secθ

$$OF = \frac{OF}{OA} = 1 \Big/ \frac{OA}{OF} = \frac{1}{\cos(90-\theta)} = \frac{1}{\sin\theta} = \text{cosec}\theta$$

$$GH = \frac{GH}{GO} = \tan(90 - \theta) = \cot\theta$$

$$AE = \frac{AE}{OA} = \tan\theta$$

$$AF = \frac{AF}{OA} = \tan(90 - \theta) = \cot\theta$$

$$OC = \frac{OC}{OD} = \frac{OA}{OB} = \frac{1}{\cos\theta} = \sec\theta$$

$$OH = \frac{OH}{OG} = \frac{1}{\cos(90-\theta)} = \frac{1}{\sin\theta} = \text{cosec}\theta$$

Identities from the diagram

1) $\frac{1}{\sin\theta} = \text{cosec}\theta$

2) $\frac{1}{\cos\theta} = \sec\theta$

3) $\frac{\sin\theta}{\cos\theta} = \tan\theta$

Trig Identities derived

1) Sin $\theta = \frac{1}{\text{cosec}\,\theta}$

2) Cos $\theta = \frac{1}{\sec\theta}$

3) tan $\theta = \frac{1}{\cot\theta}$
