Goal: To derive trigonometric ratios from given segments

**Problem**

In the figure to the right, $\odot O$ is a unit circle with $A$ being a point on the circle. Given

$$OA = 1 \text{ unit}$$

From the definitions of trigonometric ratios, the lengths of $AB$ and $OB$ are

$$AB = \sin \theta$$
$$OB = \cos \theta$$

Construct $\overline{CD}$ tangent to $\odot O$ at $D$, $C$ is on $\overrightarrow{OA}$

$\overline{GH}$ tangent to $\odot O$ at $G$, $H$ is on $\overrightarrow{OA}$

$\overline{EG}$ tangent to $\odot O$ at $E$, with $E$ of the $x$-axis and $F$ on the $y$-axis

Find:

$CD =$
$OE =$
$OF =$
$GH =$
$AE =$
$AF =$
$OC =$
$OH =$
Solution

Circle O is a unit circle
Length OA = 1

The following ratios exist

\[ AB = \frac{AB}{OA} = \sin \theta \]

\[ OB = \frac{OB}{OA} = \cos \theta \]

\[ CD = \frac{CD}{OD} = \frac{AB}{OB} = \tan \theta \]

\[ OE = \frac{OC}{OA} = \frac{1}{\cos \theta} = \sec \theta \]
OF = \frac{OF}{OA} = \frac{1}{\cos(90 - \theta)} = \frac{1}{\sin \theta} = \csc \theta

GH = \frac{GH}{GO} = \tan(90 - \theta) = \cot \theta

AE = \frac{AE}{OA} = \tan \theta

AF = \frac{AF}{OA} = \tan(90 - \theta) = \cot \theta

OC = \frac{OC}{OD} = \frac{OA}{OB} = \frac{1}{\cos \theta} = \sec \theta

OH = \frac{OH}{OG} = \frac{1}{\cos(90 - \theta)} = \frac{1}{\sin \theta} = \csc \theta

Identities from the diagram
1) \frac{1}{\sin \theta} = \csc \theta
2) \frac{1}{\cos \theta} = \sec \theta
3) \frac{\sin \theta}{\cos \theta} = \tan \theta

Trig Identities derived
1) \sin \theta = \frac{1}{\csc \theta}
2) \cos \theta = \frac{1}{\sec \theta}
3) \tan \theta = \frac{1}{\cot \theta}