CollegeBoard AP

CALCULUS

CALCULUS AB
CALCULUS BC

Course Description

MAY 2009

The College Board: Connecting Students to College Success

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 5,400 schools, colleges, universities, and other educational organizations. Each year, the College Board serves seven million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

For further information visit www.collegeboard.com.

The College Board and the Advanced Placement Program encourage teachers, AP Coordinators, and school administrators to make equitable access a guiding principle for their AP programs. The College Board is committed to the principle that all students deserve an opportunity to participate in rigorous and academically challenging courses and programs. All students who are willing to accept the challenge of a rigorous academic curriculum should be considered for admission to AP courses. The Board encourages the elimination of barriers that restrict access to AP courses for students from ethnic, racial, and socioeconomic groups that have been traditionally underrepresented in the AP Program. Schools should make every effort to ensure that their AP classes reflect the diversity of their student population.

Visit the College Board on the Web: www.collegeboard.com.

Dear Colleague:

We know that AP® is a unique collaboration among motivated students, dedicated teachers, and committed high schools, colleges, and universities. Without your contributions, the rigorous instruction that takes place in classrooms around the world would not be possible.

In 2007, approximately 1.4 million students took more than 2.5 million AP Exams. Guiding these students were talented, hardworking teachers, who are the heart and soul of the AP Program. The College Board is grateful for the dedication of AP teachers and the administrators who support them.

One example of the collaboration that makes AP possible is the AP Course Audit, the process through which college faculty review AP teachers' syllabi to ensure that both teachers and administrators are aware of the expectations colleges and universities have for AP courses. This yearlong intensive assessment involved the review and analysis of more than 134,000 syllabi to determine which courses fulfill or exceed standards for college-level curricula. In total, 14,383 secondary schools worldwide succeeded in developing one or more courses that have received authorization from the College Board.

Through the AP Audit, teachers received a number of benefits. For example, you or your colleagues told us that the AP Audit helped you to obtain more current college textbooks for your students. A significant number of teachers said they were able to prevent the reduction of lab or instructional time that was scheduled to affect their courses. Because of the audit, 22,000 teachers said they were able to incorporate advances in their discipline that had not yet been added to their curricula. The searchable AP Course Ledger is online at collegeboard.com.

The College Board remains committed to supporting the work of AP teachers. AP workshops and Summer Institutes held around the world provide stimulating professional development for more than 60,000 teachers each year. Workshops provide teachers not only with valuable course-specific information but the opportunity to interact and network with their colleagues in the AP community.

This community is extended online at AP Central® where teachers can access a wide range of resources, information, and tools to support their work in the AP classroom. In response to requests from educators to make our Web site easier to use, the College Board implemented extensive improvements to collegeboard.com. A new "K–12 Teacher" homepage makes it easier to find an array of content and services. AP Central serves as an integral part of this enhanced collegeboard.com Web site.

We appreciate all of your efforts in the AP classroom and in the courses that prepare students for the rigor and challenge of AP. It is through the dedication and hard work of educators like you that a wider range of students than ever before is being given the opportunity to succeed in AP.

Sincerely,

Gaston Capeton
President
The College Board

Contents

Welcome to the AP® Program

The Advanced Placement Program® (AP) is a collaborative effort among motivated students; dedicated teachers; and committed high schools, colleges, and universities. Since its inception in 1955, the Program has enabled millions of students to take college-level courses and exams, and to earn college credit or placement, while still in high school.

Most colleges and universities in the United States, as well as colleges and universities in more than 40 other countries, have an AP policy granting incoming students credit, placement, or both on the basis of their AP Exam grades. Many of these institutions grant up to a full year of college credit (sophomore standing) to students who earn a sufficient number of qualifying AP grades.

Each year, an increasing number of parents, students, teachers, high schools, and colleges and universities turn to the AP Program as a model of educational excellence.

More information about the AP Program is available at the back of this Course Description and at AP Central, the College Board's online home for AP professionals (apcentral.collegeboard.com). Students can find more information at the AP student site (www.collegeboard.com/apstudents).

AP Courses

Thirty-seven AP courses in a wide variety of subject areas are available now. A committee of college faculty and master AP teachers designs each AP course to cover the information, skills, and assignments found in the corresponding college course. See page 2 for a complete list of AP courses and exams.

AP Exams

Each AP course has a corresponding exam that participating schools worldwide administer in May (except for AP Studio Art, which is a portfolio assessment). AP Exams contain multiple-choice questions and a free-response section (essay, problem solving, or oral response).

AP Exams are a culminating assessment in all AP courses and are thus an integral part of the Program. As a result, many schools foster the expectation that students who enroll in an AP course will take the corresponding AP Exam. Because the College Board is committed to providing access to AP Exams for homeschooled students and students whose schools do not offer AP courses, it does not require students to take an AP course prior to taking an AP Exam.

AP Course Audit

The AP Course Audit was created at the request of secondary school and college and university members of the College Board who sought a means to provide teachers and administrators with clear guidelines on the curricular and resource requirements that must be in place for AP courses. The AP Course Audit also helps colleges and universities better interpret secondary school courses marked "AP" on students' transcripts. To receive authorization from the College Board to label a course "AP,"

schools must demonstrate how their courses meet or exceed these requirements, which colleges and universities expect to see within a college-level curriculum.

The AP Program unequivocally supports the principle that each individual school must develop its own curriculum for courses labeled "AP." Rather than mandating any one curriculum for AP courses, the AP Course Audit instead provides each AP teacher with a set of expectations that college and secondary school faculty nationwide have established for college-level courses. AP teachers are encouraged to develop or maintain their own curriculum that either includes or exceeds each of these expectations; such courses will be authorized to use the "AP" designation. Credit for the success of AP courses belongs to the individual schools and teachers that create powerful, locally designed AP curricula.

Complete information about the AP Course Audit is available at AP Central.

AP Courses and Exams

Art
Art History
Studio Art: 2-D Design
Studio Art: 3-D Design
Studio Art: Drawing

Biology

Calculus
Calculus AB
Calculus BC

Chemistry

Chinese Language and Culture

Computer Science
Computer Science A
Computer Science AB*

Economics
Macroeconomics
Microeconomics

English
English Language and Composition
English Literature and Composition

Environmental Science

French
French Language
French Literature*

German Language

Government and Politics
Comparative Government and Politics
United States Government and Politics

History
European History
United States History
World History

Human Geography

Italian Language and Culture*

Japanese Language and Culture

Latin
Latin Literature*
Latin: Vergil

Music Theory

Physics
Physics B
Physics C: Electricity and Magnetism
Physics C: Mechanics

Psychology

Spanish
Spanish Language
Spanish Literature

Statistics

---

*AP Computer Science AB, AP French Literature, and AP Latin Literature will be discontinued after the May 2009 exam administration. AP Italian may also be discontinued if external funding is not secured by May 2009. Visit AP Central for details.

AP Reading

AP Exams—with the exception of AP Studio Art, which is a portfolio assessment—consist of dozens of multiple-choice questions scored by machine, and free-response questions scored at the annual AP Reading by thousands of college faculty and expert AP teachers. AP Readers use scoring standards developed by college and university faculty who teach the corresponding college course. The AP Reading offers educators both significant professional development and the opportunity to network with colleagues. For more information about the AP Reading, or to apply to serve as a Reader, visit apcentral.collegeboard.com/readers.

AP Exam Grades

The Readers' scores on the free-response questions are combined with the results of the computer-scored multiple-choice questions; the weighted raw scores are summed to give a composite score. The composite score is then converted to a grade on AP's 5-point scale:

AP GRADE	QUALIFICATION
5	Extremely well qualified
4	Well qualified
3	Qualified
2	Possibly qualified
1	No recommendation

AP Exam grades of 5 are equivalent to A grades in the corresponding college course. AP Exam grades of 4 are equivalent to grades of A−, B+, and B in college. AP Exam grades of 3 are equivalent to grades of B−, C+, and C in college.

Credit and Placement for AP Grades

Thousands of four-year colleges grant credit, placement, or both for qualifying AP Exam grades, because these grades represent a level of achievement equivalent to that of students who take the corresponding college course. That college-level equivalency is ensured through several AP Program processes:

1. The involvement of college faculty in course and exam development and other AP activities. Currently, college faculty:
   - Serve as chairs and members of the committees that develop the Course Descriptions and exams in each AP course.
   - Are responsible for standard setting and are involved in the evaluation of student responses at the AP Reading. The Chief Reader for each AP subject is a college faculty member.
   - Teach professional development institutes for experienced and new AP teachers.
   - Serve as the senior reviewers in the annual AP Course Audit, ensuring AP teachers' syllabi meet the curriculum guidelines of college-level courses.

2. AP courses and exams are reviewed and updated regularly based on the results of curriculum surveys at up to 200 colleges and universities, collaborations among the College Board and key educational and disciplinary organizations, and the interactions of committee members with professional organizations in their discipline.
3. Periodic college comparability studies are undertaken in which the performance of college students on AP Exams is compared with that of AP students to confirm that the AP grade scale of 1 to 5 is properly aligned with current college standards.

For more information about the role of colleges and universities in the AP Program, visit the Higher Ed Services section of collegeboard.com at professionals .collegeboard.com/higher-ed.

Setting Credit and Placement Policies for AP Grades

The College Board Web site for education professionals has a section geared toward colleges and universities that provides guidance in setting AP credit and placement policies and additional resources, including links to AP research studies, released exam questions, and sample student responses at varying levels of achievement for each AP Exam. Visit professionals.collegeboard.com/higher-ed/placement/ap.

The AP Credit Policy Info online search tool provides links to credit and placement policies at more than 1,000 colleges and universities. The tool helps students find the credit hours and advanced placement they can receive for qualifying exam scores within each AP subject. AP Credit Policy Info is available at www.collegeboard.com/ap/creditpolicy.

AP Calculus

INTRODUCTION

AP courses in calculus consist of a full high school academic year of work and are comparable to calculus courses in colleges and universities. It is expected that students who take an AP course in calculus will seek college credit, college placement, or both, from institutions of higher learning.

The AP Program includes specifications for two calculus courses and the exam for each course. The two courses and the two corresponding exams are designated as Calculus AB and Calculus BC.

Calculus AB can be offered as an AP course by any school that can organize a curriculum for students with mathematical ability. This curriculum should include all the prerequisites for a year's course in calculus listed on page 7. Calculus AB is designed to be taught over a full high school academic year. It is possible to spend some time on elementary functions and still cover the Calculus AB curriculum within a year. However, if students are to be adequately prepared for the Calculus AB Exam, most of the year must be devoted to the topics in differential and integral calculus described on pages 7 to 10. These topics are the focus of the AP Exam questions.

Calculus BC can be offered by schools where students are able to complete all the prerequisites listed on page 7 before taking the course. Calculus BC is a full-year course in the calculus of functions of a single variable. It includes all topics covered in Calculus AB plus additional topics, but both courses are intended to be challenging and demanding; they require a similar depth of understanding of common topics. The topics for Calculus BC are described on pages 10 to 13. A Calculus AB subscore grade is reported based on performance on the portion of the Calculus BC Exam devoted to Calculus AB topics.

Both courses described here represent college-level mathematics for which most colleges grant advanced placement and/or credit. Most colleges and universities offer a sequence of several courses in calculus, and entering students are placed within this sequence according to the extent of their preparation, as measured by the results of an AP Exam or other criteria. Appropriate credit and placement are granted by each institution in accordance with local policies. The content of Calculus BC is designed to qualify the student for placement and credit in a course that is one course beyond that granted for Calculus AB. Many colleges provide statements regarding their AP policies in their catalogs and on their Web sites.

Secondary schools have a choice of several possible actions regarding AP Calculus. The option that is most appropriate for a particular school depends on local conditions and resources: school size, curriculum, the preparation of teachers, and the interest of students, teachers, and administrators.

Success in AP Calculus is closely tied to the preparation students have had in courses leading up to their AP courses. Students should have demonstrated mastery of material from courses covering the equivalent of four full years of high school mathematics before attempting calculus. These courses should include the study of algebra, geometry, coordinate geometry, and trigonometry, with the fourth year of

study including advanced topics in algebra, trigonometry, analytic geometry, and elementary functions. Even though schools may choose from a variety of ways to accomplish these studies—including beginning the study of high school mathematics in grade 8; encouraging the election of more than one mathematics course in grade 9, 10, or 11; or instituting a program of summer study or guided independent study—it should be emphasized that eliminating preparatory course work in order to take an AP course is not appropriate.

The AP Calculus Development Committee recommends that calculus should be taught as a college-level course. With a solid foundation in courses taken before AP, students will be prepared to handle the rigor of a course at this level. Students who take an AP Calculus course should do so with the intention of placing out of a comparable college calculus course. This may be done through the AP Exam, a college placement exam, or any other method employed by the college.

THE COURSES

Philosophy

Calculus AB and Calculus BC are primarily concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multirepresentational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important.

Calculus BC is an extension of Calculus AB rather than an enhancement; common topics require a similar depth of understanding. Both courses are intended to be challenging and demanding.

Broad concepts and widely applicable methods are emphasized. The focus of the courses is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems, or problem types. Thus, although facility with manipulation and computational competence are important outcomes, they are not the core of these courses.

Technology should be used regularly by students and teachers to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to assist in interpreting results.

Through the use of the unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all the functions listed in the prerequisites.

Goals

- Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.

- Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation and should be able to use derivatives to solve a variety of problems.
- Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and should be able to use integrals to solve a variety of problems.
- Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- Students should be able to communicate mathematics and explain solutions to problems both verbally and in written sentences.
- Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.
- Students should be able to use technology to help solve problems, experiment, interpret results, and support conclusions.
- Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- Students should develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Prerequisites

Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions at the numbers $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, and their multiples.

Topic Outline for Calculus AB

This topic outline is intended to indicate the scope of the course, but it is not necessarily the order in which the topics need to be taught. Teachers may find that topics are best taught in different orders. (See AP Central [apcentral.collegeboard.com] for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

Analysis of graphs With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process
- Calculating limits using algebra
- Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity
- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

Continuity as a property of functions

- An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

II. Derivatives

Concept of the derivative

- Derivative presented graphically, numerically, and analytically
- Derivative interpreted as an instantaneous rate of change
- Derivative defined as the limit of the difference quotient
- Relationship between differentiability and continuity

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values

Derivative as a function

- Corresponding characteristics of graphs of f and f'
- Relationship between the increasing and decreasing behavior of f and the sign of f'
- The Mean Value Theorem and its geometric interpretation
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of the graphs of f, f', and f''
- Relationship between the concavity of f and the sign of f''
- Points of inflection as places where concavity changes

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity
- Optimization, both absolute (global) and relative (local) extrema
- Modeling rates of change, including related rates problems
- Use of implicit differentiation to find the derivative of an inverse function
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Derivative rules for sums, products, and quotients of functions
- Chain rule and implicit differentiation

III. Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

- Basic properties of definite integrals (examples include additivity and linearity)

Applications of integrals Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions
- Antiderivatives by substitution of variables (including change of limits for definite integrals)

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, including applications to motion along a line
- Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth)

Numerical approximations to definite integrals Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

Topic Outline for Calculus BC

The topic outline for Calculus BC includes all Calculus AB topics. Additional topics are found in paragraphs that are marked with a plus sign (+) or an asterisk (*). The additional topics can be taught anywhere in the course that the instructor wishes. Some topics will naturally fit immediately after their Calculus AB counterparts. Other topics may fit best after the completion of the Calculus AB topic outline. (See AP Central for sample syllabi.) Although the exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

I. Functions, Graphs, and Limits

Analysis of graphs With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits)

- An intuitive understanding of the limiting process
- Calculating limits using algebra
- Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

- Understanding asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity
- Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

Continuity as a property of functions

- An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- Understanding continuity in terms of limits
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

* **Parametric, polar, and vector functions** The analysis of planar curves includes those given in parametric form, polar form, and vector form.

II. Derivatives

Concept of the derivative

- Derivative presented graphically, numerically, and analytically
- Derivative interpreted as an instantaneous rate of change
- Derivative defined as the limit of the difference quotient
- Relationship between differentiability and continuity

Derivative at a point

- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values

Derivative as a function

- Corresponding characteristics of graphs of f and f'
- Relationship between the increasing and decreasing behavior of f and the sign of f'
- The Mean Value Theorem and its geometric interpretation
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives

- Corresponding characteristics of the graphs of f, f', and f''
- Relationship between the concavity of f and the sign of f''
- Points of inflection as places where concavity changes

Applications of derivatives

- Analysis of curves, including the notions of monotonicity and concavity
+ Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration
- Optimization, both absolute (global) and relative (local) extrema
- Modeling rates of change, including related rates problems
- Use of implicit differentiation to find the derivative of an inverse function
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations
+ Numerical solution of differential equations using Euler's method
+ L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Derivative rules for sums, products, and quotients of functions
- Chain rule and implicit differentiation
+ Derivatives of parametric, polar, and vector functions

III. Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

- Basic properties of definite integrals (examples include additivity and linearity)

* **Applications of integrals** Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, the length of a curve (including a curve given in parametric form), and accumulated change from a rate of change.

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined

Techniques of antidifferentiation

- Antiderivatives following directly from derivatives of basic functions
+ Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only)
+ Improper integrals (as limits of definite integrals)

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, including applications to motion along a line
- Solving separable differential equations and using them in modeling (including the study of the equation $y' = ky$ and exponential growth)
+ Solving logistic differential equations and using them in modeling

Numerical approximations to definite integrals Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

***IV. Polynomial Approximations and Series**

* **Concept of series** A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence and divergence.

* **Series of constants**

+ Motivating examples, including decimal expansion
+ Geometric series with applications
+ The harmonic series
+ Alternating series with error bound
+ Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series
+ The ratio test for convergence and divergence
+ Comparing series to test for convergence or divergence

* **Taylor series**

+ Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve)
+ Maclaurin series and the general Taylor series centered at $x = a$
+ Maclaurin series for the functions e^x, $\sin x$, $\cos x$, and $\dfrac{1}{1-x}$
+ Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series
+ Functions defined by power series
+ Radius and interval of convergence of power series
+ Lagrange error bound for Taylor polynomials

USE OF GRAPHING CALCULATORS

Professional mathematics organizations such as the National Council of Teachers of Mathematics, the Mathematical Association of America, and the Mathematical Sciences Education Board of the National Academy of Sciences have strongly endorsed the use of calculators in mathematics instruction and testing.

The use of a graphing calculator in AP Calculus is considered an integral part of the course. Students should use this technology on a regular basis so that they become adept at using their graphing calculators. Students should also have experience with the basic paper-and-pencil techniques of calculus and be able to apply them when technological tools are unavailable or inappropriate.

The AP Calculus Development Committee understands that new calculators and computers capable of enhancing the teaching of calculus continue to be developed. There are two main concerns that the committee considers when deciding what level of technology should be required for the exams: equity issues and teacher development.

Over time, the range of capabilities of graphing calculators has increased significantly. Some calculators are much more powerful than first-generation graphing calculators and may include symbolic algebra features. Other graphing calculators are, by design, intended for students studying mathematics at lower levels than calculus. The committee can develop exams that are appropriate for any given level of technology, but it cannot develop exams that are fair to all students if the spread in the capabilities of the technology is too wide. Therefore, the committee has found it necessary to make certain requirements of the technology that will help ensure that all students have sufficient computational tools for the AP Calculus Exams. Exam restrictions should not be interpreted as restrictions on classroom activities. The committee will continue to monitor the developments of technology and will reassess the testing policy regularly.

Graphing Calculator Capabilities for the Exams

The committee develops exams based on the assumption that all students have access to four basic calculator capabilities used extensively in calculus. A graphing calculator appropriate for use on the exams is expected to have the built-in capability to:

1) plot the graph of a function within an arbitrary viewing window,
2) find the zeros of functions (solve equations numerically),
3) numerically calculate the derivative of a function, and
4) numerically calculate the value of a definite integral.

One or more of these capabilities should provide the sufficient computational tools for successful development of a solution to any exam question that requires the use of a calculator. Care is taken to ensure that the exam questions do not favor students who use graphing calculators with more extensive built-in features.

Students are expected to bring a calculator with the capabilities listed above to the exams. AP teachers should check their own students' calculators to ensure that the required conditions are met. A list of acceptable calculators can be found at AP Central. If a student wishes to use a calculator that is not on the list, the teacher must contact the AP Program (609 771-7300) before April 1 of the testing year to request written permission for the student to use the calculator on AP Exams.

Technology Restrictions on the Exams

Nongraphing scientific calculators, computers, devices with a QWERTY keyboard, and pen-input/stylus-driven devices or electronic writing pads are not permitted for use on the AP Calculus Exams.

Test administrators are required to check calculators before the exam. Therefore, it is important for each student to have an approved calculator. The student should be thoroughly familiar with the operation of the calculator he or she plans to use. Calculators may not be shared, and communication between calculators is prohibited during the exam. Students may bring to the exam one or two (but no more than two) graphing calculators from the approved list.

Calculator memories will not be cleared. Students are allowed to bring calculators containing whatever programs they want. They are expected to bring calculators that are set to radian mode.

Students must not use calculator memories to take test materials out of the room. Students should be warned that their grades will be invalidated if they attempt to remove test materials by any method.

Showing Work on the Free-Response Sections

An important goal of the free-response section of the AP Calculus Exams is to provide students with an opportunity to communicate their knowledge of correct reasoning and methods. Students are required to show their work so that AP Exam Readers can assess the students' methods and answers. To be eligible for partial credit, methods, reasoning, and conclusions should be presented clearly. Answers without supporting work may not receive credit. Students should use complete sentences in responses that include explanations or justifications.

For results obtained using one of the four required calculator capabilities listed on page 14, students are required to write the setup (i.e., the equation being solved, or the derivative or definite integral being evaluated) that leads to the solution, along with the result produced by the calculator. For example, if the student is asked to find the area of a region, the student is expected to show a definite integral (i.e., the setup) and the answer. The student need not compute the antiderivative; the calculator may be used to calculate the value of the definite integral without further explanation. For solutions obtained using a calculator capability other than one of the four required ones, students must also show the mathematical steps necessary to produce their results; a calculator result alone is not sufficient. For example, if the student is asked to find a relative minimum value of a function, the student is expected to use calculus and show the mathematical steps that lead to the answer. It is not sufficient to graph the function or use a built-in minimum finder.

When a student is asked to justify an answer, the justification must include mathematical (noncalculator) reasons, not merely calculator results. Functions, graphs, tables, or other objects that are used in a justification should be clearly labeled.

A graphing calculator is a powerful tool for exploration, but students must be cautioned that exploration is not a mathematical solution. Exploration with a graphing calculator can lead a student toward an analytical solution, and after a solution is found, a graphing calculator can often be used to check the reasonableness of the solution.

As on previous AP Exams, if a calculation is given as a decimal approximation, it should be correct to three places after the decimal point unless otherwise indicated. Students should be cautioned against rounding values in intermediate steps before a final calculation is made. Students should also be aware that there are limitations inherent in graphing calculator technology; for example, answers obtained by tracing along a graph to find roots or points of intersection might not produce the required accuracy.

Sign charts by themselves are not accepted as a sufficient response when a free-response question requires a justification for the existence of either a local or an absolute extremum of a function at a particular point in its domain. For more detailed information on this policy, read the article "On the Role of Sign Charts in AP Calculus Exams for Justifying Local or Absolute Extrema" that is available on the Home Pages for Calculus AB and Calculus BC at AP Central.

For more information on the instructions for the free-response sections, read the "Calculus FRQ Instruction Commentary" that is available on the Home Pages for Calculus AB and Calculus BC at AP Central.

THE EXAMS

The Calculus AB and BC Exams seek to assess how well a student has mastered the concepts and techniques of the subject matter of the corresponding courses. Each exam consists of two sections, as described below.

Section I: a multiple-choice section testing proficiency in a wide variety of topics

Section II: a free-response section requiring the student to demonstrate the ability to solve problems involving a more extended chain of reasoning

The time allotted for each AP Calculus Exam is 3 hours and 15 minutes. The multiple-choice section of each exam consists of 45 questions in 105 minutes. Part A of the multiple-choice section (28 questions in 55 minutes) does not allow the use of a calculator. Part B of the multiple-choice section (17 questions in 50 minutes) contains some questions for which a graphing calculator is required.

The free-response section of each exam has two parts: one part requiring graphing calculators, and a second part not allowing graphing calculators. The AP Exams are designed to accurately assess student mastery of both the concepts and techniques of calculus. The two-part format for the free-response section provides greater flexibility in the types of problems that can be given while ensuring fairness to all students taking the exam, regardless of the graphing calculator used.

The free-response section of each exam consists of 6 problems in 90 minutes. Part A of the free-response section (3 problems in 45 minutes) contains some problems or parts of problems for which a graphing calculator is required. Part B of the free-response section (3 problems in 45 minutes) does not allow the use of a calculator. During the second timed portion of the free-response section (Part B), students are permitted to continue work on problems in Part A, but they are not permitted to use a calculator during this time.

In determining the grade for each exam, the scores for Section I and Section II are given equal weight. Since the exams are designed for full coverage of the subject matter, it is not expected that all students will be able to answer all the questions.

Calculus AB Subscore Grade for the Calculus BC Exam

A Calculus AB subscore grade is reported based on performance on the portion of the exam devoted to Calculus AB topics (approximately 60 percent of the exam). The Calculus AB subscore grade is designed to give colleges and universities more information about the student. Although each college and university sets its own policy for awarding credit and/or placement for AP Exam grades, it is recommended that institutions apply the same policy to the Calculus AB subscore grade that they apply to the Calculus AB grade. Use of the subscore grade in this manner is consistent with the philosophy of the courses, since common topics are tested at the same conceptual level in both Calculus AB and Calculus BC.

Calculus AB: Section I

Section I consists of 45 multiple-choice questions. Part A contains 28 questions and does not allow the use of a calculator. Part B contains 17 questions and requires a graphing calculator for some questions. Twenty-four sample multiple-choice questions for Calculus AB are included in the following sections. Answers to the sample questions are given on page 27.

Part A Sample Multiple-Choice Questions

A calculator may not be used on this part of the exam.

Part A consists of 28 questions. In this section of the exam, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I, Part A, and a representative set of 14 questions.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. What is $\lim_{h \to 0} \dfrac{\cos\left(\frac{3\pi}{2} + h\right) - \cos\left(\frac{3\pi}{2}\right)}{h}$?

   (A) 1

   (B) $\dfrac{\sqrt{2}}{2}$

   (C) 0

   (D) -1

   (E) The limit does not exist.

2. At which of the five points on the graph in the figure at the right are $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ both negative?

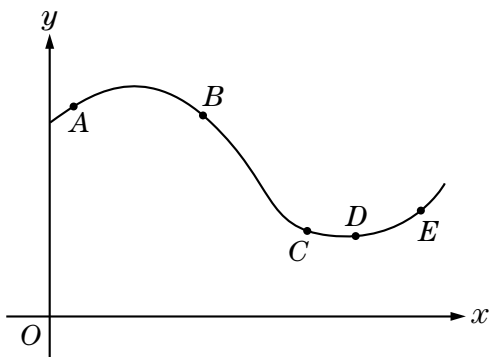
   (A) A

   (B) B

   (C) C

   (D) D

   (E) E



3. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at $(2, 1)$ is

   (A) $-\dfrac{3}{2}$

   (B) -1

   (C) $-\dfrac{5}{14}$

   (D) $-\dfrac{3}{14}$

   (E) 0

4. Let S be the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$ for $0 \le x \le 1$. What is the volume of the solid generated when S is revolved about the line $y = 3$?

(A) $\pi \int_0^1 \left(\left(3 - 2x^2\right)^2 - (3 - 2x)^2 \right) dx$

(B) $\pi \int_0^1 \left((3 - 2x)^2 - \left(3 - 2x^2\right)^2 \right) dx$

(C) $\pi \int_0^1 \left(4x^2 - 4x^4\right) dx$

(D) $\pi \int_0^2 \left(\left(3 - \frac{y}{2}\right)^2 - \left(3 - \sqrt{\frac{y}{2}}\right)^2 \right) dy$

(E) $\pi \int_0^2 \left(\left(3 - \sqrt{\frac{y}{2}}\right)^2 - \left(3 - \frac{y}{2}\right)^2 \right) dy$

5. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

(A) The function has no relative extremum.
(B) The graph of the function has one point of inflection and the function has two relative extrema.
(C) The graph of the function has two points of inflection and the function has one relative extremum.
(D) The graph of the function has two points of inflection and the function has two relative extrema.
(E) The graph of the function has two points of inflection and the function has three relative extrema.

6. If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

(A) $-2 \cos 3$
(B) $-2 \sin 3 \cos 3$
(C) $6 \cos 3$
(D) $2 \sin 3 \cos 3$
(E) $6 \sin 3 \cos 3$

7. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{4x}{y}$, where $y(2) = -2$?

(A) $y = 2x$ for $x > 0$

(B) $y = 2x - 6$ for $x \ne 3$

(C) $y = -\sqrt{4x^2 - 12}$ for $x > \sqrt{3}$

(D) $y = \sqrt{4x^2 - 12}$ for $x > \sqrt{3}$

(E) $y = -\sqrt{4x^2 - 6}$ for $x > \sqrt{1.5}$

8. What is the average rate of change of the function f given by $f(x) = x^4 - 5x$ on the closed interval $[0, 3]$?

(A) 8.5
(B) 8.7
(C) 22
(D) 33
(E) 66

9. The position of a particle moving along a line is given by $s(t) = 2t^3 - 24t^2 + 90t + 7$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

(A) $3 < t < 4$ only
(B) $t > 4$ only
(C) $t > 5$ only
(D) $0 < t < 3$ and $t > 5$
(E) $3 < t < 4$ and $t > 5$

10. $\int (x-1)\sqrt{x}\, dx =$

(A) $\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + C$

(B) $\frac{2}{3}x^{3/2} + \frac{1}{2}x^{1/2} + C$

(C) $\frac{1}{2}x^2 - x + C$

(D) $\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C$

(E) $\frac{1}{2}x^2 + 2x^{3/2} - x + C$

11. What is $\lim_{x \to \infty} \frac{x^2 - 4}{2 + x - 4x^2}$?

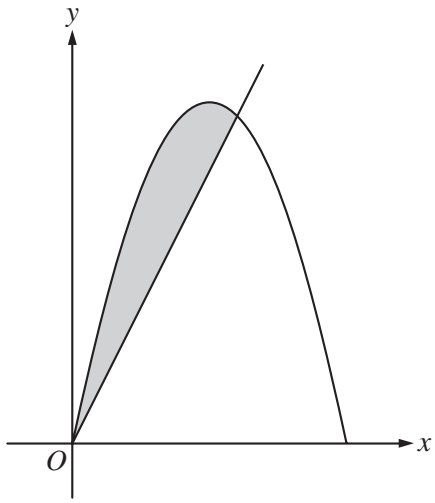
(A) -2

(B) $-\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

(E) The limit does not exist.



12. The figure above shows the graph of $y = 5x - x^2$ and the graph of the line $y = 2x$. What is the area of the shaded region?

(A) $\frac{25}{6}$

(B) $\frac{9}{2}$

(C) 9

(D) $\frac{27}{2}$

(E) $\frac{45}{2}$

13. If $y = 5 + \int_2^{2x} e^{-t^2}\, dt$, which of the following is true?

(A) $\frac{dy}{dx} = e^{-x^2}$ and $y(0) = 5$

(B) $\frac{dy}{dx} = e^{-x^2}$ and $y(1) = 5$

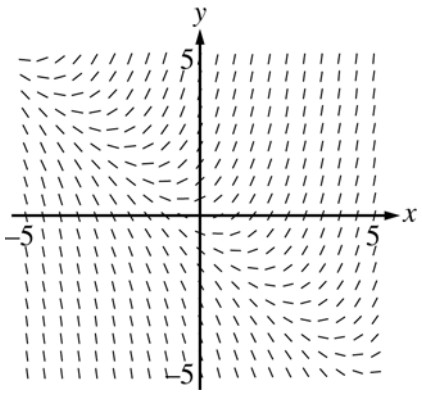
(C) $\frac{dy}{dx} = e^{-4x^2}$ and $y(1) = 5$

(D) $\frac{dy}{dx} = 2e^{-4x^2}$ and $y(0) = 5$

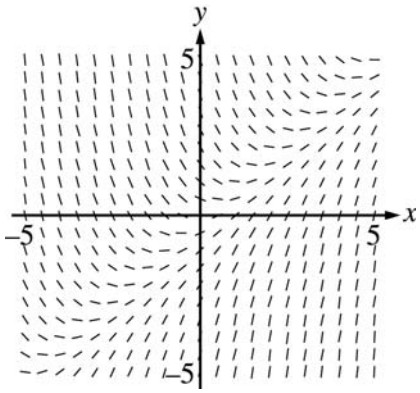
(E) $\frac{dy}{dx} = 2e^{-4x^2}$ and $y(1) = 5$

14. Which of the following is a slope field for the differential equation $\frac{dy}{dx} = \frac{x}{y}$?

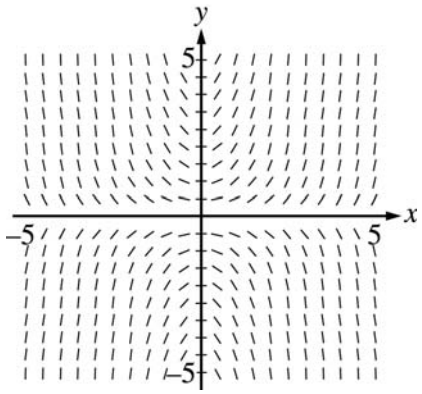
(A)



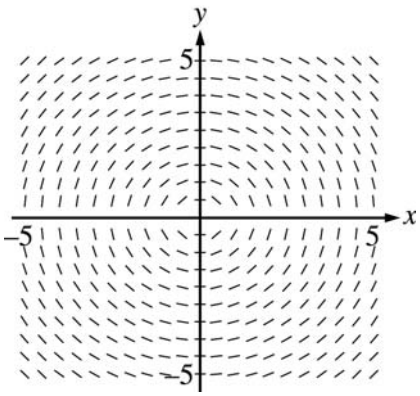
(B)



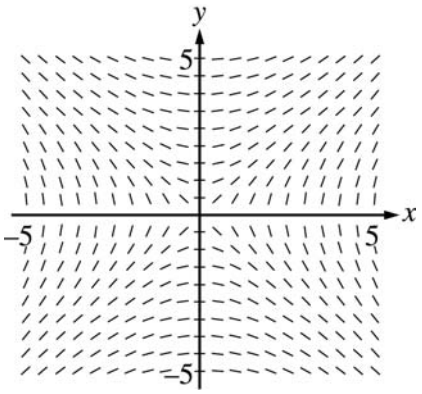
(C)



(D)



(E)



Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the exam.

Part B consists of 17 questions. In this section of the exam, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I, Part B, and a representative set of 10 questions.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

15. A particle travels along a straight line with a velocity of $v(t) = 3e^{(-t/2)}\sin(2t)$ meters per second. What is the total distance, in meters, traveled by the particle during the time interval $0 \le t \le 2$ seconds?

(A) 0.835
(B) 1.850
(C) 2.055
(D) 2.261
(E) 7.025

16. A city is built around a circular lake that has a radius of 1 mile. The population density of the city is $f(r)$ people per square mile, where r is the distance from the center of the lake, in miles. Which of the following expressions gives the number of people who live within 1 mile of the lake?

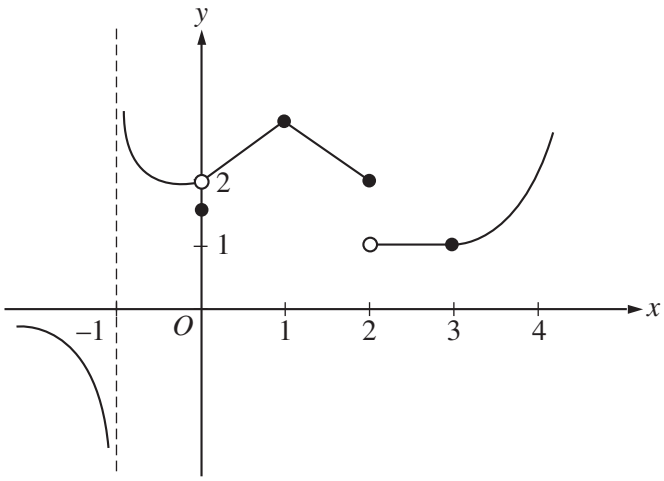
(A) $2\pi\int_0^1 r f(r)\, dr$

(B) $2\pi\int_0^1 r(1 + f(r))\, dr$

(C) $2\pi\int_0^2 r(1 + f(r))\, dr$

(D) $2\pi\int_1^2 r f(r)\, dr$

(E) $2\pi\int_1^2 r(1 + f(r))\, dr$



17. The graph of a function f is shown above. If $\lim_{x \to b} f(x)$ exists and f is not continuous at b, then $b =$

(A) –1
(B) 0
(C) 1
(D) 2
(E) 3

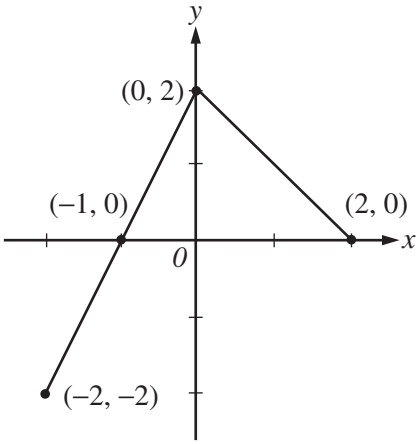
x	1.1	1.2	1.3	1.4
$f(x)$	4.18	4.38	4.56	4.73

18. Let f be a function such that $f''(x) < 0$ for all x in the closed interval [1, 2]. Selected values of f are shown in the table above. Which of the following must be true about $f'(1.2)$?

(A) $f'(1.2) < 0$
(B) $0 < f'(1.2) < 1.6$
(C) $1.6 < f'(1.2) < 1.8$
(D) $1.8 < f'(1.2) < 2.0$
(E) $f'(1.2) > 2.0$

19. Two particles start at the origin and move along the x-axis. For $0 \le t \le 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?

(A) None
(B) One
(C) Two
(D) Three
(E) Four



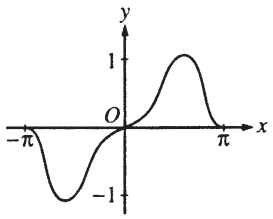
Graph of f

20. The graph of the function f shown above consists of two line segments. If g is the function defined by $g(x) = \int_0^x f(t)\,dt$, then $g(-1) =$

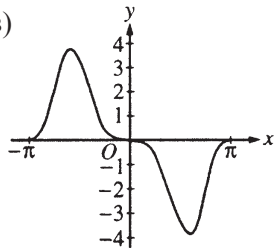
(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

21. The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval $[-\pi, \pi]$?

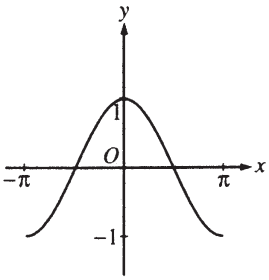
(A)



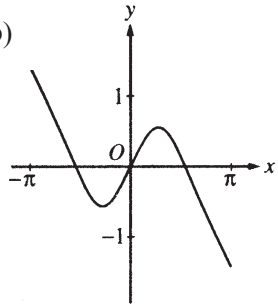
(B)



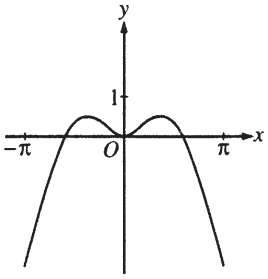
(C)



(D)



(E)



22. A differentiable function f has the property that $f(5) = 3$ and $f'(5) = 4$. What is the estimate for $f(4.8)$ using the local linear approximation for f at $x = 5$?

(A) 2.2
(B) 2.8
(C) 3.4
(D) 3.8
(E) 4.6

23. Oil is leaking from a tanker at the rate of $R(t) = 2{,}000e^{-0.2t}$ gallons per hour, where t is measured in hours. How much oil leaks out of the tanker from time $t = 0$ to $t = 10$?

(A) 54 gallons
(B) 271 gallons
(C) 865 gallons
(D) 8,647 gallons
(E) 14,778 gallons

24. If $f'(x) = \sin\left(\frac{\pi e^x}{2}\right)$ and $f(0) = 1$, then $f(2) =$

(A) -1.819
(B) -0.843
(C) -0.819
(D) 0.157
(E) 1.157

Answers to Calculus AB Multiple-Choice Questions

Part A		*Part B*	
1.	A	15.*	D
2.	B	16.	D
3.	C	17.	B
4.	A	18.	D
5.	C	19.*	D
6.	B	20.	B
†7.	C	21.	E
8.	C	22.	A
9.	E	23.*	D
10.	D	24.*	E
11.	B		
12.	B		
13.	E		
14.	E		

*Indicates a graphing calculator-active question.
†For resources on differential equations, see the Home Pages for Calculus AB and Calculus BC at AP Central.

Calculus BC: Section I

Section I consists of 45 multiple-choice questions. Part A contains 28 questions and does not allow the use of a calculator. Part B contains 17 questions and requires a graphing calculator for some questions. Twenty-four sample multiple-choice questions for Calculus BC are included in the following sections. Answers to the sample questions are given on page 39.

Part A Sample Multiple-Choice Questions

A calculator may not be used on this part of the exam.

Part A consists of 28 questions. In this section of the exam, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I, Part A, and a representative set of 14 questions.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. A curve is described by the parametric equations $x = t^2 + 2t$ and $y = t^3 + t^2$. An equation of the line tangent to the curve at the point determined by $t = 1$ is

   (A) $2x - 3y = 0$
   (B) $4x - 5y = 2$
   (C) $4x - y = 10$
   (D) $5x - 4y = 7$
   (E) $5x - y = 13$

2. If $3x^2 + 2xy + y^2 = 1$, then $\frac{dy}{dx} =$

(A) $-\frac{3x + y}{y^2}$

(B) $-\frac{3x + y}{x + y}$

(C) $\frac{1 - 3x - y}{x + y}$

(D) $-\frac{3x}{1 + y}$

(E) $-\frac{3x}{x + y}$

x	$g'(x)$
-1.0	2
-0.5	4
0.0	3
0.5	1
1.0	0
1.5	-3
2.0	-6

3. The table above gives selected values for the derivative of a function g on the interval $-1 \le x \le 2$. If $g(-1) = -2$ and Euler's method with a step-size of 1.5 is used to approximate $g(2)$, what is the resulting approximation?

(A) -6.5
(B) -1.5
(C) 1.5
(D) 2.5
(E) 3

4. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{n3^n}{x^n}$ converges?

(A) All x except $x = 0$
(B) $|x| = 3$
(C) $-3 \le x \le 3$
(D) $|x| > 3$
(E) The series diverges for all x.

5. If $\frac{d}{dx}f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx}f(h(x)) =$

(A) $g\left(x^2\right)$

(B) $2xg(x)$

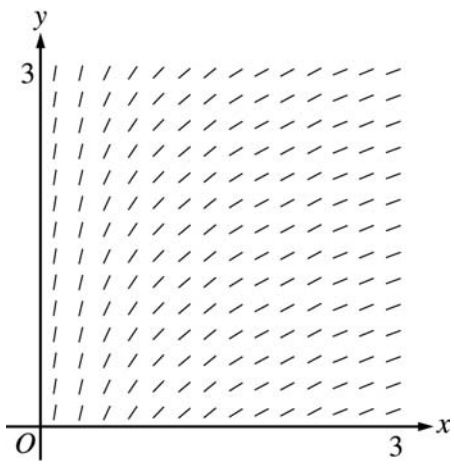
(C) $g'(x)$

(D) $2xg\left(x^2\right)$

(E) $x^2g\left(x^2\right)$

6. If F' is a continuous function for all real x, then $\lim_{h\to 0}\frac{1}{h}\int_a^{a+h}F'(x)\,dx$ is

(A) 0
(B) $F(0)$
(C) $F(a)$
(D) $F'(0)$
(E) $F'(a)$



7. The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$
(B) $y = e^x$
(C) $y = e^{-x}$
(D) $y = \cos x$
(E) $y = \ln x$

8. $\displaystyle\int_0^3 \frac{dx}{(1-x)^2}$ is

(A) $-\frac{3}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

(E) divergent

9. Which of the following series converge to 2?

I. $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n+3}$

II. $\displaystyle\sum_{n=1}^{\infty} \frac{-8}{(-3)^n}$

III. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n}$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

10. If the function f given by $f(x) = x^3$ has an average value of 9 on the closed interval $[0, k]$, then $k =$

(A) 3

(B) $3^{1/2}$

(C) $18^{1/3}$

(D) $36^{1/4}$

(E) $36^{1/3}$

11. Which of the following integrals gives the length of the graph $y = \sin\left(\sqrt{x}\right)$ between $x = a$ and $x = b$, where $0 < a < b$?

(A) $\int_a^b \sqrt{x + \cos^2\left(\sqrt{x}\right)}\, dx$

(B) $\int_a^b \sqrt{1 + \cos^2\left(\sqrt{x}\right)}\, dx$

(C) $\int_a^b \sqrt{\sin^2\left(\sqrt{x}\right) + \frac{1}{4x}\cos^2\left(\sqrt{x}\right)}\, dx$

(D) $\int_a^b \sqrt{1 + \frac{1}{4x}\cos^2\left(\sqrt{x}\right)}\, dx$

(E) $\int_a^b \sqrt{\frac{1 + \cos^2\left(\sqrt{x}\right)}{4x}}\, dx$

12. Which of the following integrals represents the area enclosed by the smaller loop of the graph of $r = 1 + 2\sin\theta$?

(A) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1 + 2\sin\theta)^2\, d\theta$

(B) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1 + 2\sin\theta)\, d\theta$

(C) $\frac{1}{2}\int_{-\pi/6}^{7\pi/6} (1 + 2\sin\theta)^2\, d\theta$

(D) $\int_{-\pi/6}^{7\pi/6} (1 + 2\sin\theta)^2\, d\theta$

(E) $\int_{7\pi/6}^{-\pi/6} (1 + 2\sin\theta)\, d\theta$

13. The third-degree Taylor polynomial about $x = 0$ of $\ln(1 - x)$ is

(A) $-x - \frac{x^2}{2} - \frac{x^3}{3}$

(B) $1 - x + \frac{x^2}{2}$

(C) $x - \frac{x^2}{2} + \frac{x^3}{3}$

(D) $-1 + x - \frac{x^2}{2}$

(E) $-x + \frac{x^2}{2} - \frac{x^3}{3}$

14. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$

(B) $e^{\tan x} + 5$

(C) $5e^{\tan x}$

(D) $\tan x + 5$

(E) $\tan x + 5e^x$

Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the exam.

Part B consists of 17 questions. In this section of the exam, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I, Part B, and a representative set of 10 questions.

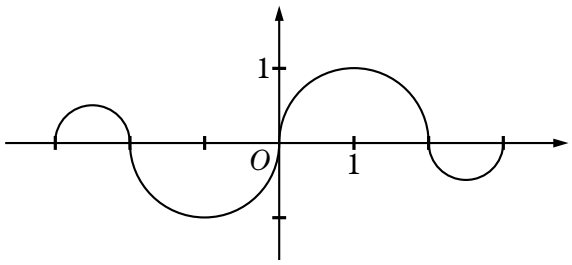
Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



Graph of f

15. The graph of the function f above consists of four semicircles. If $g(x) = \int_0^x f(t)\,dt$, where is $g(x)$ nonnegative?

(A) $[-3, 3]$
(B) $[-3, -2] \cup [0, 2]$ only
(C) $[0, 3]$ only
(D) $[0, 2]$ only
(E) $[-3, -2] \cup [0, 3]$ only

16. If f is differentiable at $x = a$, which of the following could be false?

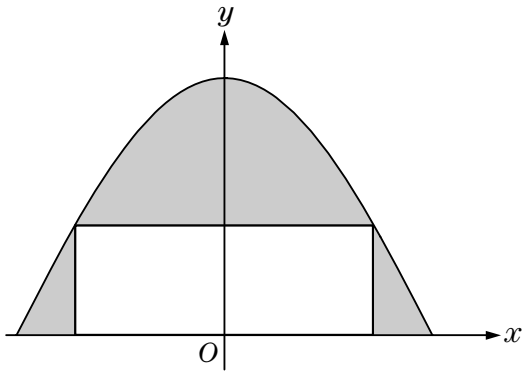
(A) f is continuous at $x = a$.

(B) $\lim_{x \to a} f(x)$ exists.

(C) $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ exists.

(D) $f'(a)$ is defined.

(E) $f''(a)$ is defined.



17. A rectangle with one side on the x-axis has its upper vertices on the graph of $y = \cos x$, as shown in the figure above. What is the minimum area of the shaded region?

(A) 0.799
(B) 0.878
(C) 1.140
(D) 1.439
(E) 2.000

18. A solid has a rectangular base that lies in the first quadrant and is bounded by the x- and y-axes and the lines $x = 2$ and $y = 1$. The height of the solid above the point (x, y) is $1 + 3x$. Which of the following is a Riemann sum approximation for the volume of the solid?

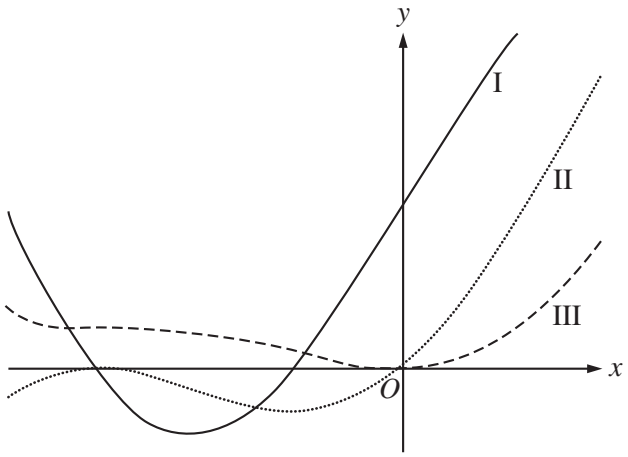
(A) $\sum_{i=1}^{n} \frac{1}{n}\left(1 + \frac{3i}{n}\right)$

(B) $2\sum_{i=1}^{n} \frac{1}{n}\left(1 + \frac{3i}{n}\right)$

(C) $2\sum_{i=1}^{n} \frac{i}{n}\left(1 + \frac{3i}{n}\right)$

(D) $\sum_{i=1}^{n} \frac{2}{n}\left(1 + \frac{6i}{n}\right)$

(E) $\sum_{i=1}^{n} \frac{2i}{n}\left(1 + \frac{6i}{n}\right)$



19. Three graphs labeled I, II, and III are shown above. One is the graph of f, one is the graph of f', and one is the graph of f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(A)	I	II	III
(B)	I	III	II
(C)	II	I	III
(D)	II	III	I
(E)	III	II	I

20. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \ln(t+1) - 2t + 1$. The total distance traveled by the particle from $t = 0$ to $t = 2$ is

(A) 0.667
(B) 0.704
(C) 1.540
(D) 2.667
(E) 2.901

21. If the function f is defined by $f(x) = \sqrt{x^3 + 2}$ and g is an antiderivative of f such that $g(3) = 5$, then $g(1) =$

(A) −3.268
(B) −1.585
(C) 1.732
(D) 6.585
(E) 11.585

22. Let g be the function given by $g(x) = \int_1^x 100\left(t^2 - 3t + 2\right)e^{-t^2}\, dt$.
Which of the following statements about g must be true?

I. g is increasing on (1, 2).

II. g is increasing on (2, 3).

III. $g(3) > 0$

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

23. For a series S, let

$$S = 1 - \frac{1}{9} + \frac{1}{2} - \frac{1}{25} + \frac{1}{4} - \frac{1}{49} + \frac{1}{8} - \frac{1}{81} + \frac{1}{16} - \frac{1}{121} + \cdots + a_n + \cdots,$$

$$\text{where } a_n = \begin{cases} \dfrac{1}{2^{(n-1)/2}} & \text{if } n \text{ is odd} \\ \dfrac{-1}{(n+1)^2} & \text{if } n \text{ is even} \end{cases}.$$

Which of the following statements are true?

I. S converges because the terms of S alternate and $\lim_{n\to\infty} a_n = 0$.

II. S diverges because it is not true that $|a_{n+1}| < |a_n|$ for all n.

III. S converges although it is not true that $|a_{n+1}| < |a_n|$ for all n.

(A) None
(B) I only
(C) II only
(D) III only
(E) I and III only

24. Let g be the function given by $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$.
For $0 \le t \le 8$, g is decreasing most rapidly when $t =$

(A) 0.949
(B) 2.017
(C) 3.106
(D) 5.965
(E) 8.000

Answers to Calculus BC Multiple-Choice Questions

Part A		*Part B*	
†1.	D	15.	A
2.	B	16.	E
†3.	D	17.*	B
†4.	D	18.	D
5.	D	19.	E
6.	E	20.*	C
7.	E	21.*	B
†8.	E	22.*	B
†9.	E	†23.	D
10.	E	24.*	B
†11.	D		
†12.	A		
†13.	A		
14.	C		

*Indicates a graphing calculator-active question.
†Indicates a Calculus BC–only topic.

Calculus AB and Calculus BC: Section II

Section II consists of six free-response problems. The problems do NOT appear in the Section II exam booklet. Part A problems are printed in the green insert* only; Part B problems are printed in a separate sealed blue insert. Each part of every problem has a designated workspace in the exam booklet. ALL WORK MUST BE SHOWN IN THE EXAM BOOKLET. (For students taking the exam at a late administration, the Part A problems are printed in the exam booklet only; the Part B problems appear in a separate sealed insert.)

The instructions below are from the 2008 exams. The free-response problems are from the 2006 exams and include information on scoring. Additional sample questions can be found at AP Central.

Instructions for Section II

Total Time	**1 hour, 30 minutes**
Number of Questions	6
Percent of Total Grade	50%
Writing Instrument	Either pencil or pen with black or dark blue ink
Weight	The questions are weighted equally, but the parts of a question are not necessarily given equal weight.
Part A	
Number of Questions	3
Time	45 minutes
Electronic Devices	Graphing calculator required
Part B	
Number of Questions	3
Time	45 minutes
Electronic Devices	None allowed

The questions for Part A are printed in the green insert and the questions for Part B are printed in the blue insert. You may use the inserts to organize your answers and for scratch work, but you must write your answers in the pink Section II booklet. No credit will be given for work written in the inserts. Write your solution to each part of each question in the space provided for that part in the Section II booklet. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be graded.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you

*Form B exams have lavender and gray inserts.

must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may keep the green insert and continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\, dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

For more information on the instructions for the free-response sections, read the "Calculus FRQ Instruction Commentary" that is available on the Home Pages for Calculus AB and Calculus BC at AP Central.

Calculus AB Sample Free-Response Questions

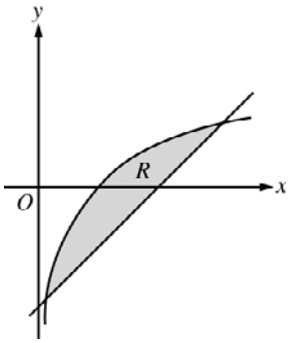
Question 1

Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.



$\ln(x) = x - 2$ when $x = 0.15859$ and 3.14619.
Let $S = 0.15859$ and $T = 3.14619$

(a) Area of $R = \int_S^T (\ln(x) - (x - 2))\, dx = 1.949$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_S^T \left((\ln(x) + 3)^2 - (x - 2 + 3)^2 \right) dx$

$= 34.198$ or 34.199

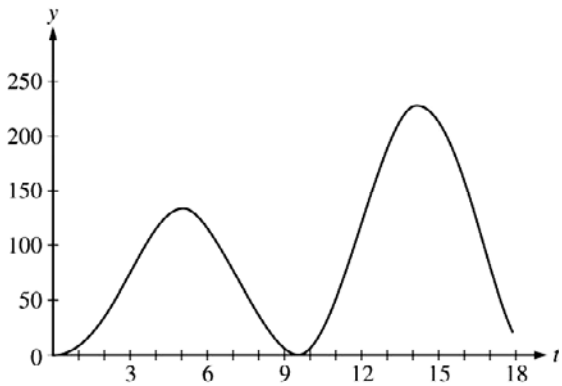
$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits, constant, and answer} \end{cases}$

(c) Volume $= \pi \int_{S-2}^{T-2} \left((y + 2)^2 - \left(e^y\right)^2 \right) dy$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

Question 2

At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.



(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

(a) $\int_0^{18} L(t)\,dt \approx 1658$ cars

$2: \begin{cases} 1: \text{setup} \\ 1: \text{answer} \end{cases}$

(b) $L(t) = 150$ when $t = 12.42831,\ 16.12166$

Let $R = 12.42831$ and $S = 16.12166$

$L(t) \ge 150$ for t in the interval $[R, S]$

$\frac{1}{S-R}\int_R^S L(t)\,dt = 199.426$ cars per hour

$3: \begin{cases} 1: t\text{-interval when } L(t) \ge 150 \\ 1: \text{average value integral} \\ 1: \text{answer with units} \end{cases}$

(c) For the product to exceed 200,000, the number of cars turning left in a two-hour interval must be greater than 400.

$\int_{13}^{15} L(t)\,dt = 431.931 > 400$

$4: \begin{cases} 1: \text{considers 400 cars} \\ 1: \text{valid interval } [h, h+2] \\ 1: \text{value of } \int_h^{h+2} L(t)\,dt \\ 1: \text{answer and explanation} \end{cases}$

OR

The number of cars turning left will be greater than 400 on a two-hour interval if $L(t) \ge 200$ on that interval.

$L(t) \ge 200$ on any two-hour subinterval of $[13.25304, 15.32386]$.

OR

$4: \begin{cases} 1: \text{considers 200 cars per hour} \\ 1: \text{solves } L(t) \ge 200 \\ 1: \text{discusses 2 hour interval} \\ 1: \text{answer and explanation} \end{cases}$

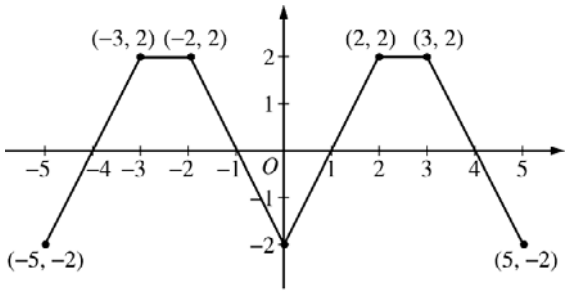
Yes, a traffic signal is required.

Question 3

The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(4)$, $g'(4)$, and $g''(4)$.

(b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.



Graph of f

(c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f. Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

(a) $g(4) = \int_0^4 f(t)\,dt = 3$

$g'(4) = f(4) = 0$

$g''(4) = f'(4) = -2$

$3 : \begin{cases} 1 : g(4) \\ 1 : g'(4) \\ 1 : g''(4) \end{cases}$

(b) g has a relative minimum at $x = 1$ because $g' = f$ changes from negative to positive at $x = 1$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(c) $g(0) = 0$ and the function values of g increase by 2 for every increase of 5 in x.

$g(10) = 2g(5) = 4$

$$g(108) = \int_0^{105} f(t)\,dt + \int_{105}^{108} f(t)\,dt$$
$$= 21g(5) + g(3) = 44$$

$g'(108) = f(108) = f(3) = 2$

An equation for the line tangent to the graph of g at $x = 108$ is $y - 44 = 2(x - 108)$.

$4 : \begin{cases} 1 : g(10) \\ 3 : \begin{cases} 1 : g(108) \\ 1 : g'(108) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$

Question 4

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(c) Rocket B is launched upward with an acceleration of $a(t) = \dfrac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

(a) Average acceleration of rocket A is

$$\frac{v(80) - v(0)}{80 - 0} = \frac{49 - 5}{80} = \frac{11}{20} \text{ ft}/\text{sec}^2$$

1 : answer

(b) Since the velocity is positive, $\int_{10}^{70} v(t)\,dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds.

A midpoint Riemann sum is
$20[v(20) + v(40) + v(60)]$
$= 20[22 + 35 + 44] = 2020$ ft

3 : { 1 : explanation; 1 : uses $v(20)$, $v(40)$, $v(60)$; 1 : value }

(c) Let $v_B(t)$ be the velocity of rocket B at time t.

$$v_B(t) = \int \frac{3}{\sqrt{t+1}}\,dt = 6\sqrt{t+1} + C$$
$$2 = v_B(0) = 6 + C$$
$$v_B(t) = 6\sqrt{t+1} - 4$$
$$v_B(80) = 50 > 49 = v(80)$$

Rocket B is traveling faster at time $t = 80$ seconds.

4 : { 1 : $6\sqrt{t+1}$; 1 : constant of integration; 1 : uses initial condition; 1 : finds $v_B(80)$, compares to $v(80)$, and draws a conclusion }

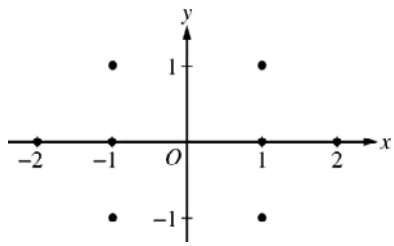
Units of ft/sec^2 in (a) and ft in (b)

1 : units in (a) and (b)

Question 5

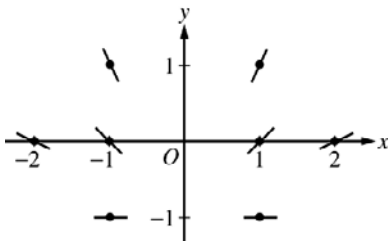
Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated. **(Note: Use the axes provided in the pink exam booklet.)**



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

(a)



2 : sign of slope at each point and relative steepness of slope lines in rows and columns

(b) $\frac{1}{1+y}\,dy = \frac{1}{x}\,dx$

$\ln|1+y| = \ln|x| + K$

$|1+y| = e^{\ln|x| + K}$

$1 + y = C|x|$

$2 = C$

$1 + y = 2|x|$

$y = 2|x| - 1$ and $x < 0$

or

$y = -2x - 1$ and $x < 0$

7 :
- 6 :
  - 1 : separates variables
  - 2 : antiderivatives
  - 1 : constant of integration
  - 1 : uses initial condition
  - 1 : solves for y
- Note: max $3/6$ [1-2-0-0-0] if no constant of integration
- Note: $0/6$ if no separation of variables
- 1 : domain

Question 6

The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \text{ and } f''(0) = 3.$$

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answers.

(b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

(a) $g'(x) = ae^{ax} + f'(x)$
$g'(0) = a - 4$

$g''(x) = a^2e^{ax} + f''(x)$
$g''(0) = a^2 + 3$

$$4 : \begin{cases} 1 : g'(x) \\ 1 : g'(0) \\ 1 : g''(x) \\ 1 : g''(0) \end{cases}$$

(b) $h'(x) = f'(x)\cos(kx) - k\sin(kx)f(x)$
$h'(0) = f'(0)\cos(0) - k\sin(0)f(0) = f'(0) = -4$
$h(0) = \cos(0)f(0) = 2$
The equation of the tangent line is $y = -4x + 2$.

$$5 : \begin{cases} 2 : h'(x) \\ 3 : \begin{cases} 1 : h'(0) \\ 1 : h(0) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$$

Calculus BC Sample Free-Response Questions

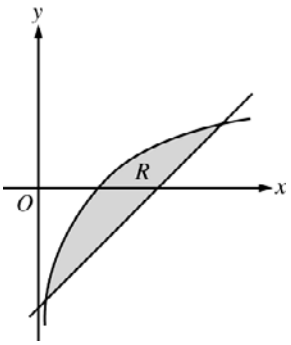
Question 1

Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.



$\ln(x) = x - 2$ when $x = 0.15859$ and 3.14619.
Let $S = 0.15859$ and $T = 3.14619$

(a) Area of $R = \int_S^T (\ln(x) - (x - 2))\, dx = 1.949$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_S^T \left((\ln(x) + 3)^2 - (x - 2 + 3)^2 \right) dx$

$= 34.198$ or 34.199

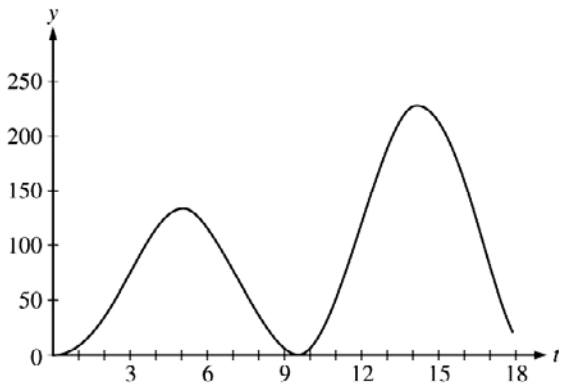
$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits, constant, and answer} \end{cases}$

(c) Volume $= \pi \int_{S-2}^{T-2} \left((y + 2)^2 - \left(e^y\right)^2 \right) dy$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

Question 2

At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.



(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

(a) $\int_0^{18} L(t)\,dt \approx 1658$ cars

$2: \begin{cases} 1: \text{setup} \\ 1: \text{answer} \end{cases}$

(b) $L(t) = 150$ when $t = 12.42831,\ 16.12166$

Let $R = 12.42831$ and $S = 16.12166$

$L(t) \ge 150$ for t in the interval $[R, S]$

$\frac{1}{S-R}\int_R^S L(t)\,dt = 199.426$ cars per hour

$3: \begin{cases} 1: t\text{-interval when } L(t) \ge 150 \\ 1: \text{average value integral} \\ 1: \text{answer with units} \end{cases}$

(c) For the product to exceed 200,000, the number of cars turning left in a two-hour interval must be greater than 400.

$\int_{13}^{15} L(t)\,dt = 431.931 > 400$

$4: \begin{cases} 1: \text{considers 400 cars} \\ 1: \text{valid interval } [h, h+2] \\ 1: \text{value of } \int_h^{h+2} L(t)\,dt \\ 1: \text{answer and explanation} \end{cases}$

OR

The number of cars turning left will be greater than 400 on a two-hour interval if $L(t) \ge 200$ on that interval.

$L(t) \ge 200$ on any two-hour subinterval of $[13.25304, 15.32386]$.

Yes, a traffic signal is required.

OR

$4: \begin{cases} 1: \text{considers 200 cars per hour} \\ 1: \text{solves } L(t) \ge 200 \\ 1: \text{discusses 2 hour interval} \\ 1: \text{answer and explanation} \end{cases}$

Question 3

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sin^{-1}\left(1 - 2e^{-t}\right) \text{ and } \frac{dy}{dt} = \frac{4t}{1+t^3}$$

for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

(a) Find the acceleration vector and the speed of the object at time $t = 2$.

(b) The curve has a vertical tangent line at one point. At what time t is the object at this point?

(c) Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of t and use it to evaluate $\lim_{t\to\infty} m(t)$.

(d) The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, an expression involving an improper integral that represents this value c.

(a) $a(2) = \langle 0.395 \text{ or } 0.396, -0.741 \text{ or } -0.740 \rangle$

Speed $= \sqrt{x'(2)^2 + y'(2)^2} = 1.207$ or 1.208

$2 : \begin{cases} 1 : \text{acceleration} \\ 1 : \text{speed} \end{cases}$

(b) $\sin^{-1}\left(1 - 2e^{-t}\right) = 0$

$1 - 2e^{-t} = 0$

$t = \ln 2 = 0.693$ and $\frac{dy}{dt} \neq 0$ when $t = \ln 2$

$2 : \begin{cases} 1 : x'(t) = 0 \\ 1 : \text{answer} \end{cases}$

(c) $m(t) = \frac{4t}{1+t^3} \cdot \frac{1}{\sin^{-1}\left(1 - 2e^{-t}\right)}$

$$\lim_{t\to\infty} m(t) = \lim_{t\to\infty}\left(\frac{4t}{1+t^3} \cdot \frac{1}{\sin^{-1}\left(1 - 2e^{-t}\right)}\right)$$

$$= 0\left(\frac{1}{\sin^{-1}(1)}\right) = 0$$

$2 : \begin{cases} 1 : m(t) \\ 1 : \text{limit value} \end{cases}$

(d) Since $\lim_{t\to\infty} x(t) = \infty$,

$$c = \lim_{t\to\infty} y(t) = -3 + \int_2^\infty \frac{4t}{1+t^3}\, dt$$

$3 : \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{initial value consistent} \\ \quad \text{with lower limit} \end{cases}$

Question 4

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(c) Rocket B is launched upward with an acceleration of $a(t) = \dfrac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

(a) Average acceleration of rocket A is

$$\frac{v(80) - v(0)}{80 - 0} = \frac{49 - 5}{80} = \frac{11}{20} \text{ ft}/\text{sec}^2$$

1 : answer

(b) Since the velocity is positive, $\int_{10}^{70} v(t)\,dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds.

A midpoint Riemann sum is
$20[v(20) + v(40) + v(60)]$
$= 20[22 + 35 + 44] = 2020$ ft

3 : { 1 : explanation; 1 : uses $v(20)$, $v(40)$, $v(60)$; 1 : value }

(c) Let $v_B(t)$ be the velocity of rocket B at time t.

$$v_B(t) = \int \frac{3}{\sqrt{t+1}}\,dt = 6\sqrt{t+1} + C$$
$$2 = v_B(0) = 6 + C$$
$$v_B(t) = 6\sqrt{t+1} - 4$$
$$v_B(80) = 50 > 49 = v(80)$$

Rocket B is traveling faster at time $t = 80$ seconds.

4 : { 1 : $6\sqrt{t+1}$; 1 : constant of integration; 1 : uses initial condition; 1 : finds $v_B(80)$, compares to $v(80)$, and draws a conclusion }

Units of ft/sec^2 in (a) and ft in (b)

1 : units in (a) and (b)

Question 5

Consider the differential equation $\frac{dy}{dx} = 5x^2 - \frac{6}{y-2}$ for $y \neq 2$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = -4$.

(a) Evaluate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $(-1, -4)$.

(b) Is it possible for the x-axis to be tangent to the graph of f at some point? Explain why or why not.

(c) Find the second-degree Taylor polynomial for f about $x = -1$.

(d) Use Euler's method, starting at $x = -1$ with two steps of equal size, to approximate $f(0)$. Show the work that leads to your answer.

(a) $\left.\frac{dy}{dx}\right|_{(-1,\,-4)} = 6$

$\frac{d^2y}{dx^2} = 10x + 6(y-2)^{-2}\frac{dy}{dx}$

$\left.\frac{d^2y}{dx^2}\right|_{(-1,\,-4)} = -10 + 6\frac{1}{(-6)^2}6 = -9$

$3 : \begin{cases} 1 : \left.\frac{dy}{dx}\right|_{(-1,\,-4)} \\ 1 : \frac{d^2y}{dx^2} \\ 1 : \left.\frac{d^2y}{dx^2}\right|_{(-1,\,-4)} \end{cases}$

(b) The x-axis will be tangent to the graph of f if $\left.\frac{dy}{dx}\right|_{(k,\,0)} = 0$.

The x-axis will never be tangent to the graph of f because $\left.\frac{dy}{dx}\right|_{(k,\,0)} = 5k^2 + 3 > 0$ for all k.

$2 : \begin{cases} 1 : \frac{dy}{dx} = 0 \text{ and } y = 0 \\ 1 : \text{answer and explanation} \end{cases}$

(c) $P(x) = -4 + 6(x+1) - \frac{9}{2}(x+1)^2$

$2 : \begin{cases} 1 : \text{quadratic and centered at } x = -1 \\ 1 : \text{coefficients} \end{cases}$

(d) $f(-1) = -4$

$f\left(-\frac{1}{2}\right) \approx -4 + \frac{1}{2}(6) = -1$

$f(0) \approx -1 + \frac{1}{2}\left(\frac{5}{4} + 2\right) = \frac{5}{8}$

$2 : \begin{cases} 1 : \text{Euler's method with 2 steps} \\ 1 : \text{Euler's approximation to } f(0) \end{cases}$

Question 6

The function f is defined by the power series

$$f(x) = -\frac{x}{2} + \frac{2x^2}{3} - \frac{3x^3}{4} + \cdots + \frac{(-1)^n nx^n}{n+1} + \cdots$$

for all real numbers x for which the series converges. The function g is defined by the power series

$$g(x) = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \cdots + \frac{(-1)^n x^n}{(2n)!} + \cdots$$

for all real numbers x for which the series converges.

(a) Find the interval of convergence of the power series for f. Justify your answer.

(b) The graph of $y = f(x) - g(x)$ passes through the point $(0, -1)$. Find $y'(0)$ and $y''(0)$. Determine whether y has a relative minimum, a relative maximum, or neither at $x = 0$. Give a reason for your answer.

(a) $\left|\dfrac{(-1)^{n+1}(n+1)x^{n+1}}{n+2} \cdot \dfrac{n+1}{(-1)^n nx^n}\right| = \dfrac{(n+1)^2}{(n+2)(n)} \cdot |x|$

$\lim\limits_{n\to\infty} \dfrac{(n+1)^2}{(n+2)(n)} \cdot |x| = |x|$

The series converges when $-1 < x < 1$.

When $x = 1$, the series is $-\frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \cdots$

This series does not converge, because the limit of the individual terms is not zero.

When $x = -1$, the series is $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots$

This series does not converge, because the limit of the individual terms is not zero.

Thus, the interval of convergence is $-1 < x < 1$.

$5 : \begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{identifies radius of convergence} \\ 1 : \text{considers both endpoints} \\ 1 : \text{analysis/conclusion for both endpoints} \end{cases}$

(b) $f'(x) = -\frac{1}{2} + \frac{4}{3}x - \frac{9}{4}x^2 + \cdots$ and $f'(0) = -\frac{1}{2}$.

$g'(x) = -\frac{1}{2!} + \frac{2}{4!}x - \frac{3}{6!}x^2 + \cdots$ and $g'(0) = -\frac{1}{2}$.

$y'(0) = f'(0) - g'(0) = 0$

$f''(0) = \frac{4}{3}$ and $g''(0) = \frac{2}{4!} = \frac{1}{12}$.

Thus, $y''(0) = \frac{4}{3} - \frac{1}{12} > 0$.

Since $y'(0) = 0$ and $y''(0) > 0$, y has a relative minimum at $x = 0$.

$4 : \begin{cases} 1 : y'(0) \\ 1 : y''(0) \\ 1 : \text{conclusion} \\ 1 : \text{reasoning} \end{cases}$

Teacher Support

AP Central® (apcentral.collegeboard.com)

You can find the following Web resources at AP Central (free registration required):

- AP Course Descriptions, AP Exam questions and scoring guidelines, sample syllabi, and feature articles.
- A searchable Institutes and Workshops database, providing information about professional development events.
- The Course Home Pages (apcentral.collegeboard.com/coursehomepages), which contain insightful articles, teaching tips, activities, lab ideas, and other course-specific content contributed by colleagues in the AP community.
- Moderated electronic discussion groups (EDGs) for each AP course, provided to facilitate the exchange of ideas and practices.

AP Publications and Other Resources

Free AP resources are available to help students, parents, AP Coordinators, and high school and college faculty learn more about the AP Program and its courses and exams. Visit apcentral.collegeboard.com/freepubs.

Teacher's Guides and Course Descriptions may be downloaded free of charge from AP Central; printed copies may be purchased through the College Board Store (store.collegeboard.com). Released Exams and other priced AP resources are available at the College Board Store.

Teacher's Guides

For those about to teach an AP course for the first time, or for experienced AP teachers who would like to get some fresh ideas for the classroom, the *Teacher's Guide* is an excellent resource. Each *Teacher's Guide* contains syllabi developed by high school teachers currently teaching the AP course and college faculty who teach the equivalent course at colleges and universities. Along with detailed course outlines and innovative teaching tips, you'll also find extensive lists of suggested teaching resources.

Course Descriptions

Course Descriptions are available for each AP subject. They provide an outline of each AP course's content, explain the kinds of skills students are expected to demonstrate in the corresponding introductory college-level course, and describe the AP Exam. Sample multiple-choice questions with an answer key and sample free-response questions are included. (The Course Description for AP Computer Science is available in PDF format only.)

Released Exams

Periodically the AP Program releases a complete copy of each exam. In addition to providing the multiple-choice questions and answers, the publication describes the process of scoring the free-response questions and includes examples of students' actual responses, the scoring standards, and commentary that explains why the responses received the scores they did.

Contact Us

National Office
Advanced Placement Program
45 Columbus Avenue
New York, NY 10023-6992
212 713-8066
E-mail: ap@collegeboard.org

AP Services
P.O. Box 6671
Princeton, NJ 08541-6671
609 771-7300
877 274-6474 (toll free in the U.S. and Canada)
E-mail: apexams@info.collegeboard.org

AP Canada Office
2950 Douglas Street, Suite 550
Victoria, BC, Canada V8T 4N4
250 472-8561
800 667-4548 (toll free in Canada only)
E-mail: gewonus@ap.ca

AP International Office
Serving all countries outside the U.S. and Canada
45 Columbus Avenue
New York, NY 10023-6992
212 373-8738
E-mail: apintl@collegeboard.org

Middle States Regional Office
Serving Delaware, District of Columbia, Maryland, New Jersey, New York, Pennsylvania, Puerto Rico, and the U.S. Virgin Islands
Two Bala Plaza, Suite 900
Bala Cynwyd, PA 19004-1501
866 392-3019
E-mail: msro@collegeboard.org

Midwestern Regional Office
Serving Illinois, Indiana, Iowa, Kansas, Michigan, Minnesota, Missouri, Nebraska, North Dakota, Ohio, South Dakota, West Virginia, and Wisconsin
6111 N. River Road, Suite 550
Rosemont, IL 60018-5158
866 392-4086
E-mail: mro@collegeboard.org

New England Regional Office
Serving Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island, and Vermont
470 Totten Pond Road
Waltham, MA 02451-1982
866 392-4089
E-mail: nero@collegeboard.org

Southern Regional Office
Serving Alabama, Florida, Georgia, Kentucky, Louisiana, Mississippi, North Carolina, South Carolina, Tennessee, and Virginia
3700 Crestwood Parkway NW, Suite 700
Duluth, GA 30096-7155
866 392-4088
E-mail: sro@collegeboard.org

Southwestern Regional Office
Serving Arkansas, New Mexico, Oklahoma, and Texas
4330 South MoPac Expressway, Suite 200
Austin, TX 78735-6735
866 392-3017
E-mail: swro@collegeboard.org

Western Regional Office
Serving Alaska, Arizona, California, Colorado, Hawaii, Idaho, Montana, Nevada, Oregon, Utah, Washington, and Wyoming
2099 Gateway Place, Suite 550
San Jose, CA 95110-1051
866 392-4078
E-mail: wro@collegeboard.org

2007-08 Development Committee and Chief Reader

Stephen Davis, Davidson College, North Carolina, *Chair*
Janet Beery, University of Redlands, California
Ruth Dover, Illinois Mathematics and Science Academy, Aurora
James Epperson, The University of Texas at Arlington
Guy Mauldin, Science Hill High School, Johnson City, Tennessee
Monique Morton, Woodrow Wilson Senior High School, Washington, D.C.
Tara Smith, University of Cincinnati, Ohio
Chief Reader: **Michael Boardman,** Pacific University, Forest Grove, Oregon
ETS Consultants: **Fred Kluempen, Craig Wright**

apcentral.collegeboard.com

I.N.070082350