Discrete Random Variables

- Finite number of values
- Displayed in histograms
- Mean/Expected Value: (Weighted Average): $\mu = \sum x \cdot P(x)$
- To be a valid Probability Distribution:
  - $0 \leq P(x) \leq 1$
  - $\sum P(x) = 1$

GPS/CCSS: MM3D1

Binomial Distribution

1) categorical variables
2) “Success” depends on how we “define” the random variable
3) $p = \text{probability of success}$
4) $(1-p) = \text{probability of failure}$
5) Assumes independent trials (p stays constant)
6) $\mu = np = \text{mean}$
7) $s = \sqrt{np(1-p)} = \text{s.d.}$
8) easily calculated probabilities
9) $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$
10) can be approximated well by the normal distribution when # of failure and success is at least 15
11) “Success” does not always mean something “positive” or “good”
12) Each trial has two outcomes
13) Fixed number of trials, $n$.

GPS/CCSS: N/A

Geometric Distribution

- $X =$ number of trials until the first success; discrete random variable
- $P(X = x) = p(1-p)^{x-1}$ where $x = 1, 2, \ldots$ and $p = \text{prob. of a success}$
- $\mu = 1/p$
- Special case of the negative binomial distribution.

GPS/CCSS: N/A
Normal Distribution (Continuous R. Variable)

- $z$-score probability
- Probability distribution $\rightarrow$ symmetric, bell-shaped graph
- Mean and Standard deviation parameters
- Empirical Rule (Image from our textbook, p. 280)

Continuous random variable has possible values that form an interval
Probability is between 0 and 1.

GPS/CCSS: MM2D1, MM3D2
Adding/Subtracting Two Independent Random Variables

- The sum of the means = mean of total \( x \pm y = \mu_x \pm \mu_y \)
- The square root of the sums of the squares of the standard deviations (of the parts) is the standard deviation of the total (or difference)

\[
\begin{align*}
\frac{2}{x+y} &= \frac{2}{x} + \frac{2}{y} \\
\frac{2}{x} \frac{2}{y} &= \frac{2}{x} + \frac{2}{y}
\end{align*}
\]

GPS/CCSS: MM3D1

Frequentist versus Bayesian Probability

<table>
<thead>
<tr>
<th>FREQUENTIST</th>
<th>BAYESIAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long run relative frequency</td>
<td>Probability is evidence based – degree belief</td>
</tr>
<tr>
<td>( \Theta ) is fixed and unknown</td>
<td>( \Theta ) must be given a distribution</td>
</tr>
<tr>
<td>( \Theta ) never gets assigned probability</td>
<td>( \Theta ) has probability but no agreement on dist.</td>
</tr>
</tbody>
</table>

GPS/CCSS: N/A

Confidence Interval for a Proportion

Estimate \( \pm \) margin of error \( \rightarrow \) sample proportion \( \pm z \) (standard error of sample proportion)

Contains a range of plausible values for the population proportion at a specified confidence level

If using 95% confidence level, in repeated sampling, if we build an interval using the above procedure, we expect 95% of the intervals to capture the population proportion.

GPS/CCSS: MM4D3