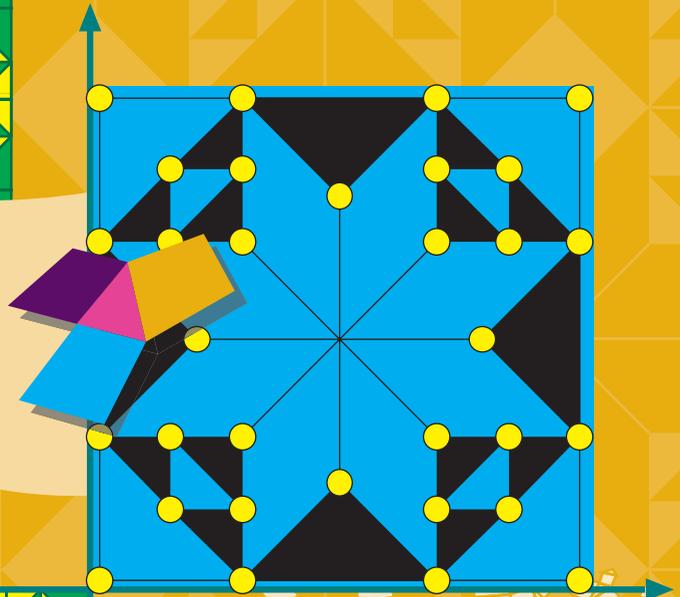


101 PROJECT IDEAS

for **THE GEOMETER'S SKETCHPAD[®]**



Dynamic Geometry[®] Activities



Key Curriculum Press
Key College Publishing

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Editor and Art Designer: Steven Chanan

Co-editor: Dan Bennett

Contributors: Masha Albrecht, Dan Bennett, Steven Chanan, Bill Finzer, Dan Lufkin, John Olive, John Owens, Ralph Pantozzi, James M. Parks, Cathi Sanders, Daniel Scher, Audrey Weeks, Janet Zahumeny

Sketchpad Design and Implementation: Nick Jackiw

Production Editor: Jennifer Strada

Art and Design Coordinator: Caroline Ayres

Production and Manufacturing Manager: Diana Jean Parks

Layout and Cover Designer: Kirk Mills

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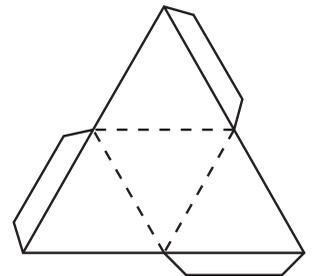
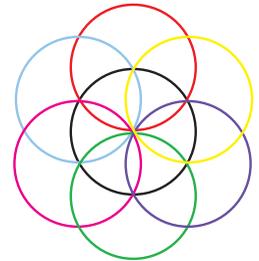
101 Project Ideas for The Geometer's Sketchpad®

There are a million and one things you can do with The Geometer's Sketchpad, and this little book covers just a few. If you're not particularly experienced with Sketchpad™, you may want to start by working through the Guided Tours in the Learning Guide that came with the program. The projects in this book are designed to be challenging—they are **project ideas** after all. So, don't worry if in some cases you need to ask for help from friends or teachers. If you get really stuck, or just want to see other possible solutions to selected projects, then visit the 101 Project Ideas area of The Geometer's Sketchpad Resource Center at www.keypress.com/sketchpad/. And please do feel free to send along your project ideas for inclusion in possible sequels!

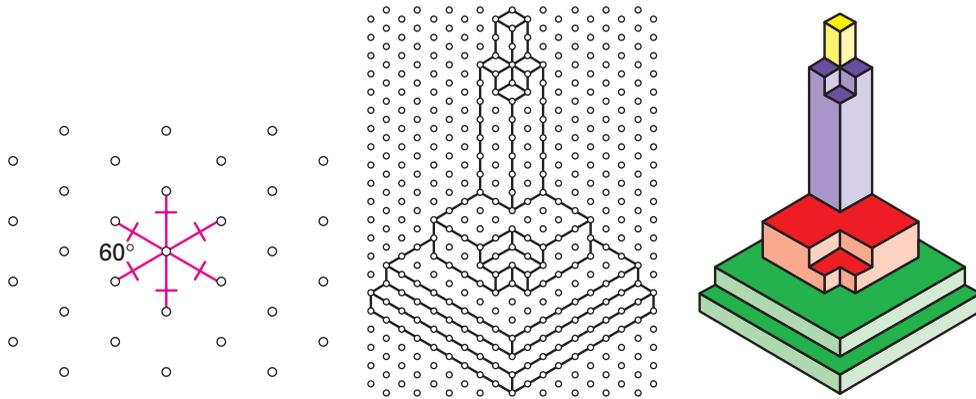
The projects in this book are arranged by category—Circles, Fractals, Real World Modeling, and so on—but keep in mind that most projects could have been placed in any of several categories. Also, within a given section, projects are arranged in an approximate order of difficulty. In other words, the first project idea listed under Triangles should be a lot easier to pull off than the last project listed in that section. Furthermore, don't feel that you need to do a project exactly the way it's described or shown—let your own creativity be your guide!

Art/Animation

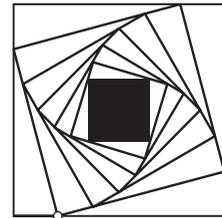
1. A simple **daisy design** is shown at right. Create your own daisy designs using only the Compass tool.
2. Reproduce a favorite optical illusion in Sketchpad.
3. Using reflections and possibly other transformations, create an animated kaleidoscope.
4. A two-dimensional template for folding a three-dimensional solid is called a **net**. Construct nets for each of the five platonic solids, complete with folding tabs. (A net for a tetrahedron is shown at right.) Then print out your nets and build the solids!
5. Fold a favorite origami figure, and then create a step-by-step set of illustrated instructions in Sketchpad that someone else could follow. To find a good origami figure to use, go to a local arts and crafts store or search the Web. To see an example of an excellent implementation of this idea (and to see some other great student Sketchpad projects!), check out www.k12.hi.us/~csanders/SW.html.



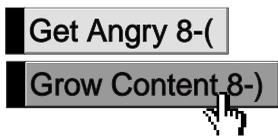
6. An isometric grid is a grid of dots in which each dot is equidistant from the six dots closest to it, as shown below left. Create an isometric grid and use it to draw 3-D skyscrapers or anything else you can imagine.



7. The logo for Key Curriculum Press's Interactive Mathematics Program® textbook series is shown at right (with a point added). Construct this logo in Sketchpad so that the ratio of a side of a square to the side of the next smallest square remains constant. Animate the point shown for a really cool effect.



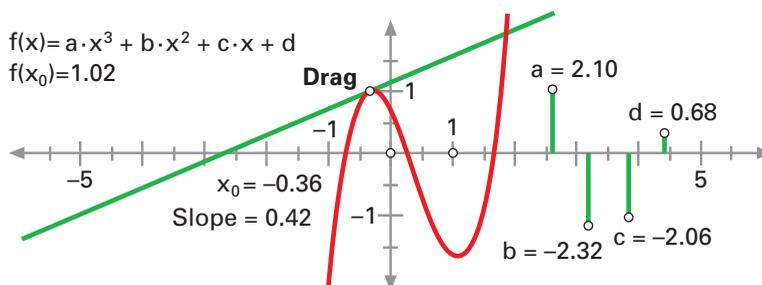
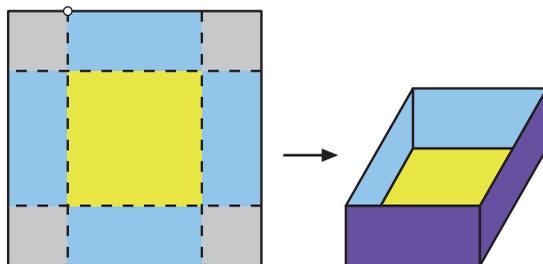
8. Draw a house or a building, perhaps where you live, in two-point perspective. There should be "control points," or "sliders," for changing the perspective.
9. Create two or more figures using segments, arcs, circles, or whatever. The figures can be letters, geometric figures, or, as below, caricatured faces. Now, using Movement buttons (or, for more control and sophistication, a slider controlling a dilation from one state to the other), set things up so that one figure "morphs" into the other, as shown below.



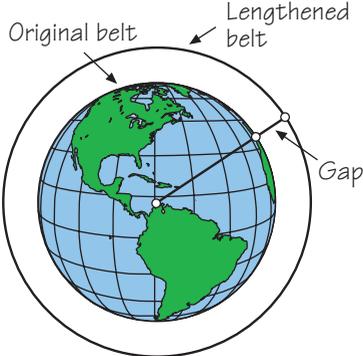
Calculus

In order to do several of the projects in this section, you will need to know how to plot functions based on constructed sliders or using the New Parameter command. See **Appendix 1: Plotting in Sketchpad** and **Appendix 2: Sliders** for more information.

10. Imagine that you want to create an open box by cutting equal squares from the four corners of a rectangular sheet of cardboard and then folding, as shown at right. Model this situation in Sketchpad. (You don't necessarily need to show the folded box, but this is a nice touch.) Graph the relationship between the length of an edge of a cut-out square and the volume of the resulting box. What size square maximizes the box's volume?
11. Create a sketch that shows that the slope of a secant line to a curve gets closer and closer to the slope of the tangent line as the two points defining the secant line get closer and closer together.
12. Create a sketch that shows the graph of a general cubic function $y = ax^3 + bx^2 + cx + d$ and the graphs of its first and second derivatives, y' and y'' . Sliders for a , b , c , and d should control the original function, and as they are dragged, all three graphs should change accordingly.
13. Create a sketch with action buttons to display the left-hand rectangles, right-hand rectangles, or trapezoids that approximate the area under a portion of a curve. The sketch should also display the estimated area under the curve in that region for the given shapes.
14. Create a sketch that shows the tangent line to the general cubic curve $y = ax^3 + bx^2 + cx + d$ (or any other graph of your choice) at a certain point. The user should be able to change the graph using sliders for a , b , c , and d , and move the point of tangency around by dragging it. Your final sketch might resemble the one below.



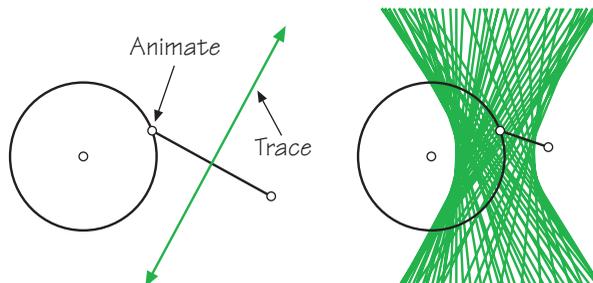
Circles

15. Construct a circle and an inscribed angle. Measure the angle and the arc it intercepts. Animate the vertex of the inscribed angle to demonstrate that every inscribed angle that intercepts this arc has the same measure.
16. Draw two circles and connect them with a segment (don't connect the segment to either circle's radius point). What does the midpoint of your segment trace when you animate the segment's endpoints around the circles? Experiment with different relative sizes of the circles, distances between them, and speeds of the two points. Then, instead of two circles, you might try animating around other shapes, such as square or triangle interiors.
17. Construct a circle and a **secant** line (a line that intersects the circle in two points). Construct a radius to one of the intersection points, and measure the angle formed by the radius and the secant line. Drag one of the intersection points closer to the other until they coincide (and the secant line becomes a **tangent** line). Describe what you observe about the angle formed by a tangent and the radius drawn to the point of tangency.
18. Imagine a "belt" wrapped snugly around the earth (assume the earth is a perfect sphere and the belt is wrapped around a great circle). Now cut the belt and add 10 feet to its length. Stretch it back into a (slightly larger) circle and position it so that it's a uniform distance above the earth's surface. How large is the gap between the earth and the belt? Start by making a guess as to what you think the answer would be in the actual situation described. Then explore the question by modeling the situation in Sketchpad using circles of various sizes. Does the size of the original sphere matter? What is the mathematical relationship between the added length and the resulting gap?
 
19. Imagine that you are playing pool on a circular table (with no pockets and only one ball!). Create a sketch in which you control the initial ball placement and initial shot direction and that shows the path the ball will take for some (relatively large) number of bounces.
20. Create a Custom tool that, given a circle and a point outside the circle, will construct both tangent lines to the circle through the point.

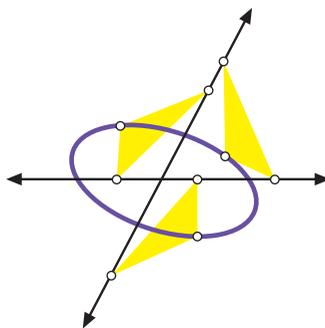
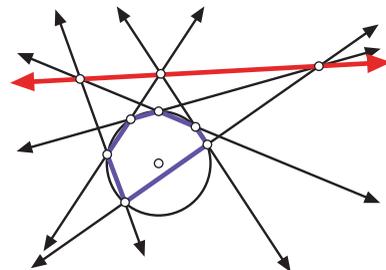
Conic Sections

21. Construct hyperbolic/elliptical envelopes as follows (see the figure on the next page): Construct a circle and a segment with one endpoint on the circle (not at the circle's radius point). Now construct the perpendicular bisector of the segment and trace it as the endpoint on the circle is animated. (Alternatively, you can use the Locus command.) What you get is either a **hyperbolic envelope** or an **elliptical envelope**, depending on where the segment's free endpoint is located. Drag parts of your

sketch around to see how the shape of the resulting envelope is affected. As a follow-up, read **Folding an Ellipse** and **Folding a Hyperbola** (pp. 22–23) in **More Joy of Mathematics** by Theoni Pappas (or find information elsewhere on how to fold these figures on paper with a circle and a dot). Try to explain the connection between these paper-folding activities and your Sketchpad sketch.



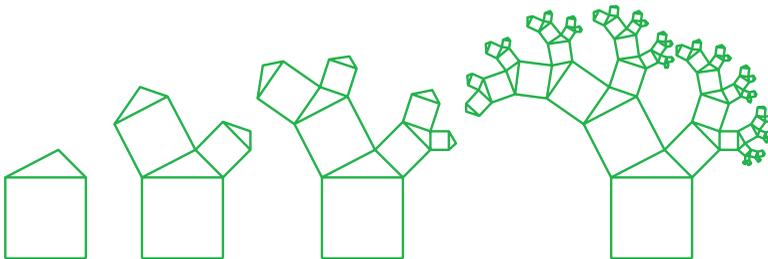
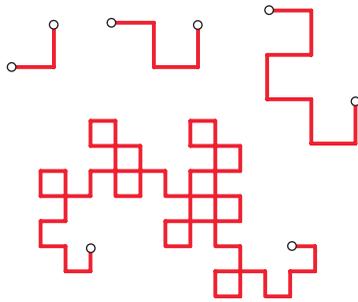
22. A parabola can be defined as the locus of points equidistant from a given point and a given line. An ellipse can be defined as the locus of points, the **sum** of whose distances to two given points is a constant. A hyperbola can be defined as the locus of points, the **difference** of whose distances to two given points is a constant. Construct one or all of these curves in Sketchpad based on these definitions.
23. Demonstrate **Pascal's theorem**. First, inscribe a hexagon in a conic section (circle, parabola, ellipse, or hyperbola). Then extend the hexagon's sides and find the intersection points of pairs of lines from **opposite** sides. Pascal's theorem states that these three points will always lie along a straight line.
24. Construct an ellipse using Leonardo da Vinci's method: Construct a pair of intersecting lines and attach one vertex of a triangle of given dimensions to one line and the second vertex to the other. As the triangle's vertices move through every possible position on these lines (without the triangle changing shape or size), the third vertex traces out an ellipse.



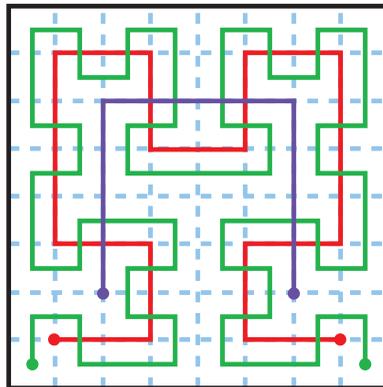
25. **Super-challenge:** Given a triangle and an arbitrary point inside it, construct an ellipse that has that point as one of its foci and that is inscribed in the triangle.

Fractals

26. If you've read the book **Jurassic Park**, you're familiar with the **Dragon Fractal** (it appears at the beginning of each chapter at higher and higher iterations). It's constructed as follows: Start with two congruent segments at 90° to each other (as in the top left figure at right). Now rotate the entire figure 90° around one of the endpoints. Take the new figure and rotate it 90° around one of its endpoints. Continue this process as many times as you'd like. Try using the Iterate command to quickly generate higher-level versions of this fractal. First-, second-, third-, and sixth-level Dragon Fractals are shown above right.
27. A **Pythagorean Tree** is a fractal that is constructed as shown below. Create a Custom tool for constructing Pythagorean Trees or use the Iterate command to quickly generate higher-level versions.

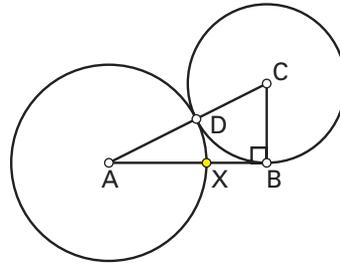
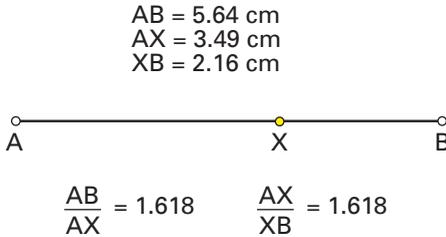


28. **Super-challenge:** The first three iterations of the Hilbert space-filling curve construction are shown below (first iteration in purple, second in red, third in green). Do a little research on the Hilbert curve, and then use the Iterate command to construct a Hilbert curve of a given iteration. The problem is, no one we've talked to knows for sure whether or not this is even **possible** in Sketchpad. If you do figure it out, please let us know!



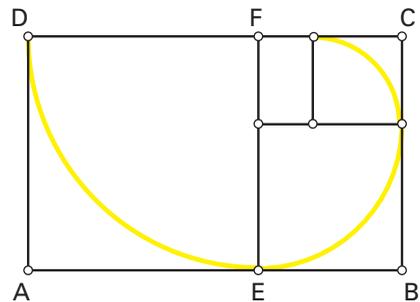
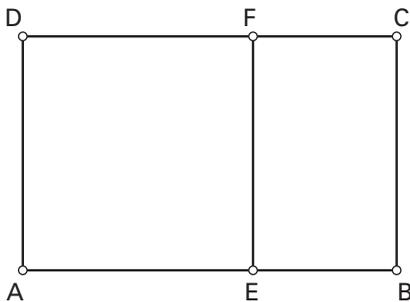
The Golden Ratio

29. On a segment AB , point X is called the **Golden Cut** if the ratio $AB/AX = AX/XB$ (where $AX > XB$), as shown below left. This ratio is called the **Golden Ratio** (≈ 1.618). The figure below right shows a simple construction of the golden cut. Use this construction to create a Custom tool for constructing the golden cut of a segment.



In right triangle ABC , $m \overline{BC} = 1/2 m \overline{AB}$.

30. A **Golden Rectangle** is a rectangle in which the ratio of the length to the width is the golden ratio. Using your Custom tool from Project 29, construct a golden rectangle. Now use this construction to create a Custom tool for constructing golden rectangles.
31. Golden rectangles have the special property that when a square is cut off, the remaining shape is a smaller golden rectangle, as shown below left. A close approximation to a **Golden Spiral** can be constructed by repeating this process over and over again on smaller and smaller rectangles, and then drawing quarter-circles inside the squares. The first three steps of this process are shown below right. With the help of your Custom tool from Project 30, construct a golden spiral. You may even want to turn this into a Custom tool for constructing golden spirals.



$$\frac{AB}{BC} = 1.618 \quad \frac{BC}{CF} = 1.618$$

Graphing/Coordinate Geometry

In order to do several of the projects in this section, you will need to know how to plot functions based on constructed sliders or using the New Parameter command. See **Appendix 1: Plotting in Sketchpad** and **Appendix 2: Sliders** for more information.

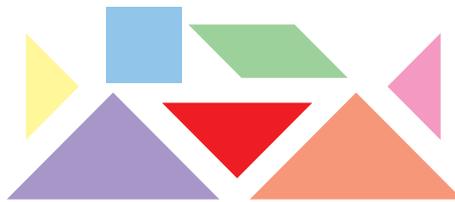
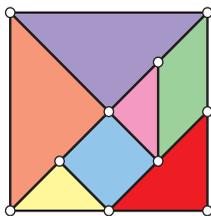
- 32.** Create a sketch that illustrates the effects of changing m (the slope) and b (the y -intercept) in the slope-intercept form of the equation of a line: $y = mx + b$. One way to do this is to show the axes and construct a line with one control point on the y -axis (this is the line's y -intercept). Measure the coordinates of that point, the slope of the line, and the line's equation. Try dragging the various parts of the sketch and see what happens.
- 33.** Make a sketch that graphs the parabola $y = ax^2 + bx + c$ where a , b , and c are controlled by sliders. You can, of course, apply this idea to other families of curves, such as:
- $$y = a(x - h)^2 + k \text{ (parabolas given in "vertex form")}$$
- $$y = a(x - b)(x - c)(x - d) \text{ (cubics given in "root form")}$$
- $$y = a \cdot \sin[b(x - c)] + d$$
- $$y = a \cdot |b(x - c)| + d$$
- 34.** Once you've done Project 33 (for a parabola in standard form), try this one: The vertex of the parabola $y = ax^2 + bx + c$ traces another curve when b is allowed to vary (while a and c remain constant). Create a sketch that traces this curve. What is the equation of this function in terms of a , b , and c from the original parabola?
- 35.** Graph one branch of a rectangular hyperbola by doing the following: Construct a rectangle of constant **area**—one whose dimensions can change but whose area remains constant—and plot its length and width as x and y (or just trace the corner point whose x - and y -coordinates can both change). Hint: Research something called the "Power of a Point" theorem to discover one way to construct a rectangle of constant area.
- 36.** Explore the polar curves $r = a + b\sin(c \cdot \theta)$ and $r = a + b\cos(c \cdot \theta)$ by putting Sketchpad into polar mode (set Grid Form in the Graph menu to Polar), setting up sliders for the parameters a , b , and c , and then graphing. (You may want to limit c to integer values using the Calculator's **round** function. Then again, you may not!) Summarize the effect that each parameter has in the resulting sketch.

Lines and Angles

37. Create a sketch showing the relationships among the angles formed when a pair of parallel lines is cut by a transversal: corresponding angles, alternate interior angles, and so on.
38. Proposition 2 in Euclid's **Elements** reads as follows:
 Given a segment and a point not on the segment, construct a segment congruent to the given segment, with the given point as an endpoint.
 Look up Euclid's construction and see if you can recreate it using only Sketchpad's freehand tools (tools from the Toolbox—no menu commands!).
39. Demonstrate a way to divide a segment into n equal parts, where n is any whole number. Here's the catch: You're only allowed to use the freehand tools (tools from the Toolbox) and the commands in the Construct menu.

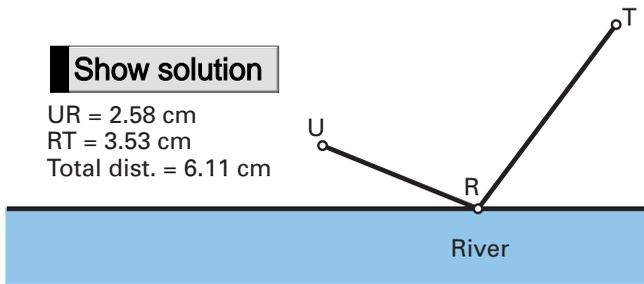
Miscellaneous

40. Choose a theorem from your geometry text, perhaps one you feel would be confusing to most people. Create a sketch that clearly explains the theorem using captions, animations, or whatever else you decide is appropriate.
41. "Reverse engineer" a favorite sketch. In other words, find a finished sketch (preferably one that seems at or just **slightly** beyond your current level of expertise), play around with it to see how it works, then try to create a replica. If you get stuck, the Show All Hidden command and Object Properties can both help. One good place to look for sketches is the Samples folder that came with your software. Another place to look is the Web: Start at www.keypress.com/sketchpad/sketchlinks.html and go from there, or type "Sketchpad" into your favorite search engine.
42. Construct a set of "tangrams," as shown below. Once constructed within a square, copy and paste the various interiors so that they can be moved freely. Use the Rotate Arrow tool to turn pieces. Can you put the pieces back into a square using a different arrangement than the one shown below? There are many, many books and Web sites dedicated to tangrams where you can find further activities.

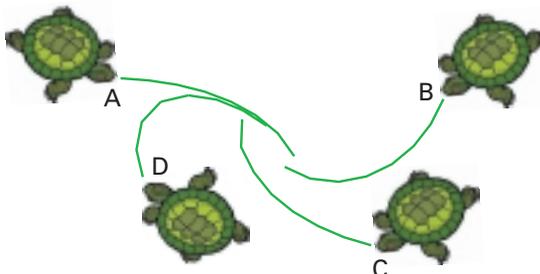


43. In the well-known "Burning Tent Problem," you (point U —see drawing on the next page) are returning to your campsite from a hike along the river when you notice that your tent (point T) is on fire. You realize that you need to run to a point along the river (point R), fill up the bucket you conveniently have with water, then run to the tent to put out the fire. It would be pretty stupid to stop and do a geometry problem at a

time like this, but for the sake of argument, what would be the best point R for you to pick to minimize your total running distance? Explore this problem in Sketchpad with various placements of U and T . There's an elegant way to find the optimal R —can you discover it? (Search for “burning tent problem” on the Web if you need some help.)



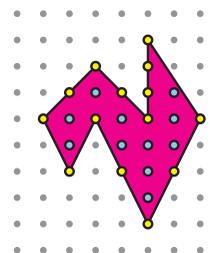
44. Research the half-plane model of non-Euclidean geometry. Create Custom tools for constructing “lines” in this model and investigate other properties of the model.
45. Four turtles start at points A, B, C, and D. During each time interval, each turtle takes a small step toward the next turtle (i.e., A toward B, B toward C, C toward D, and D toward A). Using four Move buttons (combined into one Presentation button), create a sketch that models the motion of the four turtles.



Polygons

46. In this project, you'll use a Sketchpad virtual geoboard to explore an interesting theorem called **Pick's theorem**. Open *Dot Paper.gsp* from the Sketches folder inside the Samples folder next to your Sketchpad application. Now construct polygons such as the one shown at right (all vertices will automatically be on grid dots). There is an interesting relationship between the number of dots lying along the border (yellow—14 here), the number of dots on the interior (blue—11 here), and the area of the polygon (17 here). Can you discover it?
47. A **regular polygon** has congruent sides and congruent angles. Experiment with various ways to construct regular polygons in Sketchpad. (You may want to save some of these constructions as Custom tools to save yourself time in the future.)

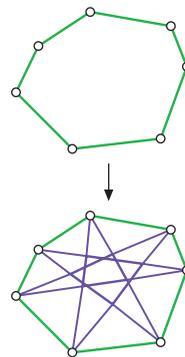
Area p1 = 17.00 cm²



48. Investigate angle sums in stars. No, we're not talking about stars in the sky, or even movie stars—we mean stars formed as follows: Starting with a polygon of at least 5 sides, draw diagonals between every 2nd vertex (or 3rd, as at right, or 4th, or whateverth vertex).

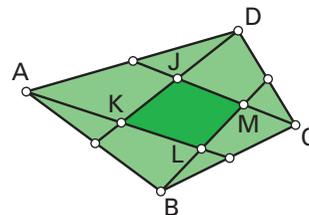
What patterns can you find in star-point angle sums for stars with different numbers of points and for stars in which points are connected in different ways?

49. Can you construct a quadrilateral that is equilateral but not necessarily equiangular? Equiangular but not necessarily equilateral? How about for pentagons? Hexagons? Bigger-number-gons?



Quadrilaterals

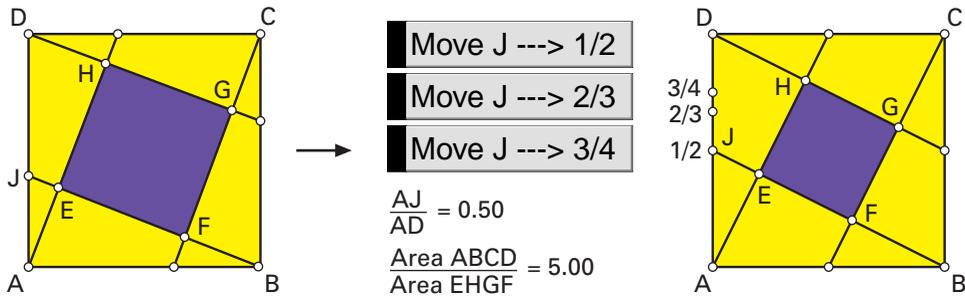
50. Find as many different ways as you can to construct special quadrilaterals: parallelograms, isosceles trapezoids, kites, rectangles, rhombuses, and squares. Describe the properties you used in each construction. (You may want to save some of these constructions as Custom tools to save yourself time in the future.) As a follow-up activity, try the following: Construct several of these figures in the same sketch. Drag each figure until it **appears** to be a square. Move the figures around until you can't remember which is which, and save the sketch. Now make sure all the labels are showing and, using only Object Properties, try to figure out which quadrilateral is which.
51. Construct a quadrilateral so that the only property it **must** have is that its diagonals are the same length as each other. What special kinds of quadrilaterals could this shape be?
52. Here's an investigation that illustrates the importance of dragging vigorously before you make a conjecture. On the Units panel of Preferences, set Scalar Precision to hundredths. Construct a generic quadrilateral and the midpoints of its sides. Connect each vertex to the midpoint of an opposite side in consecutive order to form an inner quadrilateral. Measure the areas of the inner and original quadrilateral and calculate the ratio of these areas. What conjecture are you tempted to make? Change Scalar Precision to thousandths and drag until you find counterexamples. As an added challenge, can you devise another investigation that *almost* results in a conjecture?



$$\frac{\text{Area KLMJ}}{\text{Area ABCD}} = 0.20$$

53. Construct a quadrilateral and the perpendicular bisectors of its sides. The intersections of these bisectors, taken in order, form a second quadrilateral. Construct the perpendicular bisectors of the sides of the second quadrilateral to form a third quadrilateral. Investigate relationships between the original and the third quadrilaterals.

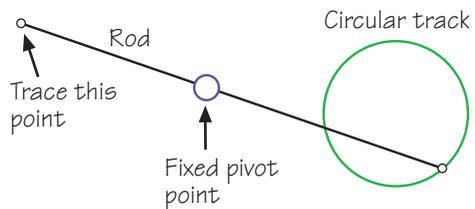
54. Construct a square-within-a-square, as suggested by the figure below left, and investigate how the ratio of areas of the larger square to the smaller square relates to the position of point J . In particular, there is an interesting number pattern that emerges when point J is positioned $1/2$, $2/3$, $3/4$, etc., of the way from point A to point D , as shown below right. (The best way to get J to these exact places is using Movement buttons, as shown.) Can you discover the pattern?



Real World Modeling

55. Do some research on solar/lunar eclipses, and create a sketch that models one or both in Sketchpad.
56. A pantograph is a mechanical device that is used to copy text or images at a larger or smaller scale. Do some research on pantographs (there's some good information on the Web!) and then construct an accurate model in Sketchpad. It should be a "working" model in the sense that when one of two traced points is dragged, the other point should trace out a scale copy of the original trace, and the "arms" on the model should move as they would on the actual device.
57. In 1956, the famous philosopher Ludwig Wittgenstein proposed a thought experiment that can be paraphrased as follows:

Imagine that a rod of a given length is attached at one end to a circular track and passes through a fixed pivot point. As the one endpoint moves along the circular track, what kind of curve does the other endpoint describe?

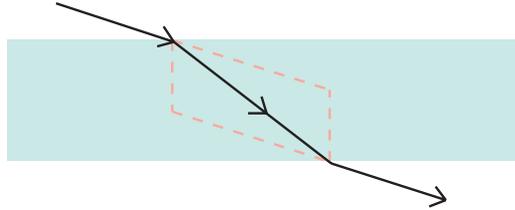


Model this situation in Sketchpad, and then investigate how your results depend on the size of the circular track, the length of the rod, and the location of the pivot point.

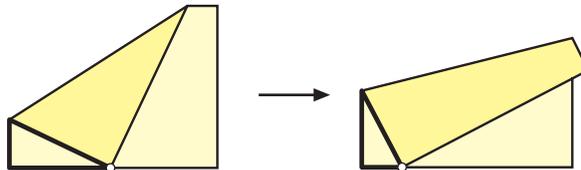
58. A power company wants to select the most efficient location to build a power station to supply three cities (A , B , and C). They want to build it at the spot where the sum of the distances to the three cities is a minimum (to minimize the length of power lines needed). Use Sketchpad to investigate where the power plant should be located relative to the three cities (let A , B , and C be the three vertices of a

triangle—start exploring with these configured as an acute triangle). Can you **construct** the optimal point based on A , B , and C ? Does your construction always work? (For a big hint, see Project 98.)

59. When a ray of light passes from one medium into another, its angle changes according to the properties of the medium. The medium's index of refraction (a numerical value) determines the angle. The relationship is described by Snell's Law. Look up Snell's Law in a physics book and create a sketch that dynamically shows the path of a ray of light through a medium for a given index of refraction (set, perhaps, with a slider).

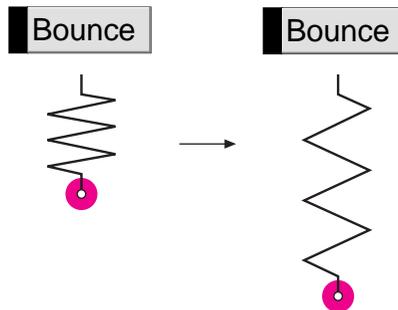


60. Model a wheel with spokes rolling down the road. Once you've done this, you may even want to try modeling an entire car or cart rolling down the road.
61. Construct a model of a clock. It should be a "working" model in the sense that when one of the hands is dragged (or animated), the other should move as it would on a real clock.
62. Create a model of a spinning Ferris wheel. Of course, as the wheel spins, the chairs must always remain vertical!
63. Model a sewing machine with a needle that goes up and down. For an even greater challenge, research the actual internal mechanism that causes the needle to move up and down, and include that in your sketch.
64. When the top-left corner of a piece of paper is folded down to touch the bottom edge, a triangle is formed in the lower-left corner, as shown below. Model this situation in Sketchpad (the most challenging thing is for there to be a smooth transition from the state shown below left to that shown below right as the point is dragged!). Now use your sketch to answer the following question: What fold angle produces the maximum area of the lower-left triangle?



Further challenge: Your sketch probably falls apart when the point is dragged too close to the lower-right corner. Extend your sketch to show (and smoothly transition into) this third possible state. Can you construct a sketch in which the corner point can be dragged all the way around the paper's edge?

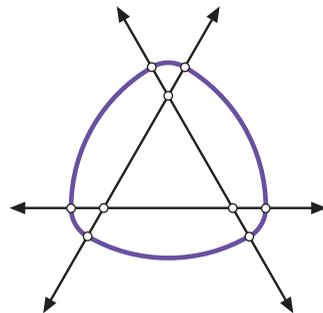
65. Create an animated model of a car's windshield wipers. Experiment with different starting points of each blade. For an even greater challenge, research the actual internal mechanism that causes the blades to move back and forth, and include that in your sketch.
66. If you've done Project 19, you've already experienced one version of billiards in Sketchpad. Another possibility would be to place two "balls" on a rectangular table surface, and create a moveable vector that would control the angle at which the first ball is hit. An action button might send the shot into motion. The goal is to have the first ball bounce off the top wall, side wall, and bottom wall before hitting the second ball. Of course, you are free to modify the set-up and have the goal be whatever you want!
67. Construct a model of the solar system in Sketchpad (don't try to make it to scale!).
68. Construct a bouncing spring (it might look something like the sketch below). Make the motion of the weight at the end of the spring as realistic as possible.



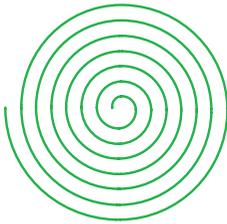
69. Construct a pendulum that swings. Make the motion realistic, and make the period of the pendulum dependent on its length in a realistic way.

Special Curves

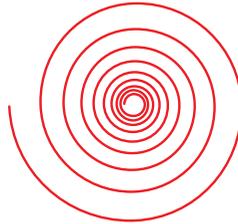
70. The curve shown at right, called a **Reuleaux triangle**, is a **curve of constant width** (the distance between any pair of parallel lines tangent to it is constant). It is made with arcs of circles centered at the vertices of the inner triangle. Construct a Reuleaux triangle in Sketchpad and explain why it has constant width. Can you devise an animation that dynamically illustrates this fact? Experiment with constructing other curves of constant width in Sketchpad.



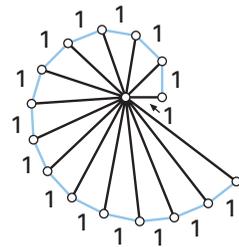
71. If you've done Project 31, you've already constructed a spiral in Sketchpad, namely the **Golden Spiral**. There are, of course, many other types of spirals. In a **Record Groove Spiral**, each turn of the spiral is the same distance from the previous turn. In a **Self-Similar Spiral**, the spiral shrinks by the same ratio each turn. In the **Square Root Spiral** construction, shown below right, every triangle is a right triangle and each outer (blue) segment—and the horizontal segment near the center—is the same length, “1 unit.” (Why is “Square Root Spiral” an appropriate name for this figure?) Approximate one or more of these spirals in Sketchpad using segments or arcs. As a further challenge (and real time-saver if you're familiar with this command), use the Iterate command to generate these spirals for any arbitrary length.



Record groove spiral

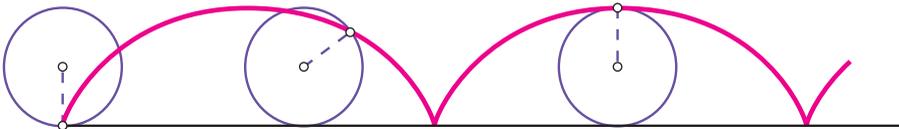


Self-similar spiral



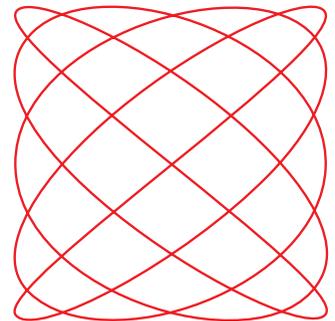
Square root spiral

72. As a wheel moves along the ground, a point on the outside of the wheel traces out a curve called a **cycloid**, as shown below. Construct a cycloid in Sketchpad, or set up an animation that traces one out.



73. Explore the so-called **Lissajou curves**, namely the graphs of the parametric equations $x(t) = \cos(a \cdot t)$ and $y(t) = \sin(b \cdot t)$ where a and b are integers. (The Lissajou curve for $a = 5$ and $b = 4$ is shown at right.)

74. An **epicycloid** is the locus of a point on a circle as that circle rolls around the **outside** of a given circle. A **hypocycloid** is the locus of a point on a circle as that circle rolls around the **inside** of a given circle. Create these curves using traced points and animations. Investigate the relationship between the number of loops on the cycloids (epi or hypo) and the circumferences of the two circles. For a greater challenge, construct both of these curves using Sketchpad's Locus command. (Hint: Create an arc on the given circle that measures the distance traveled by a point on the rolling circle. Use this distance and the circumference of the rolling circle to define an angle of rotation for the point on the rolling circle.) Further challenge: Create a sketch that constructs the locus of either the epicycloid or the hypocycloid as the center of the rolling circle is dragged from outside to inside the given circle and back!

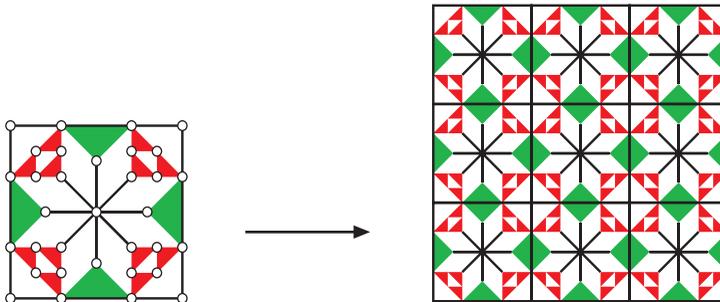


TechnoSketchpad

75. Download and learn about JavaSketchpad by visiting the site www.keypress.com/sketchpad/java_gsp/. Then recreate one of your favorite sketches in JavaSketchpad and put it on your or your school's Web site.
76. Take your favorite existing sketch and turn it into a multimedia presentation. One approach would be to work directly within Sketchpad. You could use Document Options to create a multi-page "sketchbook" with Link buttons connecting pages to each other and to the Web. Another approach would be to work in a multimedia environment such as Powerpoint. You could import graphics (saved as Metafiles in Windows or PICTs in Mac) from Sketchpad, or create links to existing Sketchpad sketches.

Transformations and Tessellations

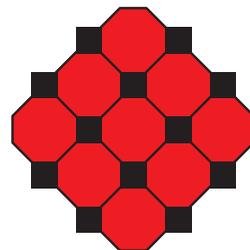
77. Parquet, linoleum, or tile floors are often designed by creating simple, symmetrical patterns using basic shapes on a square. When the squares are lined up, the patterns combine in a pleasing way, often leaving the original square hardly visible, as shown below. Design your own floor pattern in Sketchpad.



Design from Sykes' **Source Book of Problems for Geometry**

78. Demonstrate two different ways to tile the plane with copies of any arbitrary triangle.
79. Investigate the following question: Is it possible to tile the plane with copies of any arbitrary quadrilateral?
80. Create a sketch in which an object (a polygon interior, perhaps) is reflected across one line, and then the image is reflected across a second line. What single transformation would move the original object to the second image if the lines intersect? What if the lines are parallel?
81. Tessellations in which tiles are congruent regular polygons whose edges exactly match are called **regular tessellations**. Create sketches demonstrating **all** the possible regular tessellations. Explain why these are the **only** regular tessellations that are possible.

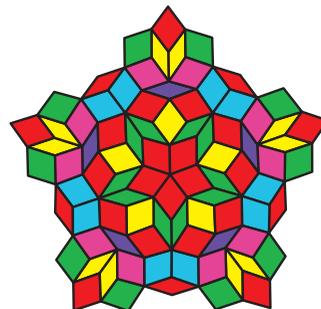
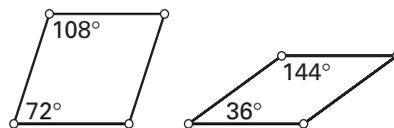
82. A **semiregular tessellation** involves a combination of two or more types of regular polygons that meet in the same pattern at each vertex. In the example at right, two octagons and a square meet at each vertex. There are exactly eight semiregular tessellations (including this one). Construct as many of them as you can in Sketchpad.



83. Do some research on convex pentagons that tile the plane. (A great place to look is Ivars Peterson's book **Islands of Truth: A Mathematical Mystery Cruise**.) Demonstrate at least one such tiling in Sketchpad.

84. Choose an M. C. Escher drawing (such as **Study of Regular Division of the Plane with Reptiles**) that involves tiling the plane. Reproduce the drawing using translations and other transformations.

85. Using the two types of rhombuses shown at right, it's possible to tile the plane in many different ways, all of which show some five-fold symmetry, and none of which have a perfectly repeating (periodic) pattern. These tilings are called **Penrose tilings**, named after the physicist who discovered them, Roger Penrose. Create a Custom tool for each rhombus, then play around with tiling the plane with them. To learn more about these tilings, check out one of the many articles, books, or Web sites dedicated to them. In particular, read the section **From Here to Infinity** (pp. 86–95) in Ivars Peterson's excellent book **Islands of Truth: A Mathematical Mystery Cruise**.

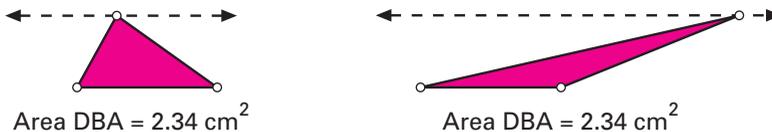


86. Construct three arbitrary lines and a polygon interior. Reflect the interior over one line, then reflect the image over a second line, and then reflect the second image over the third line. (You may want to name the lines 1, 2, and 3 to keep track of your reflection order.) The final image is a glide-reflection of the pre-image. Can you construct the components of this glide-reflection (single mirror line and vector) so that it's equivalent to reflection over each of the three lines? As a further challenge, can you construct the vector using **only** the three lines?

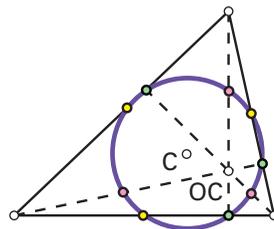
Triangles

87. Create a sketch demonstrating that the sum of the angles in a triangle always equals 180° . Bonus challenge: Can you create a convincing demonstration that doesn't rely on actually measuring any angles?

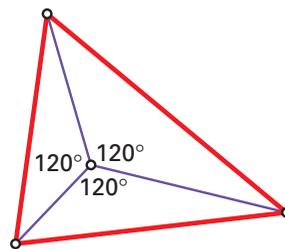
88. Construct a sketch demonstrating that the area of a triangle remains unchanged as the top vertex is dragged along a line parallel to the base.



89. Find as many different ways as you can to construct special triangles: isosceles, equilateral, and right. Describe the properties you used in each construction.
90. Create a sketch that demonstrates why Side-Side-Angle (SSA) is **not** a congruence postulate.
91. Construct a triangle and its three altitudes. The altitudes intersect in a single point, called the **orthocenter**. Now hide the altitudes and construct the three segments connecting the orthocenter to each vertex of the original triangle. Three new triangles are formed (one may be larger than the original triangle, depending on whether or not the original was acute). Construct the orthocenters of these three new triangles and describe what you observe. Can you explain why this is so?
92. There are hundreds and hundreds of different proofs of the Pythagorean theorem. Find one that you like and turn it into a Sketchpad presentation.
93. On each side of an arbitrary triangle, construct an equilateral triangle and its center. Connect the equilateral triangle centers and observe the triangle formed. You should discover what's known as **Napoleon's theorem**. Does this work if the three equilateral triangles are constructed toward the interior of the original triangle? See what else you can discover by connecting points in this construction in different ways.
94. Create Custom tools that construct and label a triangle's centroid, orthocenter, incenter, and circumcenter, given the triangle's three vertices. Now apply all four Custom tools to one triangle. You should find that three of the points always lie along a line (called the **Euler line**). Which of the four points does not lie on the Euler line?
95. Demonstrate that in any triangle, there is a circle that passes through the midpoints of the three sides, the three points where the altitudes touch their respective bases, and the three midpoints of the segments joining the vertices to the orthocenter (the **orthocenter** is the point where the three altitudes meet). This circle is called the **Nine Point Circle**. How does the center of the Nine Point Circle relate to the Euler line discovered in Project 94 (and, more specifically, the Euler segment that joins the orthocenter to the circumcenter)?

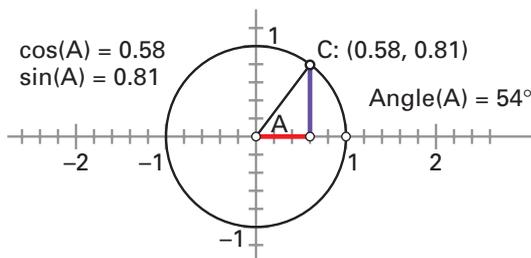


96. Draw three arbitrary segments. Can you construct a triangle with sides the same lengths as the lengths of the three segments? Beware, because it's not always possible! In fact, if a segment is randomly subdivided into three smaller segments, the probability that those sub-segments can form a triangle is only 1 in 4. As a follow-up project, can you devise a way to verify experimentally this 1-in-4 probability? Can you construct a sketch that demonstrates **why** this might be the case?
97. A triangle is said to be **inscribed** in another triangle if each of its vertices lies on a side of the larger triangle. Use Sketchpad to find the inscribed triangle of minimum perimeter for a given triangle. (For a good discussion of this problem—and sketches to download!—check out forum.swarthmore.edu/sketchpad/maa96/parks/.)
98. A triangle's Fermat point is the point that, when connected to each vertex, forms three 120° angles, as shown at right. Construct the Fermat point for a given triangle. This can be tricky, because not all triangles even have a Fermat point (what distinguishes the ones that don't?). As a further challenge, make a Custom tool that constructs the Fermat point given the three vertices of a triangle. (Hint: If you spent time on Project 93, you may have already stumbled onto the easiest Fermat point construction!)



Trigonometry

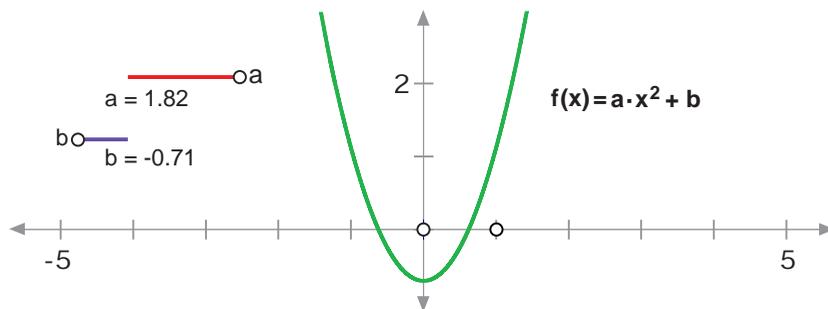
99. Construct a right triangle. Measure its three side lengths and one of its acute angles. Use these measures to create a table of trig ratios—sine, cosine, and tangent—for angles in 5° intervals from 5° to 85° . Compare the values in this table with values given by the Calculator.
100. Create a sketch that demonstrates the relationships between the unit circle and the sine and cosine functions. Bonus challenge: There is a triangle, closely related to the triangle shown below, whose height is equal in magnitude to the tangent of angle A. Find this triangle (hint: it has to do with the other meaning of the word “tangent”) and include it in your sketch.



101. Create an animation that traces out a sine wave.

Appendix 1: Plotting in Sketchpad

Several activities in this book ask you to plot families of curves—in other words, plots in which the parameters (such as a and b in the graph below) can be dynamically changed using sliders. The figure below shows the basics of a dynamic Sketchpad function plot (in this case, the plot of $y = ax^2 + b$).



Here's how to make the above plot:

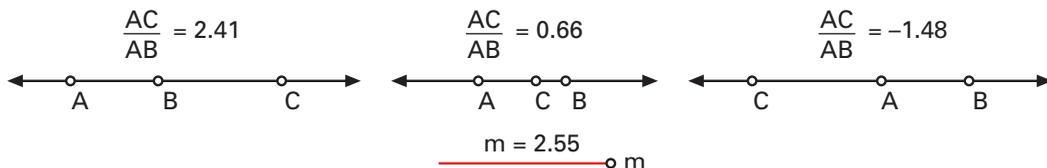
- Construct “sliders” to control dynamic parameters a and b . (See **Appendix 2: Sliders** below for more information.)
- Select the ratio measurements for a and b and then choose Plot New Function from the Graph menu.
- Enter the equation $y = a \cdot x^2 + b$ in the New Function dialog box. You'll find a , b , and x in the Values pop-up menu. (An alternative is to click on the measurements for a and b in the sketch.) Click OK.

Note: The steps needed to graph polar functions are more or less the same as those listed above. Just choose $r = f(\theta)$ from the Equation pop-up menu in the New Function dialog box.

Appendix 2: Sliders

The easiest way to set up sliders is to use the ones provided with your software. Open *Sliders.gsp* from the Custom Tools folder inside the Samples folder next to your Sketchpad application. All the information you'll need is given there.

If you'd like to construct your own sliders, do the following: Construct three points on a straight object, select them left to right (or down to up), then choose Ratio from the Measure menu. If the three points are (in order) A , B , and C , then Sketchpad will return the value AC/AB , which will be negative if B and C are on opposite sides of A . To create a finished slider, construct a segment between A and C , then hide A , B , and the line, and relabel the calculation to match your parameter, as shown below.



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