This write-up is in response to the following prompt:

*Write parametric equations of a line segment through (7,5) with slope of 3. Graph the line segment using your equations. As a line segment, it will have end points. Explore how you would choose endpoints of such that the two distances from (7,5) are 2 units and 3 units.*

Having rarely used parametric equations in my mathematics classes, this was an interesting write-up for me. I started writing my equations using the point my line must go through as an origin of sorts and a consideration of slope as "rise over run":

After this successful first attempt, I moved on to the second part of the prompt: choosing

\[
\begin{pmatrix}
  x \\
  y
\end{pmatrix} = \begin{pmatrix}
  7 + t \\
  5 + 3t
\end{pmatrix}
\]

\[y = \frac{5}{7} x\]

Figure 1: The line \( y = \frac{5}{7} x \) was used to check that the parametric equations were correct.
endpoints that are 2 and 3 units away from (7,5).
I first created the following diagram: Then, using the Pythagorean Theorem, I determined

\[ x_1^2 + (3x_1)^2 = 4 \Rightarrow x_1 = \frac{2\sqrt{10}}{10} \]

\[ x_2^2 + (3x_1)^2 = 25 \Rightarrow x_2 = \frac{5\sqrt{10}}{10} \]

I then edited my graph to include from which I could gather that

\[ x_1 = \frac{2\sqrt{10}}{10}, \quad x_2 = \frac{3\sqrt{10}}{10} \]
Therefore, the most simple change to the parametrized line was to simply change adding $t$ from any range to adding $\frac{\sqrt{10}}{10} t$ on the range $[-3, 2]$.

Could we parametrize this segment differently, though? Perhaps we could work backwards to determine what the endpoints that are 2 and 3 units away from $(7, 5)$ are before parametrizing the general line through $(7, 5)$. With all of the legwork done above, we can identify the endpoints as

$\left(7 + \frac{2\sqrt{10}}{10}, 5 + \frac{6\sqrt{10}}{10}\right)$ and $\left(7 - \frac{3\sqrt{10}}{10}, 5 - \frac{9\sqrt{10}}{10}\right)$

Thus, if we use the lower left point as our origin, we can calculate the differences in the $x$ and $y$ directions as

$x_2 = \frac{5\sqrt{10}}{10}$, $3x_2 = \frac{15\sqrt{10}}{10}$

and parametrize as follows for $t \in \left[0, \frac{5\sqrt{10}}{10}\right]$, with $\frac{5\sqrt{10}}{10} \approx 1.58113883$.

$$
\begin{bmatrix}
    x \\
    y 
\end{bmatrix} = \begin{bmatrix}
    7 - 3\frac{\sqrt{10}}{10} + t \\
    5 - 9\frac{\sqrt{10}}{10} + 3t
\end{bmatrix}
$$