We are here going to investigate some elegant curves that result from simple functions in polar coordinates. In particular, we are going to look at polar roses, or curves of the following form:

\[ r = a \cos (k \theta) \]
\[ r = a \sin (k \theta) \]
\[ r = a \cos (k \theta) + b \]
\[ r = a \sin (k \theta) + b \]
\[ r = \frac{c}{r = a \cos (k \theta) + b \sin (k \theta)} \]

First, let us look at varying values of \(a, k\) for \(r = a \cos (k \theta)\). When \(k = 1\) and we vary \(a\), we get circles with center at \((\frac{a}{2}, 0)\) and radius \(\frac{a}{2}\) as follows.

![Figure 1: \(r = a \cos (\theta)\) when a=1, 2, 3, 4](image)

When \(a\) is fixed and \(k\) varies, we begin to understand the origins of the name "rose curve." Let us look at a few small integer values of \(k\).

For \(k = 2\) and \(k = 4\), the graphs of \(r = \cos (k \theta)\) resemble roses and have 4 and 8 pedals.
This pattern continues for $k$ even - there are $2k$ pedals on the rose. For $k$ odd, we get a rose with $k$ pedals.

(When $k$ is a rational fraction $\frac{m}{n}$, the rose takes an extended domain of $[0, 2n\pi]$ to develop fully. If the domain is the $[0, 2\pi]$ interval that is sufficient for $k \in \mathbb{Z}$, the graph of $a \cos(k\theta)$ will appear truncated. We will not delve into this here.) Now, rather than consider these same variables $a$ & $k$ for curves with equation $r = a \sin(k\theta)$, we can just observe that in polar coordinates, $\sin(\theta)$ is just a rotation of $\cos(\theta)$ by 90 degrees (as shown.
below for the most basic equations). Let us now examine the above equations when a constant $b$ is added to the trigonometric functions. We will only look at $r = a \cos(k\theta)$ for the time being, as we above observed that $\sin(\theta)$ is simply a $90^\circ$ rotation of $\cos(\theta)$. Figure 4 shows that for $a = 1$, $k = 1$, the cosine curve approaches a circle of infinite radius as $b \to \infty$. 

\begin{align*}
\text{Purple: } r &= \cos(\theta) \\
\text{Red: } r &= \sin(\theta)
\end{align*}
Similarly, when we vary $b$ for a fixed $k \neq 1$, a larger value of $b$ only stretches the graph of $\cos(k\theta)$ into, eventually, a theoretical circle centered at the origin with infinite radius. Observe. In particular, how $r = n \cos(n\theta) + 100$ behaves for even large $n$. (Note: the graph is not "complete" because $n = 9.7 \notin \mathbb{N}$.

If we fix $b$ such that $a = b = k$, we get what is called the "$k$-leaf rose." The rose has $k$ pedals, each with length $k$; the rose is symmetrical across the $y$-axis if $k$ is even. Let us

![Graph](image)

Figure 4: $r = n \cos(n\theta) + n, n = 10$

lastly look at a few examples of graphs of the form $r = \frac{c}{a \cos(k\theta) + b \sin(k\theta)}$. (Note that the
presence of $k$ ensures that the roses that would be produced by the trigonometric functions would have the same number of pedals.)

Figure 5: $a=b=c=k=1$

Figure 6: $a=b=c=1, k=2$. The graph is four lines and two hyperbolas.

It appears that whether $k$ is even or odd again has an impact on the shape of the graph. We expect that, for $k = 3$, we will have 3 hyperbolas with a collective 3 axes of symmetry. For variables $a$ & $b$, it is intuitively clear that when $a = b$, the only impact that the size
of $a, b$ will have is to inversely translate the graph in a vertical direction; the larger $a(= b)$ is, $r$ will be translated in the negative direction by $\frac{1}{n}$.

When $a \neq b$, the line $r = \frac{c}{a \cos (k\theta) + b \sin (k\theta)}$ will no longer be equivalent in polar coordinates to $y = -x + 1$, but rather $y = \frac{a}{b} x + \frac{1}{b}(c)$. Observe an example: Lastly, let us consider all of the above considerations at once. We know that $a, b$ determine the slope and $y$-intercept of the line formed by $r$ when $k = 1$. Similarly, $c$ determines a vertical translation of the line. When $k \neq 1$, however, we have hyperbolas that should intuitively rotate as $k$ increases and have similar translations and stretches for varying $a, b, c$. Below
is an example with $a = b = c = k = 7$.

Figure 9: There are 7 axes of symmetry for 7 hybrid parabola/hyperbola curves.

Below is an animation of varying $n$, where $a = b = c = k = n$. The curves are indeed lovely.