Sublime Triangle Problem

Problem: The Sublime Triangle is an isosceles triangle with angles of measure 36, 72, and 72 degrees. It is the only triangle with angle measures in the ratio 1:2:2.

Prove that the ratio of a lateral side of a sublime triangle to its base is the golden ratio, \( \varphi = \frac{1 + \sqrt{5}}{2} \).

Solution: Since \( \angle ABC \approx \angle ACD \), \( \angle BAC \approx \angle CAD \) and side \( AC \) is common. We have \( \triangle ABC \) and \( \triangle ACD \) are similar isosceles triangles.

Now, \( \frac{a+b}{a} = \frac{a}{b} = \frac{1+\sqrt{5}}{2} = \varphi \)

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\frac{a + b}{a} = \frac{a}{b} \\
\Rightarrow \frac{a + b}{b} = \frac{a}{b} \\
\Rightarrow \frac{\varphi + 1}{\varphi} = \varphi \\
\Rightarrow \varphi^2 = \varphi + 1 \\
\Rightarrow \varphi^2 - \varphi - 1 = 0
\]

Applying quadratic formula, \( \varphi = \frac{1+\sqrt{5}}{2} \).