Exploration Topic:

a) Construct parametric equation of a line segment through (7,5) with slope of 3.
b) Graph the line segment with the equation.
c) Explore ways of choosing endpoints such that the two distances from (7,5) are 2 units and 3 units.

There are many ways to write the parametric equations of a line segment for given slope and point. In this case, I will use the coordinates of the point given as the constant terms in the parametric equation. Hence, 7 would be the constant for \( x(t) \) and 5 is the constant for \( y(t) \). Now, for slope 3, for each unit that \( x \)-coordinate moves, the \( y \)-coordinate moves 3 units. So we can form the parametric equations for the case given by

\[
\begin{align*}
x(t) &= t + 7 \\
y(t) &= 3t + 5
\end{align*}
\]

Note: these parametric equations initial point \((t=0)\) is the point given \((7,5)\).
c) Now, let’s find the values of t such that the distance from \((x(t), y(t))\) to \((7,5)\) is the desired value.

For general equation, using the distance formula, we get:

\[
 r = \sqrt{(x(t) - 7)^2 + (y(t) - 5)^2}
\]

\[
 r = \sqrt{(t + 7 - 7)^2 + (3t + 5 - 5)^2}
\]

\[
 r = \sqrt{t^2 + (3t)^2}
\]

\[
 r = \sqrt{10t^2}
\]

\[
 r = t\sqrt{10}
\]

\[
 t = \frac{r}{\sqrt{10}}
\]

For the first endpoint, it is located 2 units from \((7,5)\). So, using the above formula for \(r = 2\), we get the \(t\) value

\[
 t = \frac{2}{\sqrt{10}}
\]

Now, using this value in the parametric equation would give us an endpoint in the segment line 2 units from \((7,5)\). We can also substitute the additive inverse of the value to get the same distance but in opposite direction. The later will be done for the case when \(r = 3\).

Replacing the value of \(t\) found in the parametric equations, we get:

\[
 x\left(\frac{2}{\sqrt{10}}\right) = \frac{2}{\sqrt{10}} + 7 = \frac{\sqrt{10} + 35}{5}
\]

\[
 y\left(\frac{2}{\sqrt{10}}\right) = 3\left(\frac{2}{\sqrt{10}}\right) + 5 = \frac{6}{\sqrt{10}} + 5 = \frac{3\sqrt{10} + 25}{5}
\]

Hence,

\[
 \left(\frac{\sqrt{10} + 35}{5}, \frac{3\sqrt{10} + 25}{5}\right)
\]

For the second case \(r = 3\), replacing the value of \(t\) found in the parametric equations, we get:
\[ x\left( -\frac{3}{\sqrt{10}} \right) = -\frac{3}{\sqrt{10}} + 7 = \frac{-3\sqrt{10}}{10} + 7 = \frac{-3\sqrt{10} + 70}{10} \]

\[ y\left( -\frac{3}{\sqrt{10}} \right) = 3\left( -\frac{3}{\sqrt{10}} \right) + 5 = -\frac{9}{\sqrt{10}} + 5 = -\frac{9\sqrt{10}}{10} + 5 = \frac{-9\sqrt{10} + 50}{10} \]

Hence,

\[ \left( \frac{-3\sqrt{10} + 70}{10}, \frac{-9\sqrt{10} + 50}{10} \right) \]